Strain energy density to assess mode II fracture in U-notched disk-type graphite plates
AR Torabi and F Berto
DOI: 10.1177/1056789513519349

The online version of this article can be found at:
http://ijd.sagepub.com/content/23/7/917

Published by:
SAGE
http://www.sagepublications.com

Additional services and information for International Journal of Damage Mechanics can be found at:

Email Alerts: http://ijd.sagepub.com/cgi/alerts
Subscriptions: http://ijd.sagepub.com/subscriptions
Reprints: http://www.sagepub.com/journalsReprints.nav
Permissions: http://www.sagepub.com/journalsPermissions.nav
Citations: http://ijd.sagepub.com/content/23/7/917.refs.html

>> Version of Record - Aug 14, 2014
OnlineFirst Version of Record - Jan 16, 2014
What is This?
Strain energy density to assess mode II fracture in U-notched disk-type graphite plates

AR Torabi¹ and F Berto²

Abstract
The brittle fracture criterion, namely the strain energy density (SED) over a control volume, which embraces the notch edge, is utilized in the present research to assess the experimentally obtained fracture loads of several U-notched Brazilian disk (UNBD) specimens made of a type of commercial graphite under pure mode II loading. The results show that the SED criterion could successfully predict the fracture loads of graphite specimens for different notch tip radii with an average discrepancy of about ±10%. It is proved in this investigation that not only the SED criterion works well on brittle fracture of notched graphite components under pure mode I, mixed mode I/II and pure mode III loading conditions, but also under pure mode II loading.

Keywords
Strain energy density (SED), U-notch, brittle fracture, mode II loading, graphite, U-notched Brazilian disk (UNBD)

Introduction
Graphite materials have a wide range of engineering applications in different industries such as aerospace, chemical, nuclear, steel-making industries, etc. Such applications include mainly thermal, chemical, electrical and mechanical applications. In aerospace industries, there are many products in which graphite materials are utilized. For instance, graphite shields are employed to protect metallic components from melting as a result of aerodynamic heating.

The most common application of graphite as an electrical material is in the production of carbon brushes in electric motors. Mechanical applications of graphite materials involve many components

¹Fracture Research Laboratory, Faculty of New Science and Technologies, University of Tehran, Tehran, Iran
²Department of Management and Engineering, University of Padova, Stradella, Vicenza, Italy

Corresponding author:
AR Torabi. Fracture Research Laboratory, Faculty of New Science and Technologies, University of Tehran, P.O. Box 13741-4395, Tehran, Iran.
Email: a_torabi@ut.ac.ir
such as thrust bearings, journal bearings, piston rings and vanes. Graphite seals are also utilized in the shafts and fuel pumps of many jet aero-engines.

In the presence of stress concentrators like notches, which are well-known man-made stress raisers, the risk of nucleating crack(s) from notch border increases dramatically needing reliable failure assessment. As an example, one can mention notches introduced in graphite components for joining thermally protective components to the main metallic and/or composite parts of the engineering structures.

Crack growth and fracture in brittle and quasi-brittle graphite materials have been investigated in the past since 1970 by several researchers under different loading conditions at various temperatures (Ayatollahi and Aliha, 2008; Bruno and Latella, 2006; Etter et al., 2004; Jae et al., 2004; Lomakin et al., 1975; Mostafavi and Marrow, 2012; Mostafavi et al., 2013; Nakhodchi et al., 2013; Sato et al., 1981; Shi et al., 2008; Yamauchi et al., 2001). Such fracture investigations include mainly mode I, mixed mode I/II and pure mode II loading conditions at room and elevated temperatures.

Dealing with notches introduced in graphite, a few researchers have already studied the notch sensitivity of various graphite materials (see e.g. Bazaj and Cox, 1969 and Kawakami, 1985). Nowadays, the fracture assessment of notched brittle and quasi-brittle materials are carried out by means of the principles of the notch fracture mechanics (NFM).

