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What is This?
Brittle fracture in V-notches with end holes

AR Torabi and SH Amininejad

Abstract
In the present research, two brittle fracture criteria were developed in terms of the notch stress intensity factors to predict mode I fracture in engineering components weakened by V-notches with end holes (VO-notches). The criteria were based on the point stress and the mean stress failure concepts. To evaluate the validity of the criteria, first, 36 new fracture tests were conducted on a new notched specimen, namely the Brazilian disk containing central V-notch with end hole (VO-BD specimen) made of polymethyl-metacrylate. Three notch angles and four notch radii were considered in the experiments. Then, the experimentally obtained fracture loads were converted to the corresponding mode I notch fracture toughness values by mean of the finite element method in order to compare the test results with the theories. It was found that very good agreement exists generally between the experimental and theoretical results. Also, found in this research was that by increasing the VO-notch angle, the accuracy of the mean stress criterion decreases, particularly for larger notch radii. Although the accuracy of the criteria depends significantly on the notch geometry, except for the angle 90°, the average discrepancies of the mean stress criterion were less than 8.5% as well as those of the point stress criterion for the entire angles showing acceptable accuracy of the predictions.

Keywords
V-notch with end hole (VO-notch), brittle fracture, polymethyl-metacrylate, notch fracture toughness, point stress, mean stress

Introduction
Notches are man-made discontinuities in engineering components. Due to the stress concentration at the neighborhood of notches, cracks usually appear in this region. Hence, during periodic inspections, cracks detected should be removed from the components to avoid catastrophic damages like sudden fracture, etc. Several methods such as welding have been normally considered for removing and repairing the cracks. In some practical applications, for cracks with small size, a common
repairing method is certainly drilling a circular hole with the radius normally equal to the crack length and removing the crack. This hole erases the crack and hence, changes the initial geometry of the notch. For V-notches, the repairing method explained changes the initial V-notch to a V-notch with end hole (herein after referred to as VO-notch) making different stress concentration and hence different fracture behavior. Thus, the fracture resistance of the new VO-notch should be separately evaluated with the aim to prove the safety of the VO-notched member. For brittle components, fracture occurs suddenly with usually no caution and results in catastrophic damages to the structure. Therefore, the fracture assessment of VO-notched members is much serious for brittle and quasi-brittle materials.

Dealing with notches, several fracture investigations have been performed. In general, the assessments have been performed mainly by energy- and stress-based failure concepts, both experimentally and theoretically. Altogether, six famous failure criteria exist in open literature for investigating fracture in notched components. These criteria are the strain energy density (SED) (Berto et al., 2011; Lazzarin and Zambardi, 2001; Lazzarin et al., 2003, 2009; Yosibash et al., 2004) the generalized J-integral (Barati and Alizadeh, 2011; Barati et al., 2009; Becker et al., 2012; Berto and Lazzarin, 2007; Livieri 2003, 2008; Matvienko and Morozov, 2004), the finite fracture mechanics (FFM) (Carpinteri et al., 2008), the cohesive zone model (CZM) (Gomez and Elices, 2003; Gomez et al., 2000), the point stress (PS), and the mean stress (MS) (Ayatollahi and Torabi, 2010a, 2010b; Torabi, 2012, 2013a, 2013b, 2013c; Torabi et al., 2013). Under mode I loading, the CZM, FFM, PS, and MS criteria represent the onset of fracture with $K_I = K_{Ic}$ for sharp cracks. $K_I, K_{Ic}$, $K_{Ic}$, and $K_{Ic}$ are the stress intensity factor, the plane-strain fracture toughness of material, the notch stress intensity factor (NSIF), and the notch fracture toughness (NFT), respectively.

Gomez and Elices (2003) suggested a closed-form expression to estimate mode I NFT of sharp V-notches by means of the CZM concept. They verified their predications by means of several mode I fracture test results obtained from the V-notched samples made of polymethyl-metacrylate (PMMA) (Gomez and Elices, 2003). Carpinteri et al. (2008) proposed an expression based on the FFM failure concept for predicting well the load-carrying capacity of V-notched rectangular beams made of PMMA and subjected to three-point bending. The more recent works in which the mode I fracture of notched components has been estimated by closed-form NFTs have dealt with the PS and the MS criteria. Ayatollahi and Torabi (2010a) suggested two Irwin-like NFT expressions on the basis of the PS and MS models. They verified the validity of the expressions by means of several fracture test results on V-notched samples made of PMMA and ceramic materials and obtained successful predictions. Moreover, in Ayatollahi and Torabi (2010b), they made use of the MS criterion to estimate mode I fracture in polycrystalline graphite specimens weakened by rounded-tip V-notches. As a more applied work, the ultimate tensile loads have been successfully predicted for ductile steel bolts containing V-shaped threads by means of the simple NFT expression of the MS criterion combined with the equivalent material concept (Torabi, 2013c).

