Development of a thermodynamic model for hydrogen and hydrogen containing mixtures

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A R T I C L E   I N F O

Article history:
Received 20 June 2014
Received in revised form 6 August 2014
Accepted 7 August 2014
Available online 17 August 2014

Keywords:
Hydrogen
Thermodynamic
Equation of state
Model development
Black Hole Optimization

A B S T R A C T

Based on Virial Equation of State (V-EOS), a thermodynamic model was developed in this study. The applicability and performance of proposed modified-Virial EOS (mV-EOS) were assessed for calculation of thermodynamic properties of hydrogen and then extended to the hydrogen containing mixtures. For comparison purpose, a number of cubic equations of state were used. Four statistical parameters were defined as goodness criteria which can be regarded as Objective Functions (OF) introduced to optimization method i.e. Black Hole Method (BH). According to obtained results, the model was able to predict the studied thermodynamic properties with desirable accuracy.

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1. Introduction

Electronics and power generation, food industry, pharmaceuticals, aerospace, fuel cells and etc. are examples of the wide application of hydrogen in recent years [1–3]. The primary sources of hydrogen production are steam reforming of natural gases and oil mixtures [4,5]. For production and process design, accurate prediction of hydrogen and hydrogen containing mixtures properties are highly important which can be done by means of equations of state (EOSs) [6–9]. Except low temperature and high density conditions, hydrogen shows classical behavior [10].

Literatures review shows that substantial reports devoted to the application of Equations of State, especially Cubic Equations of State (CEOS), for prediction or correlation of hydrogen and hydrogen containing mixtures properties. The simple form and robustness of Cubic Equations of state made them acceptably predictive and thus popular however they have limitation on the range they cover. The most application of Cubic Equations of State is in the correlation of the vapor pressures of pure substances and prediction of properties along the vapor–liquid coexistence curve [11,12].

One of the most applied equations used in petroleum and natural gas industries is the Benedict–Webb–Rubin equation (BWR) which has been developed on the basis of Virial Equation of State [13] by addition of some empirical terms. Thus, in following paragraphs, the development of a new equation of state based on Virial Equation of State will be illustrated following a brief introduction of some Cubic Equation of States studied in this paper. The method of calculations and measures of goodness of the fit are introduced. Also, for the optimization of measures, we employed a new optimization method i.e. Black Hole (BH) method which is completely new in such problems [14] – as far as we know. The feasibility and applicability of new model together with selected models will be discussed in details. Then, the extension of model to hydrogen containing mixtures will be presented.

2. Theoretical

2.1. Cubic Equations of State

The Generic Cubic Equation of State (GEOS) is defined by Eq. (1) [15]:

\[
P = \frac{RT}{V - b} - \frac{a(T)}{(V + \varepsilon b)(V + \sigma b)}
\]  

(1)
here $\varepsilon$ and $\sigma$ are pure numbers – same for all substances – for each equation of state. Based on the substance of under study, parameters $a(T)$ and $b$ will have different values. There are two methods for determination of the unknown constants corresponding to each substance: (i) Data fitting using available PVT data and (ii) estimation from the critical temperature and pressure values [15]. The concept behind method (ii) is that the critical isotherm shows a horizontal inflection in critical point, from which one can conclude:

$$
\left( \frac{\partial P}{\partial V} \right)_{T,cr} = 0, \quad \left( \frac{\partial^2 P}{\partial V^2} \right)_{T,cr} = 0
$$

(2)

Applying these two conditions (Eq. (2)) in critical point, one would be able to calculate the unknown constrains of Eq. (1) as presented in Eqs. (3) and (4) [15]:

$$
a(T) = \Psi \frac{\alpha(T_c)R^2T^2}{P_c}, \quad \alpha(T_c) = \Psi \frac{R^2T^2}{P_c}
$$

(3)

$$
b = \Omega \frac{RT_c}{P_c}
$$

(4)

where $\Psi$ and $\Omega$ are pure numbers and independent of the substance which are determined for each equation of state based on the values assigned to $\varepsilon$ and $\sigma$. The function $\alpha(T_c)$ is an empirical expression and specific to each equation of state.

Using Eq. (1), the widely available equations of state can be implemented in a computer script/program for the property calculations. Table 1 summarizes the equations of state used [11,12] in current study to be compared to new proposed model.

### 2.2. Modified Virial Equation of State (mV-EOS)

The Virial Equation of State has following two forms [15]:

$$
Z = 1 + B'P + C'T^2 + \ldots
$$

(5)

$$
Z = 1 + \frac{B}{V} + \frac{C}{V^2} + \ldots
$$

(6)

In fact, the idea of expressing equations of state of gases and liquids by means of series was presented by Kamerlingh Onnes for the first time, and the Virial Equation of State, up to three terms, is a truncated and reduced form of that series which is sufficient in most engineering applications.