Ayatollahi and Torabi (2010a) investigated the fracture phenomenon in V-notched graphite components by means of NFM. They have tested numerous laboratory-scaled specimens of various notch angles and different notch tip radii under mode I loading conditions; which have been completely different in overall geometry, made of a type of very brittle polycrystalline graphite material. The notch fracture toughness values have been experimentally measured and the experimental values have been successfully predicted by using the well-established brittle fracture criterion namely the mean stress (MS) model (Ayatollahi and Torabi, 2010a). In addition to mode I fracture assessment, a wide range fracture experiments have been carried out on the same graphite material by using V-notched Brazilian disc (V-BD) specimens weakened by V-shaped notches under mixed mode I/II and pure mode II loading (Ayatollahi and Torabi, 2011). The experimental results have been well estimated by means of the fracture curves of the V-notched maximum tangential stress (V-MTS) criterion (Ayatollahi and Torabi, 2011). Ayatollahi et al. (2011a) have utilized the strain energy density (SED) fracture criterion to assess mixed mode fracture test results and found very good agreement between the experimental and theoretical results. Dealing with out-of-plane fracture many experiments have been performed by Berto et al. (2012a) on round bars of polycrystalline isostatic graphite weakened by sharp and rounded-tip V-notches under torsion loading. The test results have also been satisfactorily predicted by means of the SED criterion (Berto et al., 2012a).

A recent investigation deals with brittle fracture of U-notched graphite materials under mixed mode loading (Berto et al., 2012c). Many fracture experiments have been carried out on centrally U-notched rectangular isostatic graphite plates under pure mode I and mixed mode I/II loading conditions. The SED failure model has also been utilized to predict successfully the experimental fracture loads (Berto et al., 2012c). In Torabi (2013a), the point stress (PS) and the MS failure models have been employed to predict the experimental results on mode I fracture of U-notched graphite specimens published in Berto et al. (2012c). It was revealed that the MS criterion could predict the test results quite satisfactory. In Torabi (2013b), the U-notched maximum tangential stress (UMTS) criterion has been used to estimate satisfactorily the experimental results on the same U-notched graphite specimens reported by Berto et al. (2012c) considering all the cases related to mixed mode I/II loading conditions. More recently, a new set of experimental results have been provided by Torabi et al. (2013a) summarizing the fracture of U-notched Brazilian disk
(UNBD) specimens made of polycrystalline graphite under pure mode I loading. Again, the MS criterion has been revealed to be a useful failure model in the theoretical predictions (Torabi et al., 2013a).

As repaired U-notches (e.g. by removing a short crack initiated from the U-notch border), key-hole notches have attracted new interests in the context of brittle fracture of notched domains. In this area, a recent work has been performed both experimentally and theoretically on isostatic graphite plates containing a central slit with two key-hole ends under mixed mode I/II loading conditions (Lazzarin et al., 2013). The failure theory has been the SED criterion capable of predicting successfully the experimentally obtained fracture loads (Lazzarin et al., 2013). A large bulk of tests has been performed under pure compression dealing with V-notches with end holes (Berto et al., 2014).

Dealing with pure mode II fracture in graphite materials weakened by U-shaped notches, a research paper has been more recently published by Torabi et al. (2013b) in which the experimentally obtained mode II fracture toughness and the fracture initiation angle of UNBD specimens made of coarse-grained polycrystalline graphite have been well predicted by means of two failure criteria namely the UMTS and the U-notched mean stress (UMS).

The main goal of the present work is to verify the suitability of the SED criterion in predicting the experimental fracture loads of some recent UNBD graphite specimens (Torabi et al., 2013b) under pure mode II loading. A comparison between the experimental and theoretical results reveals that the fracture loads can be successfully predicted by means of the SED criterion with an average discrepancy of about ±10%. This research shows that the SED criterion is not only effective in predicting brittle fracture of notched graphite components under mode I, mixed mode I/II and mode III loading conditions, but also under pure mode II loading. This fact confirms the choice of maintaining, as first engineering approximation, the same control volume in the case of mode I and mode II loadings.

**Experimental results on mode II fracture reported in literature**

**Material**

In Torabi et al. (2013b), a type of commercial course-grain polycrystalline graphite has been utilized in fracture experiments with the properties presented in Table 1.