Dealing with V-notches with end holes (VO-notches), very few papers have been recently published both on formulating the stress field and investigating the brittle fracture. Zappalorto and Lazzarin (2011) formulated the stress fields around VO-notches for various notch geometries under mode I, mode II, and mode III loading conditions. The accuracy of the suggested stress fields has been verified by using finite element (FE) models showing a very good agreement. When the notch flanks become parallel, the solution matches previous solutions (Kullmer, 1994; Radaj et al., 2001) for the key-hole notch. In another paper Lazzarin et al. (2011) computed the NSIFs for blunt notches, focusing some analyzes just on VO-notches under mode II loading. In a study by Berto and Zappalorto (2011), the fictitious notch radius concept has been utilized for fracture assessment...
of VO-notches. Other research dealing with brittle fracture of VO-notches has been done by Lazzarin et al. (2013). In this research, the key-hole notch, as a specific type of VO-notch having zero notch angle, has been studied. In Lazzarin et al. (2013) brittle fracture in rectangular plates containing central key-hole notches of various tip radii and made of isostatic graphite has been investigated, under pure mode I and mixed mode I/II loading conditions. That study documented a very good agreement between the experimental fracture loads and the theoretical ones by means of the SED criterion applied to a material-dependent control volume (Lazzarin et al., 2013).

The most recent works on brittle fracture of VO-notched members are those published by Berto et al. (2013) and Torabi and Ayatollahi (2014) both under pure compression (negative mode I loading). In Berto et al. (2013) a large bulk of experimental results has been provided dealing with compressive brittle fracture of isostatic graphite plates weakened by two edge VO-notches. The load-carrying capacity of the tested specimens has been successfully estimated by means of the local SED criterion (Berto et al., 2013). The same experimental results have also been successfully predicted by Torabi and Ayatollahi (2014) using two stress-based closed-form failure criteria, namely the PS and the MS criteria. At the best of authors’ knowledge, no paper or technical report has been published in open literature dealing with tensile brittle fracture in V-notches with end holes.

In the present research, two closed-form expressions were developed by means of the PS and the MS failure concepts to estimate the mode I NFT of brittle components weakened by VO-notches. These theoretical expressions were verified by using 36 new experimental results obtained from mode I fracture tests on the Brazilian disk specimens made of PMMA and weakened by central VO-notches for various notch tip radii and different notch angles. It was found that except an especial case for the MS criterion, the mean relative deviations between the experimental results and the theoretical predictions of both the criteria were less than 8.5% making an evidence for the success of the proposed models.

**Closed-form brittle fracture criteria**

In this section, the elastic stress distribution around a V-notch with end hole (VO-notch) suggested by Zappalorto and Lazzarin (2011) is considered to formulate two closed-form stress-based brittle fracture criteria on the basis of the PS and the MS failure concepts. Since only mode I loading conditions are investigated in the present research, the tangential stress distribution on a VO-notch bisector line \((\theta = 0\) in Figure 1) can be written as (Zappalorto and Lazzarin, 2011)

\[
\sigma_{\theta \theta}(r, 0) = \frac{K_{VO}^I}{A \sqrt{2\pi}} r^{-B} \left[ C + (5\lambda_1 - 2\lambda_1^2 - 1) \left( \frac{\rho}{r} \right)^E \right. + (4 - 6\lambda_1 + 2\lambda_1^2) \left( \frac{\rho}{r} \right)^F + \phi_1 + \phi_1 B \left( \frac{\rho}{r} \right)^E + \phi_1 D \left( \frac{\rho}{r} \right)^2C \left. \right]
\]

In equation (1), \(\sigma_{\theta \theta}(r, 0), K_{VO}^I, \rho,\) and \(r\) are the tangential stress on the notch bisector line, the mode I NSIF for VO-notch, the notch tip radius, and the polar distance measured from the coordinate origin (i.e. the hole center in Figure 1), respectively. The constants \(\lambda_1\) and \(\phi_1\) depend on the notch angle which is presented in Zappalorto and Lazzarin (2011). Other parameters are listed in Table 1. In Figure 1, a VO-notch is schematically presented together with its polar and Cartesian coordinate systems.