<table>
<thead>
<tr>
<th>Material property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus, $E$ (GPa)</td>
<td>8.05</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.2</td>
</tr>
<tr>
<td>Ultimate tensile strength (MPa)</td>
<td>27.5</td>
</tr>
<tr>
<td>Plane-strain fracture toughness (MPa m$^{0.5}$)</td>
<td>1.0</td>
</tr>
<tr>
<td>Bulk density (kg/m$^3$)</td>
<td>1710</td>
</tr>
<tr>
<td>Mean grain size ($\mu$m)</td>
<td>320</td>
</tr>
<tr>
<td>Porosity (%)</td>
<td>9</td>
</tr>
</tbody>
</table>

In Torabi et al. (2013b), a type of commercial course-grain polycrystalline graphite has been utilized in fracture experiments with the properties presented in Table 1.
Specimen

The UNBD specimen, originally suggested and used by Ayatollahi and Torabi (2010b), has been more recently utilized in Torabi et al. (2013b) for mode II fracture experiments. Figure 1 shows the UNBD specimen, schematically.

In Figure 1, $\beta$ is the angle between the notch bisector line and the loading direction. $D$ and $P$ denote the disc diameter and the applied compressive load, respectively. When the direction of the applied load $P$ is along the notch bisector line (i.e. $\beta = 0$), the U-notches are subjected to pure mode I loading. As the angle $\beta$ enhances gradually, the loading condition varies from pure mode I towards pure mode II. For a particular angle, called $\beta_{II}$, pure mode II loading is achieved. The angle $\beta_{II}$ depends on the notch length and the notch tip radius (Torabi et al., 2013b). The diameter, the overall slit length (the tip-to-tip distance) and the thickness of the UNBD specimens were equal to 60 mm, 18 mm and 10 mm, respectively (Torabi et al., 2013b). The notch tip radii ($\rho$) were equal to 0.5, 1, 2 and 4 mm (Torabi et al., 2013b). For this geometrical configuration, the angle which assures a pure mode II stress field near the notch tip (i.e. $\tau_{r0}$ is the only non-null stress component along the notch bisector line) is $\beta = 30^\circ$.

The experimentally obtained fracture loads of UNBD graphite specimens are presented in Table 2 (Torabi et al., 2013b).

Since completely linear load-displacement curves have been obtained from the fracture tests in Torabi et al. (2013b), it seems principally permissible to apply the SED brittle fracture criterion for predicting the experimental results. Therefore, the SED model is elaborated in the next section and then utilized to predict the test results.

Brittle fracture model based on the SED over a control volume

The most important point for designers is certainly the existence of appropriate failure models to predict the load-carrying capacity of graphite components weakened by notches, e.g. U-shaped
With the aim to provide such models, a strain-energy-density based criterion is elaborated in this section by which the experimental fracture loads of UNBD graphite specimens, described in Table 2, can be estimated very well.

The SED factor $S$ (Sih, 1974) was defined for sharp cracks by Sih as the product of the SED by a specified critical distance measured from the crack tip. Fracture was thought of as controlled by a critical value $S_c$, whereas the crack growth direction was determined by imposing a minimum condition on the factor $S$.

Sih’s model is a point-wise criterion while the averaged SED criterion as presented in Berto and Lazzarin (2009), Lazzarin and Berto (2005) and Lazzarin and Zambardi (2001) suggests that brittle fracture takes place when the mean value of the SED over a known control volume is equal to a critical value $W_c$. This critical value varies from material to material but it is independent of the notch geometry. The control volume, reminiscent of Neuber’s concept of elementary structural volume (Neuber, 1958), is thought of as dependent on the ultimate tensile strength and the fracture toughness $K_{IC}$ in the case of brittle or quasi-brittle materials subjected to static and monotonic loads. Such a method was formalized and applied first to sharp V-notches under mode I and mixed I/II loading (Lazzarin and Zambardi, 2001) and later extended to blunt U- and V-notches (Berto and Lazzarin, 2009; Lazzarin and Berto, 2005).

For sharp cracks, the critical volume is a circle of radius $R_c$ centered at the crack tip (Figure 2a). Under plane-strain conditions, the critical length, $R_c$, can be evaluated by the following expression (Yosibash et al., 2004):

$$R_c = \frac{(1 + \nu)(5 - 8\nu)}{4\pi} \left(\frac{K_{IC}}{\sigma_f}\right)^2$$ (1)

In equation (1), $K_{IC}$ is the fracture toughness, $\nu$ the Poisson’s ratio and $\sigma_f$ the ultimate tensile strength of material.