In the next subsections, the PS and the MS brittle fracture models are formulated for VO-notches under pure mode I loading conditions and two closed-form expressions are developed for predicting mode I NFT values.
The PS failure criterion proposes that the onset of brittle fracture in a notched component under mode I loading is when the tensile stress at a specified critical distance ahead of the notch on the bisector line attains a critical value. On the other hand, it states that at the fracture instance, the NSIF should be equal to its critical value, so-called the NFT. Figure 2 represents schematically a VO-notch together with the critical distances related to the PS criterion. The $r_c$ and $r_{c,vo}$ are the critical distances measured from the notch tip and from the hole center, respectively. As seen in Figure 2, only $r_c$ is physically meaningful because it lies on the material. The relation between $r_c$ and $r_{c,vo}$ is simply $r_{c,vo} = \rho + r_c$.

One can compute the critical distance $r_c$ from equation (2). This critical distance for the VO-notch is assumed to be equal to that for sharp cracks. This assumption has already provided very good predictions to the extensive experimental results obtained from different notched specimens made of various brittle materials (see for example Ayatollahi and Torabi, 2010a; Torabi, 2013a; Torabi et al., 2013, etc.). In equation (2), $\sigma_c$ is assumed to be equal to the ultimate tensile strength of material ($\sigma_u$) and $K_{IC}$ is the plane-strain fracture toughness.

$$r_c = \frac{1}{2\pi}\left[\frac{K_{IC}}{\sigma_c}\right]^2$$

Table 1. Parameters used in equation (1).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$1 + \lambda_1 + \phi_1$</td>
</tr>
<tr>
<td>$B$</td>
<td>$1 - \lambda_1$</td>
</tr>
<tr>
<td>$C$</td>
<td>$1 + \lambda_1$</td>
</tr>
<tr>
<td>$D$</td>
<td>$2 + \lambda_1$</td>
</tr>
<tr>
<td>$E$</td>
<td>$2\lambda_1$</td>
</tr>
<tr>
<td>$F$</td>
<td>$2\lambda_1 + 1$</td>
</tr>
</tbody>
</table>

Figure 1. VO-notch together with its polar and Cartesian coordinate systems.
The two main requirements of the PS criterion can be mathematically written as

\[
\sigma_{\infty}(r_c, 0) = (\sigma_{\infty})_c = \sigma_c = \sigma_u
\]

\[
K_{f}^{VO} = K_{f}^{VO}
\]

By considering equation (1) and applying the conditions above into it, we have

\[
(\sigma_{\infty})_c = \frac{K_{f}^{VO} r_{c,VO}}{A \sqrt{2\pi}} \left[ C + (5\lambda_1 - 2\lambda_1^2 - 1) \left( \frac{\rho}{r_{c,VO}} \right)^E + (4 - 6\lambda_1 + 2\lambda_1^2) \left( \frac{\rho}{r_{c,VO}} \right)^F + \phi_1 + \phi_1 B \left( \frac{\rho}{r_{c,VO}} \right)^E + \phi_1 D \left( \frac{\rho}{r_{c,VO}} \right)^{2C} \right]
\]

Extracting equation (4) gives the NFT expressions as \( r_{c,VO} = \rho + r_c \)

\[
K_{f}^{VO} = \frac{\sqrt{2\pi} A \sigma_u (\rho + r_c)^B}{f(\rho, r_c)}
\]

where

\[
f(\rho, r_c) = C + (5\lambda_1 - 2\lambda_1^2 - 1) \left( \frac{\rho}{\rho + r_c} \right)^E + (4 - 6\lambda_1 + 2\lambda_1^2) \left( \frac{\rho}{\rho + r_c} \right)^F + \phi_1 + \phi_1 B \left( \frac{\rho}{\rho + r_c} \right)^E + \phi_1 D \left( \frac{\rho}{\rho + r_c} \right)^{2C}
\]

**MS criterion**

According to the MS criterion, brittle fracture takes place under mode I loading conditions when the average value of the tangential stress over a critical distance ahead of the notch on the bisector line

\[
\frac{\rho}{r_{c,VO}}
\]

Figure 2. Typical VO-notch together with the critical distances of the PS criterion.
attains its critical value. Usually, this critical value is assumed to be equal to the ultimate tensile strength of material. By paying attention to the conditions mentioned earlier, it is clear that the MS failure concept is very similar to the PS concept, except that the MS over a specified critical distance is considered as the governing failure parameter. Figure 3 depicts a VO-notch together with the critical distances of the MS criterion. The critical distances \( d_c \) and \( d_{c,vo} \) are measured from the notch tip and from the hole center, respectively. Similar to the PS criterion, \( d_c \) is also considered to be equal to the critical distance of sharp cracks (see equation (6)) which is equal to \( 4r_c \) (see equation (2)). From Figure 3, being valid \( d_{c,vo} = \rho + d_c \). In equation (7), the mathematical definition of the MS criterion is presented

\[
d_c = \frac{\pi}{2} \left[ \frac{K_{lc}}{\sigma_c} \right]^2
\]

\[
\frac{1}{d_c} \int_{r=\rho}^{r=\rho+d_c} \sigma_{\|}(r,0)dr = (\sigma_{\|})_c = \sigma_c = \sigma_u
\]

By substituting equation (1) into equation (7) and integrating, one can obtain the MS NFT expression as (note that \( K_{Ic}^{VO} \) is equal to \( K_{lc}^{VO} \) at fracture)

\[
K_{lc}^{VO} = \frac{\sqrt{2\pi} A \sigma_u d_c}{f(\rho, d_c)}
\]
where

\[
f(\rho, d_c) = \left(\frac{\rho + d_c}{\lambda_1} - \rho^\lambda_1\right) + \left(\frac{\rho + d_c}{\lambda_1} - \rho^\lambda_1\right) - 5\rho^F\left[\left(\frac{\rho + d_c}{\lambda_1} - \rho^\lambda_1\right) + E\rho^F\left(\frac{\rho + d_c}{\lambda_1} - \rho^\lambda_1\right)ight]
\]

\[
+ \rho^E\left[\left(\frac{\rho + d_c}{\lambda_1} - \rho^\lambda_1\right) - \frac{4\rho^E}{C}\left[\left(\rho + d_c\right)^{-\lambda_1} - \rho^{-\lambda_1}\right] + \frac{6\rho^E}{C}\left[\left(\rho + d_c\right)^{-\lambda_1} - \rho^{-\lambda_1}\right] + \frac{6\rho^E}{C}\left[\left(\rho + d_c\right)^{-\lambda_1} - \rho^{-\lambda_1}\right]
\]

\[
- \frac{2\rho^2}{C}\left[\left(\rho + d_c\right)^{-\lambda_1} - \rho^{-\lambda_1}\right] - \frac{2\rho^2}{D}\left[\left(\rho + d_c\right)^{-\lambda_1} - \rho^{-\lambda_1}\right] - \frac{2\rho^2}{D}\left[\left(\rho + d_c\right)^{-\lambda_1} - \rho^{-\lambda_1}\right]
\]

\[
- \frac{2\rho^2}{D}\left[\left(\rho + d_c\right)^{-\lambda_1} - \rho^{-\lambda_1}\right] - \frac{2\rho^2}{D}\left[\left(\rho + d_c\right)^{-\lambda_1} - \rho^{-\lambda_1}\right] - \frac{2\rho^2}{D}\left[\left(\rho + d_c\right)^{-\lambda_1} - \rho^{-\lambda_1}\right]
\]

Experiments

It is very important to evaluate whether the NFT expressions obtained in the previous section are accurate enough. For this goal, a bulk of experimental results is needed. Since no experimental data have been found in the open literature, we carried out mode I fracture experiments on VO-notched specimens. The details of the experiments are presented herein.

Material

The material utilized in the fracture tests was the PMMA with the mechanical properties detailed in Table 2. This material is one of the most common materials in experimental fracture assessments that shows approximately brittle behavior even at room temperature. Moreover, it is homogeneous and exhibits isotropic behavior; appropriate properties for verifying the theoretical predictions elaborated in the previous section. The same material has been previously utilized by Ayatollahi and Torabi (2010c, 2010d) for fracture experiments.