<table>
<thead>
<tr>
<th>Loading angle $\beta$ ($^\circ$)</th>
<th>$\rho$ (mm)</th>
<th>Fracture load $P_f$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.5</td>
<td>3120</td>
</tr>
<tr>
<td>30</td>
<td>0.5</td>
<td>3210</td>
</tr>
<tr>
<td>30</td>
<td>0.5</td>
<td>3439</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>3520</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>3466</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>3499</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>3700</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>3857</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>3378</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>3519</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>3620</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>3369</td>
</tr>
</tbody>
</table>

Table 2. The experimentally obtained fracture loads of UNBD graphite specimens under pure mode II loading conditions (Torabi et al., 2013b).
For a sharp V-notch, the critical volume becomes a circular sector of radius \( R_c \) centered at the notch tip (Figure 2b) while for a blunt V-notch under mode I loading, the volume assumes the crescent shape shown in Figure 2c, where \( R_c \) is the depth measured along the notch bisector line. The outer radius of the crescent shape is equal to \( R_c + r_0 \), being \( r_0 \) the distance between the notch tip and the origin of the local coordinate system (see Figure 2). Such a distance depends on the V-notch opening angle \( 2\alpha \), according to the expression \( r_0 = \rho (\pi - 2\alpha) / (2\pi - 2\alpha) \) (Lazzarin et al., 2011).

Under mixed mode loading, the critical volume is no longer centered on the notch tip, but rather on the point where the principal stress reaches its maximum value along the border of the notch. It was fundamentally assumed that the crescent shape volume rotates rigidly under mixed mode, with no change in shape and size. This is the governing idea of the ‘equivalent local mode I’ approach, as suggested and applied to U- and V-notches (Berto and Lazzarin, 2009; Lazzarin et al., 2009).

When the area embraces the semicircular edge of the notch (and not its rectilinear flanks), the mean value of SED can be expressed in the following form (Lazzarin and Berto, 2005):

\[
\overline{W}_1 = \frac{F(2\alpha) \times H \left( 2\alpha, \frac{R_c}{\rho} \right) \times \sigma_{\text{tip}}^2}{E}
\]  

(2)

where \( F(2\alpha) \) depends on previously defined parameters

\[
F(2\alpha) = \left( \frac{g - 1}{q} \right)^{2(1-\lambda_1)} \left[ \frac{\sqrt{2\pi}}{1 + \omega_1} \right]^2
\]

(3)

which are reported in the last column of Table 3. \( H \) is summarized for U-notches in Table 4 as a function of the ratio \( R_c/\rho \) and for different values of the Poisson’s ratio.

By simply using the definition of the mode I notch stress intensity factor (NSIF) for rounded-tip V-notches, a simple relationship between the stress at the notch tip \( \sigma_{\text{tip}} \) and \( K_{1\rho} \) can be obtained as follows with \( F(2\alpha) \) defined by Lazzarin and Berto (2005):

\[
K_{1\rho} = \sqrt{2\pi \frac{\sigma_{\text{tip}}}{1 + \omega_1}} \left( \frac{q - 1}{q - \rho} \right)^{1-\lambda_1} = \sqrt{F(2\alpha)\sigma_{\text{tip}}^{1-\lambda_1}}
\]

(4)
The expression above links the generalized NSIF to the maximum stress at the notch tip and can be obtained by equating the expression of the stress component \( \sigma_{\theta_0} \), written as a function of \( K_1 \rho \), with \( \sigma_{\text{tip}} \) along the notch bisector line at \( r = r_0 \) (i.e. at the tip of the notch).

Then, it is possible to rewrite equation (2) in a more compact form:

\[
W_1 = H (2\alpha, \frac{R_c}{\rho}) \times \frac{K_1^2 \rho}{E} \times \frac{1}{\rho^{2(1-\lambda_1)}}
\]  

Equation (5) can be used to evaluate the SED under mode I loading conditions, once \( K_1 \rho \) is known.

For a sharp V-notch subjected to pure mode II loading, the local energy can be written as (Ayatollahi et al., 2001):

\[
\bar{W}_2 = \frac{e_2^2}{E} \times \frac{K_2^2}{R_c^{2(1-\lambda_2)}}
\]  

where \( K_2 \) is the mode II NSIF. Equation (6) can be utilized to compute the SED under mode II loading once \( K_2 \) is known. For a blunt V-notch (also a U-notch as the reduced case), similar closed-form expression can be derived to compute the mode II SED.