Specimen

The test sample utilized in the present research was the well-known Brazilian disk specimen containing central VO-notches (VO-BD specimen) that is subjected to remote diametral compression. As can be seen in Figure 4, the VO ends of the slit experience pure mode I loading conditions. It has been shown in several publications (see for example, Ayatollahi and Torabi, 2010c) that by changing

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>E</td>
<td>2.96 GPa</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>(\nu)</td>
<td>0.38</td>
</tr>
<tr>
<td>Tensile strength</td>
<td>(\sigma_u)</td>
<td>70.5 MPa</td>
</tr>
<tr>
<td>Plane-strain fracture toughness</td>
<td>(K_{IC})</td>
<td>1.96 MPam^{0.5}</td>
</tr>
</tbody>
</table>
the angle between the loading direction and the slit bisector line (in Figure 4 it is equal to zero), mixed mode I/II loading conditions can be provided around the notch. Since the theoretical NFT expressions were developed under pure mode I loading, only mode I loading was considered in the experiments.

The disk diameter ($D$), the overall slit length ($d$), and the thickness were equal to 80, 40, and 10 mm, respectively. To fabricate the samples, first, a large PMMA plate of 10 mm thick was provided. Then, the sketch of each specimen was drawn and gave to a high-precision 2D Computer numerical control (CNC) water jet cutting machine for fabrication. The cut surfaces were precisely polished by means of abrasive paper. Three notch angles ($30^\circ$, $60^\circ$, and $90^\circ$) and four notch radii (0.5, 1, 2, and 4 mm) were considered in the experiments. Each test was repeated three times with the aim to check the repeatability of the experiments. Totally, 36 fracture tests were carried out in this investigation. Figures 5 and 6 show a VO-BD specimen during mode I fracture test and after fracture, respectively.

The test speed was selected to be 0.5 mm/min providing monotonic loading conditions. The experimentally recorded fracture loads are summarized in Table 3 for the notch angles of $30^\circ$, $60^\circ$, and $90^\circ$ and various notch radii.

It is worth noting that the load–displacement plots recorded from the tests were all linear up to final breakage (see Figure 7). Also, fracture happened abruptly without a visible phase tied to crack nucleation. Hence, it was acceptable to utilize the experimental results presented earlier for verifying the theoretical NFT expressions developed in “Closed-form brittle fracture criteria” section on the basis of the linear elastic notch fracture mechanics.

In the next section, it is attempted to predict the load-carrying capacity of each tested VO-BD specimen by means of the PS and the MS criteria.
Figure 5. The VO-BD specimen under mode I fracture test.

Figure 6. The VO-BD specimen after fracture.
Results and discussion

As can be found in open literature, the predictions of the most failure criteria for brittle fracture of notched components are presented in terms of the critical NSIFs, i.e. the NFTs. It is because the NFT values do not depend on the geometrical shape of the notched member. This advantage

Table 3. The experimentally recorded fracture loads of the tested VO-BD specimens for different notch angles and various notch radii.

<table>
<thead>
<tr>
<th>ρ (mm)</th>
<th>P_1 (N)</th>
<th>P_2 (N)</th>
<th>P_3 (N)</th>
<th>P_{mean} (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2α = 30°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>5635</td>
<td>5277</td>
<td>5724</td>
<td>5545.3</td>
</tr>
<tr>
<td>1</td>
<td>5203</td>
<td>5612</td>
<td>5128</td>
<td>5314.3</td>
</tr>
<tr>
<td>2</td>
<td>6497</td>
<td>6081</td>
<td>6389</td>
<td>6322.3</td>
</tr>
<tr>
<td>4</td>
<td>6997</td>
<td>6530</td>
<td>6730</td>
<td>6752.3</td>
</tr>
<tr>
<td>2α = 60°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>3857</td>
<td>3790</td>
<td>3566</td>
<td>3737.6</td>
</tr>
<tr>
<td>1</td>
<td>4073</td>
<td>4089</td>
<td>4533</td>
<td>4231.6</td>
</tr>
<tr>
<td>2</td>
<td>5325</td>
<td>5440</td>
<td>4754</td>
<td>5173</td>
</tr>
<tr>
<td>4</td>
<td>5793</td>
<td>5866</td>
<td>5939</td>
<td>5866</td>
</tr>
<tr>
<td>2α = 90°</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>2290</td>
<td>2481</td>
<td>2360</td>
<td>2377</td>
</tr>
<tr>
<td>1</td>
<td>2570</td>
<td>2473</td>
<td>2385</td>
<td>2476</td>
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<tr>
<td>2</td>
<td>2827</td>
<td>3013</td>
<td>3013</td>
<td>2951</td>
</tr>
<tr>
<td>4</td>
<td>3779</td>
<td>3947</td>
<td>3971</td>
<td>3899</td>
</tr>
</tbody>
</table>

Figure 7. A load–displacement curve for a VO-BD PMMA specimen tested at room temperature.
provides an excellent convenience for designers to employ simple theoretical NFTs for designing various notched components and structures made of the same material and weakened by the same notch geometry. In order to investigate the validity of the NFT expressions of the PS and the MS criteria, the experimental fracture loads presented in Table 3 should be first converted to the corresponding NFT values. For this purpose, the closed-form in-plane stress field solutions for VO-notches suggested by Zappalorto and Lazzarin (2011) were considered. As can be seen in equation (9), the closed-form expression of the mode I NSIF for a VO-notch ($K_{VO}^{I}$) depends on the geometrical factors and the tangential stress at the notch bisector line ($\sigma_{t0}(r,0)$) (Zappalorto and Lazzarin, 2011)

$$K_{VO}^{I} = \frac{\sqrt{2\pi r^{1-\lambda_1}} \sigma_{t0}(r,0)[1 + \lambda_1 + \phi_1]}{[g_1 + g_2 \bigg(\frac{r}{\rho}\bigg)^{2\lambda_1} + g_3 \bigg(\frac{r}{\rho}\bigg)^{1+2\lambda_1} + g_4 \bigg(\frac{r}{\rho}\bigg)^{2+2\lambda_1}]}$$

(9)

The parameters $g_i$ ($i = 1, 2, 3, 4$) have been reported in Zappalorto and Lazzarin (2011) which depend on the notch angle. Note that one can use in equation (9) the tangential stress at the notch tip (i.e. at $r = \rho$) and obtain a simple approximate formula for the mode I NSIF as follows

$$K_{VO}^{I} = \frac{\sqrt{2\pi \rho^{1-\lambda_1}} \sigma_{max}[1 + \lambda_1 + \phi_1]}{[g_1 + g_2 + g_3 + g_4]}$$

(10)

Note that the NSIF can also be computed by taking an average from the NSIF values calculated at different points on the notch bisector line close to the notch tip having various distances ($r$) and stresses (see equation (9)). However, it has been shown in a few publications (see for example, Lazzarin et al., 2013) that the NSIF values at different distances close to the notch tip are very close to that at the notch tip. Therefore, it is expected that the NSIFs obtained from equation (10) are able to control accurately the local stresses over a finite volume of material ahead of the notch tip.

For computing the experimental NFT values by means of equation (10), the first step is to compute the tangential stresses corresponding to the fracture loads (see Table 3). So, FE analysis was performed and the maximum tangential stress at each VO-notch tip was determined. Then, the $\sigma_{max}$ values were substituted into equation (10) to obtain the critical NSIFs (i.e. the experimental NFTs). Figure 8 shows a meshed VO-BD specimen modeled in the FE software. Very fine meshes were utilized near the VO-notch tip because of the presence of high stress gradient.

The variations of the theoretical NFTs for the PS and the MS models versus the notch radius are plotted in Figure 9 for different notch angles together with the experimental results of the VO-BD PMMA specimens. In Table 4, the relative deviations between the theoretical and the average experimental results are presented.

It is clear in Figure 9 that the theoretical mode I NFT curves of the PS and MS criteria enhance gradually, as the notch radius increases. This behavior is also seen generally in the experimental results. As shown in the figure, the predictions of both the models converge slightly together by increasing the notch radius. Accordingly, it is expected that for very large radii, the theoretical curves approach very closely together. Of course, this claim should be verified by extra analyzes and/or experiments. Another visible point in the figure is that the MS model NFT curves locate always beyond those of the PS model and therefore, the MS model estimates generally greater values...
than the PS model. It is worth mentioning that dissimilar to the fracture load that depends upon the overall geometry of VO-notched member, $K_{vo}^{Ic}$ depends only on the material properties, the notch angle, and the notch radius. It is obvious that the same VO-notch inserted in structural components made of the same material but with a different geometry results in the same $K_{vo}^{Ic}$ but different loads to fracture. Much more complex is the problem of VO-notches scaled in geometrical proportion.