The problem was widely discussed considering different combinations of mode mixity (Berto and Lazzarin, 2009; Lazzarin et al., 2009).

\begin{table}
\caption{Parameters of the stress distributions.}
\begin{tabular}{cccc}

\hline
2\( \alpha \) (rad) & \( q \) & \( \lambda_1 \) & \( \omega_1 \) & \( F(2\alpha) \) \\
\hline
0 & 2.0000 & 0.5 & 1 & 0.7850 \\
\( \pi/6 \) & 1.8333 & 0.5014 & 1.034 & 0.6917 \\
\( \pi/4 \) & 1.7500 & 0.5050 & 1.014 & 0.6692 \\
3\( \pi/4 \) & 1.2500 & 0.6736 & 0.432 & 1.0717 \\
\hline
\end{tabular}
\end{table}

\begin{table}
\caption{\( H \) values for U-notched specimens.}
\begin{tabular}{ccccc}

\hline
\( R_c/\rho \) & \( \nu = 0.1 \) & \( \nu = 0.15 \) & \( H \nu = 0.2 \) & \( \nu = 0.25 \) & \( \nu = 0.3 \) \\
\hline
0.0005 & 0.6294 & 0.6215 & 0.6104 & 0.5960 & 0.5785 \\
0.001 & 0.6286 & 0.6207 & 0.6095 & 0.5952 & 0.5777 \\
0.005 & 0.6225 & 0.6145 & 0.6033 & 0.5889 & 0.5714 \\
0.01 & 0.6149 & 0.6068 & 0.5956 & 0.5813 & 0.5638 \\
0.05 & 0.5599 & 0.5515 & 0.5401 & 0.5258 & 0.5086 \\
0.1 & 0.5028 & 0.4942 & 0.4828 & 0.4687 & 0.4518 \\
0.3 & 0.3528 & 0.3445 & 0.3341 & 0.3216 & 0.3069 \\
0.5 & 0.2672 & 0.2599 & 0.2508 & 0.2401 & 0.2276 \\
1 & 0.1590 & 0.1537 & 0.1473 & 0.1399 & 0.1314 \\
\hline
\end{tabular}
\end{table}
The expression for U-notches under mixed mode is analogous to that valid for notches in mode I only updating $\sigma_{\text{tip}}$ with $\sigma_{\text{max}}$ which is the maximum stress along the notch edge outside the notch bisector line:

$$\overline{W} = H^*(2\alpha, \frac{R_c}{\rho}) \times \frac{\pi \sigma_{\text{max}}^2}{4E}$$  \hspace{1cm} (7)$$

where $\sigma_{\text{max}}$ is the maximum value of the principal stress along the notch edge and $H^*$ depends again on the normalized radius $R_c/\rho$, the Poisson’s ratio $\nu$ and the loading conditions. For different configurations of mode mixity, the function $H$, analytically obtained under mode I loading, was shown to be very close to $H^*$. This idea of equivalent local mode I was discussed in previous works (Berto and Lazzarin, 2009, 2014; Lazzarin et al., 2009).

However, as an alternative, avoiding any simplifying assumption, the SED values can be directly derived from finite element (FE) models without using any NSIF-based formulas. The advantage of the direct evaluation of the SED from a FE model is that the value of this parameter is mesh-independent as described in Lazzarin and Berto (2005), Lazzarin et al. (2009). A very coarse mesh can be adopted for the SED evaluation contrary to the mesh required to evaluate the NSIFs or other stress based parameters (Lazzarin et al., 2008, 2010). A recent application of the SED approach deals also with high temperature fatigue tests (Berto et al., 2014). This parameter is also able to take into account all three-dimensional effects described in Berto et al. (2011, 2012b), Harding et al. (2010) and Kotousov et al. (2010) automatically.

**SED approach for fracture analysis of UNBD graphite specimens**

The fracture criterion described in the previous section is employed here to predict the fracture loads obtained from the experiments performed on the UNBD graphite specimens. In order to compute the SED values under mode II loading, first a FE model was created for each graphite specimen. A typical mesh used in the numerical analyses is shown in Figure 3a. The averaged SED criterion states that brittle fracture occurs when the mean value of the SED over a control volume, $\overline{W}$, is equal to a critical value $W_c$, which depends on the material but not on notch geometry (Lazzarin and Berto, 2005; Lazzarin and Zambardi, 2001). This critical value can be determined from the ultimate tensile strength $\sigma_t$ according to Beltrami’s expression:

$$W_c = \frac{\sigma_t^2}{2E}$$  \hspace{1cm} (8)$$

In parallel, the control volume definition via the control radius $R_c$ requires the knowledge of the fracture toughness $K_{IC}$ and the Poisson’s ratio $\nu$, see equation (1). The critical load that is sustainable by a notched component can be predicted by imposing $\overline{W}$ equal to the critical value $W_c$. This value is considered here constant under mode I, mode II and in plane mixed-mode conditions. This assumption has been extensively verified for a number of different brittle and quasi-brittle materials (Berto and Lazzarin, 2009, 2014; Lazzarin and Berto, 2005; Lazzarin and Zambardi, 2001; Lazzarin et al., 2009).

As presented in Table 1, the properties of the graphite material used in Torabi et al. (2013b) are: $\sigma_t = 27.5$ MPa, $K_{IC} = 1$ MPa$\sqrt{m}$, Poisson’s ratio $\nu = 0.2$. As a result, the critical SED for the reported
The SED occurring inside the control volume embracing the edges of U-notches has been calculated numerically by using the FE code ANSYS. For each geometry, a model was created which

graphite material is $W_c = 0.0469$ MJ/m$^3$ whereas the radius of the control volume is $R_c = 0.429$ mm considering realistic plane-strain conditions (Berto et al., 2012a).

The SED occurring inside the control volume embracing the edges of U-notches has been calculated numerically by using the FE code ANSYS. For each geometry, a model was created which
requires an accurate definition of the control volume where the SED should be averaged (see Figure 3). The entire analyses were performed by using eight-node elements under plane-strain conditions.

Figure 3 refers to the case $\rho = 0.5\text{ mm}$. The SED contour lines inside the control volume are shown in Figure 3b. Notice that the SED is not symmetric with respect to the notch bisector line because of the presence of pure shear (i.e. mode II) deformation. However, the symmetry exists with respect to the normal line to the notch edge in the point where the maximum principal stress is experienced. This line can be considered as a virtual bisector line for the notch when it is subjected to mode II loading.

Table 5 summarizes the outlines of the experimental, numerical and theoretical findings for UNBD graphite specimens with four different notch tip radii ($\rho = 0.5, 1, 2, 4\text{ mm}$) reported in literature and re-analyzed by means of SED. Particularly, Table 5 summarizes the experimental loads to fracture ($P$) for every notch radius $\rho$ compared with the theoretical values ($P_{\text{th}}$) based on the SED evaluation. $P_{\text{th}}$ is the theoretical load obtained by keeping a constant averaged energy equal to 0.0469 MJ/m$^3$ over the control volume. $\bar{W}$ can be evaluated by using equation (7) or alternatively directly by numerical models as widely discussed in (Berto and Lazzarin, 2014).

The sixth column of the table presents the relative deviation between the mean values of the experimental fracture loads and the theoretical ones evaluated by means of SED. It is clearly seen in Table 5 that except for the notch radius of 0.5 mm, the discrepancies are less than 5% demonstrating the effectiveness of the SED criterion under mode II loading conditions. The discrepancy for $\rho = 0.5\text{ mm}$ was found to be about 17.5% which falls inside a relatively satisfactory scatter band of ±20%. This moderate agreement may or may not be attributed to possible inaccuracy in the experiments of Torabi et al. (2013b) due to some manufacturing problems for the specimens with small notch radii.

Another possible reason for this discrepancy could be the issue commonly called geometry effect in mixed mode I/II fracture of brittle materials that can be relevant for smaller notch tip radii close.