Dealing with the PS and MS concepts, perhaps one asks that: **why the stress values at or over the critical distances are considered as the governing failure parameters in the models instead of the maximum stress values at the notch tip?** A reasonable answer to this question can be provided thanks to the stress concentration around the notch tip. Table 5 presents the maximum tensile stresses at the notch tip $\sigma_{max}$ (i.e. at the distance $r = \rho$) associated with the fracture loads. It is seen in Table 5 that except for the notch radius of $\rho = 4$ mm, the maximum tensile stresses at fracture exceed significantly the ultimate tensile strength of the tested PMMA (i.e. 70.5 MPa in Table 2). In other words, when the notch radius decreases, the difference between the maximum stress and the ultimate tensile strength of material increases. This means that the maximum stress at the notch tip cannot generally be considered as a determinative parameter for predicting the load-carrying capacity of the notched components. Since the stress falls from a maximum at the notch tip to the lower values ahead of the notch tip, as the distance from the notch tip enhances, one should consider the stress at or over a specified critical distance as the failure parameter. Shortly speaking, based on Table 5, when the notch radius decreases, designer should consider the critical distances in the formulations of the MS and PS criteria. For the notch tip radius equal to 4 mm, as the notch angle enhances, one can ignore the critical distance in the fracture models and he/she authorized to use accurately the stress at the notch tip that can be monitored and checked with the material tensile strength. As is well known, the
stress distribution around blunt notches is not usually influenced by large notch radii. Thus, for the VO-notched PMMA specimens tested in this research, it is expected that the critical distances are not required to be incorporated in the models for the notch radii larger than 4 mm.

As presented in Table 4, the maximum value of the mean discrepancies between the experimental and theoretical results are achieved for the notch opening angle equal to 90° for both the PS and MS.

Table 4. The relative deviations between the theoretical and the average experimental results.

<table>
<thead>
<tr>
<th>ρ (mm)</th>
<th>2α = 30°</th>
<th>2α = 60°</th>
<th>2α = 90°</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dev. for PS (%)</td>
<td>Dev. for MS (%)</td>
<td>Dev. for PS (%)</td>
</tr>
<tr>
<td>0.5</td>
<td>16</td>
<td>0.1</td>
<td>7.2</td>
</tr>
<tr>
<td>1</td>
<td>3.6</td>
<td>10</td>
<td>5.5</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>0.1</td>
<td>6.5</td>
</tr>
<tr>
<td>4</td>
<td>4.1</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>Mean Dev. (%)</td>
<td><strong>7.9</strong></td>
<td><strong>2.6</strong></td>
<td><strong>4.8</strong></td>
</tr>
</tbody>
</table>

Figure 9. The variations of the theoretical NFT versus the notch tip radius together with the experimental results for different notch angles. (a) 2α = 30°, (b) 2α = 60°, and (c) 2α = 90°.
models. For this angle, the PS criterion gives generally better predictions than the MS criterion. Also, the minimum average discrepancies between the experimental and theoretical results are found for the notch angle of 30° for the MS criterion. As can be seen in Table 4, by increasing the notch angle, the accuracy of the MS criterion decreases and the mean discrepancy between the experimental and theoretical results rises. This result is probably because of the enhancement of the difference between the VO-notch geometry and that of the blunt V-notch. Torabi (2013c) showed that for circular notches (O-notches), the MS model does not work well, dissimilar to V and U-notches. Since for great notch angles and particularly for large notch radii, the circular end of the VO-notch dominates, the maximum deviation takes place from the blunt V-notch (the VO-notch behavior tends from V-notch to O-notch) resulting in less accuracy of the MS criterion. Totally, except a special case for the MS criterion, the average discrepancies of the PS and MS criteria in comparison with the experimental results are less than 8.5% stating that both the criteria are accurate enough to be used in NFT predictions for V-notches with end holes.

It should be remembered that PMMA is a homogenous polymer with very different mechanical properties at various temperatures. Its strength and brittleness increases as temperature decreases. It has been reported in several publications that PMMA exhibits linear elastic behavior up to final breakage in the universal tension test at temperatures below –60°C (see for instance, Gomez et al., 2009). At room temperature, PMMA experiences relatively large plastic deformations during the test of un-notched specimens and fractures rapidly in a brittle manner. When a notch is exerted in the PMMA component, the plastic deformations are localized at the notch tip vicinity which reduces significantly the effects of plasticity in the fracture phenomenon. Therefore, the PMMA component fractures suddenly in a brittle manner with small plastic deformations. Based on the phrase earlier, it is expected that the present fracture test results on VO-BD PMMA specimens obtained at room temperature could be valid for verifying the brittle fracture criteria resulted from a linear elastic stress distribution (see equation (1)). Despite the natural scatter in the experimental data, a portion of the relative deviations between the mean experimental results and the theoretical predictions can be definitely attributed to the plastic deformations of PMMA at room temperature. Hence, it is expected that much better correlation would be found if the theoretical results are compared with other engineering materials exhibiting ideally brittle behavior, including PMMA at low temperatures.

As seen in the past sections, a correlation between a VO-notch and a sharp crack was considered to make a convenient and quick estimation to the instance of mode I brittle fracture in engineering components containing VO-notches. In illustrating the difference between the crack and the notch case, it should be noticed that the appearance of the criteria for both the crack and notch is very similar, but they have fundamentally different physical meanings. The model $K_I = K_I$ is for the onset

<table>
<thead>
<tr>
<th>$\rho$ (mm)</th>
<th>$\sigma_{\text{max}}$ for 2$\alpha = 30$ (MPa)</th>
<th>$\sigma_{\text{max}}$ for 2$\alpha = 60$ (MPa)</th>
<th>$\sigma_{\text{max}}$ for 2$\alpha = 90$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>122.2</td>
<td>111.2</td>
<td>117.9</td>
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<tr>
<td>1</td>
<td>89.8</td>
<td>91.9</td>
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</tr>
<tr>
<td>2</td>
<td>85.4</td>
<td>84.2</td>
<td>72.6</td>
</tr>
<tr>
<td>4</td>
<td>78.1</td>
<td>74.6</td>
<td>66.1</td>
</tr>
</tbody>
</table>

Table 5. The $\sigma_{\text{max}}$ values at fracture for VO-notched PMMA specimens.
of crack propagation in brittle components under static and monotonic loadings. Since the material is brittle, the time for crack propagation is negligible and therefore, it is reasonable to consider the instance of crack growth, the same with the onset of abrupt fracture. Dissimilar to crack, the criterion $K_{VO}^I = K_{V}^{OP}$ is for predicting the instance of crack nucleation from the VO-notch border. Again, final fracture would occur rapidly after crack initiation.

It should be highlighted that the critical distances were assumed in the present research to be equal for sharp crack and VO-notch without actual demonstration. Although satisfactory predictions were found from this assumption, one can utilize some methods to prove actually the validity of the assumption. One way would be to study in detail the displacement field on the surface of the sample by Digital image correlation (DIC) and then to locally evaluate the local material parameters existing in both configurations (see Lubineau (2009) and Moussawi et al. (2013) describing such technique). This way, one can prove that the hole actually does not affect the state of the material at failure far from the hole.

A subject for future works may be the use of the other famous fracture criteria such as the CZM (Gomez and Elices, 2003; Gomez et al., 2000), the FFM (Carpinteri et al., 2008), and the SED (Berto et al., 2011; Lazzarin and Zambardi, 2001; Lazzarin et al., 2009, 2003; Yosibash et al., 2004). This suggestion is because the predictions of these criteria are in the form of closed-form NFT expressions utilizing only the ultimate tensile strength and the fracture toughness of material. Such criteria would probably give acceptable predictions to the experimental results on PMMA samples weakened by V-notches with end holes.

Conclusions

The present investigation deals with brittle fracture in V-notches with end holes under pure mode I loading conditions. In the theoretical part, two closed-form expressions were developed for predicting the mode I NFT in the components weakened by V-notches with end holes, based on the PS and the MS failure models. In the NFT expressions, two material properties were involved, namely the ultimate tensile strength and the plane-strain fracture toughness. The predicted NFT expressions were compared with 36 fracture test results obtained from the Brazilian disks weakened by central VO-notches and made of PMMA. Satisfactory agreement was found between the theoretical and the experimental results. The comparison revealed that except a special case for the MS criterion, the average discrepancies between the theoretical expressions and the experimental results were less than about 8.5% demonstrating generally the effectiveness of the criteria. Moreover, it was found that the MS criterion provides generally higher NFT estimates than the PS criterion meaning that the PS model is more conservative. The novel finding of the present research was that the critical distance could be ignored in the models for the notch radius equal to 4 mm and for larger notch radii. Since the NFT does not depend on the overall geometry of the notched member, it is expected that the suggested expressions can be used for different structural components weakened by VO-notches.

References