<table>
<thead>
<tr>
<th>Angle (°)</th>
<th>Notch radius (mm)</th>
<th>$\bar{W}$ (MJ/m$^3$)</th>
<th>Experimental $P$ (N)</th>
<th>Theoretical $P_{\text{th}}$ (N)</th>
<th>(%)</th>
<th>$\sqrt{\bar{W}/W_c}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>0.5</td>
<td>0.0637</td>
<td>3120</td>
<td>2677.9</td>
<td>1.165</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>0.5</td>
<td>0.0674</td>
<td>3210</td>
<td>2677.9</td>
<td>17.5</td>
<td>1.199</td>
</tr>
<tr>
<td>30</td>
<td>0.5</td>
<td>0.0774</td>
<td>3439</td>
<td>2677.9</td>
<td>1.284</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>0.0523</td>
<td>3520</td>
<td>3334.1</td>
<td>1.056</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>0.0507</td>
<td>3466</td>
<td>3334.1</td>
<td>4.5</td>
<td>1.040</td>
</tr>
<tr>
<td>30</td>
<td>1</td>
<td>0.0517</td>
<td>3499</td>
<td>3334.1</td>
<td>1.049</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.0534</td>
<td>3700</td>
<td>3466.4</td>
<td>1.067</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.0581</td>
<td>3857</td>
<td>3466.4</td>
<td>5</td>
<td>1.113</td>
</tr>
<tr>
<td>30</td>
<td>2</td>
<td>0.0445</td>
<td>3378</td>
<td>3466.4</td>
<td>0.974</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>0.0477</td>
<td>3519</td>
<td>3487.8</td>
<td>1.009</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>0.0505</td>
<td>3620</td>
<td>3487.8</td>
<td>0.4</td>
<td>1.038</td>
</tr>
<tr>
<td>30</td>
<td>4</td>
<td>0.0438</td>
<td>3369</td>
<td>3487.8</td>
<td>0.966</td>
<td></td>
</tr>
</tbody>
</table>
to the crack case (see Ayatollahi and Aliha, 2005, 2006, 2011, Ayatollahi et al., 2011b; Ayatollahi and Sistaninia, 2011; Smith et al., 2001).

The results are given also in graphical form in Figure 4 where the experimental values of the fracture loads (open dots) have been compared with the theoretical predictions based on the constancy of the SED in the control volume (solid line). The plots are given for the UNBD graphite specimens as a function of the notch radius $\rho$. The trend of the theoretically estimated loads is in very good agreement with the experimental ones.

A synthesis in terms of the square root value of the local energy averaged over the control volume (of radius $R_c$), normalized with respect to the critical energy of the material as a function of the notch tip radius is shown in Figure 5. The plotted parameter is proportional to the fracture load. The goal is to study the influence of the notch tip radius on the fracture prediction on the basis of SED. From the figure, it is obvious that all the values fall inside a scatter ranging from 0.80 to 1.20 with the majority of the data inside 0.90 to 1.10. The synthesis confirms also the choice of the control volume which seems to be suitable to characterize the material behavior under pure mode II loading.

![Figure 4](image1.png)

**Figure 4.** Fracture prediction based on SED.

![Figure 5](image2.png)

**Figure 5.** Synthesis of brittle fracture data from UNBD graphite specimens.
The scatter of the experimental data presented here is in very good agreement with the recent database in terms of SED reported in Berto and Lazzarin (2009) and Lazzarin et al. (2009).

It is ultimately reminded that U-shaped notches are widely utilized in the design and manufacturing of engineering components made of graphite materials. Notches make these components vulnerable to brittle fracture as a result of high stress concentration around the notch border. Since performing fracture experiments on the full-scale U-notched components or relevant laboratory specimens is often costly, it is preferred to determine the fracture resistance of the notched graphite parts by an appropriate fracture criterion. Generally, if a validated failure model is available, one can predict the instance of fracture in complicated U-notched components without requiring expensive and time-consuming mode II fracture tests. It was found in the present research that a fracture criterion based on the SED averaged over a control volume can be used reliably to estimate the fracture initiation in polycrystalline graphite components subjected to pure in-plane shear loading.

Conclusions

The well-established SED criterion was utilized in the present research to predict the experimentally recorded fracture loads of several U-notched disk-type graphite plates, reported in literature, under pure mode II loading conditions. Four different notch tip radii were considered in this investigation. A total average accuracy of 92% was found for SED model consisting of excellent predictions for the notch tip radii $\rho = 1, 2, 4 \text{ mm}$ and moderately satisfactory prediction for $\rho = 0.5 \text{ mm}$ demonstrating the reliability and the effectiveness of the SED criterion. It was shown in the present research that not only SED works well on brittle fracture of notched graphite components under pure mode I, mixed mode I/II and pure mode III loading conditions (reported already in open literature), but also under pure mode II loading.

Funding

The author(s) received no financial support for the research and/or authorship of this article.

References