For development of new modified model (mV-EOS), the Eq. (5) has been used. Here for $B'$ and $C'$ these two equations exist [15]:

$$
B' = \frac{B}{RT}
$$

(7)

$$
C' = \frac{C - B^2}{(RT)^2}
$$

(8)

where $B$ and $C$ are second and third Virial coefficients as defined in Eqs. (9) and (10):

$$
B = \frac{RT_c}{P_c} (b^0 + \omega b^1)
$$

(9)

$$
C = \frac{(RT_c/P_c)^2 (c^0 + \omega c^1)}{2}
$$

(10)

By inserting Eqs. (9) and (10) in Eq. (5), Eq. (5) can be reformed in following form:

$$
Z = 1 + \left( \frac{RT_c/P_c (b^0 + \omega b^1)}{RT} \right) P + \left( \frac{(RT_c/P_c)^2 (c^0 + \omega c^1) - \frac{(RT_c/P_c)^3 (b^0 + \omega b^1)}{2}}{RT^2} \right) P^2 + \ldots
$$

(11)

which can be rewritten in reduced form as Eq. (12):

$$
Z = 1 + \left( \frac{P}{P_c} \right) (b^0 + \omega b^1) + \left( \frac{P}{P_c} \right)^2 \left[ (c^0 + \omega c^1) - (b^0 + \omega b^1)^2 \right] + \ldots
$$

$$
= 1 + a_1 \left( \frac{P}{P_c} \right) + a_2 \left( \frac{P}{P_c} \right)^2 + \omega \left[ a_1 \left( \frac{P}{P_c} \right) + a_2 \left( \frac{P}{P_c} \right)^2 \right] + \ldots
$$

(12)

As this expression is in reduced form, one then can investigate the applicability of the model for other gases and their mixture.

For $b^0$, $b^1$, $c^0$, $c^1$, and $\omega$, following Equations exist [16]:

$$
b^0 = 0.083 - \frac{0.422}{T_r^{1.6}}
$$

(13)

$$
b^1 = 0.139 - \frac{0.172}{T_r^{4.2}}
$$

(14)

$$
c^0 = 0.01407 + \frac{0.02432}{T_r} - \frac{0.00313}{T_r^{10.5}}
$$

(15)

$$
c^1 = -0.02676 + \frac{0.05539}{T_r^{2.7}} - \frac{0.00242}{T_r^{10.5}}
$$

(16)

$$
\omega = \frac{3}{7} \frac{\theta}{1 - \theta} \log P_c - 1
$$

(17)

In these equations, $\theta = \frac{T_b}{T}$ and $T_b$ is the normal boiling point temperature. Eqs. (13)–(16) are all only function of reduced temperature ($T_r$) and Eq. (17) is a function of $P_c$ and $T_r$. Thus, the proposed model has following main equation as Eq. (18):

$$
\left\{ \begin{array}{l}
Z = 1 + f^{(0)} + f^{(1)}A_i + Error \\
f^{(0)} = A_iM_i + A_iM_i^2 \\
f^{(1)} = A_iM_i + A_iM_i^2
\end{array} \right.
$$

(18)

where $M_i = \frac{P_c}{P_r}$, $A_i(T_r)$, $i = 1, \ldots, 4$

$$\left\{ \begin{array}{l}
A_5 = \frac{1}{7} \frac{T_r}{1 - \theta} \log P_c - 1
\end{array} \right.
$$

(19)

In Section 3, our task is to find the optimum order/degree of polynomials of the proposed model’s functional coefficients which minimize Error. Finding these degrees, the model (mV-EOS) establishment is finalized.

### 3. Calculations

To measure how close is the predicted values to real and experimental values of a property of interest, there's always a required to need to define a proper parameter. Common statistical parameter are Accumulative Absolute Relative Deviation (AARD) and Individual Absolute Relative Deviation (IARD), however, in this research, four more reliable statistical parameters were utilized to measure the agreement between predictions and experimental data together with an overview of how much the fitting is successful by the model. These parameters are described in following paragraphs, followed by the method of minimization employed in this study.
Table 1
The details of cubic equations of state used (Eq. (1)).

<table>
<thead>
<tr>
<th>EOS</th>
<th>$\psi$</th>
<th>$\Omega$</th>
<th>$\varepsilon$</th>
<th>$\sigma$</th>
<th>$\alpha(T_e)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RK</td>
<td>0.42748</td>
<td>0.08664</td>
<td>0</td>
<td>1</td>
<td>$T_e^{-0.5}$</td>
</tr>
<tr>
<td>SRK</td>
<td>0.42748</td>
<td>0.08664</td>
<td>0</td>
<td>1</td>
<td>$\alpha_{SRK} = \left[ 1 + \left( 0.480 + 1.574\omega - 0.176\omega^2 \right) \left( 1 - T_e^{-1} \right) \right]^2$</td>
</tr>
<tr>
<td>PR</td>
<td>0.45724</td>
<td>0.07780</td>
<td>$1 - \sqrt{2}$</td>
<td>$1 + \sqrt{2}$</td>
<td>$\alpha_{PR} = \left[ 1 + \left( 0.37464 + 1.54226\omega - 0.26992\omega^2 \right) \left( 1 - T_e^{-1} \right) \right]^2$</td>
</tr>
<tr>
<td>RKTCC</td>
<td>0.42748023354</td>
<td>0.086640349965</td>
<td>0</td>
<td>1</td>
<td>$\alpha_{RKTCC} = \left[ 1 + \left( 0.480 + 1.574\omega - 0.176\omega^2 \right) \left( 1 - T_e^{-1} \right) \right]^2$, $T_e \leq 1$</td>
</tr>
<tr>
<td>RKB</td>
<td>0.42747</td>
<td>0.08664</td>
<td>0</td>
<td>1</td>
<td>$\alpha_{RKB} = \left[ 0.480 + 1.574\omega - 0.176\omega^2 \right] + \left[ 0.480 + 1.574\omega - 0.176\omega^2 \right]$, $T_e &gt; 1$</td>
</tr>
<tr>
<td>PRTCC</td>
<td>0.457235528921</td>
<td>0.0777960739039</td>
<td>$1 - \sqrt{2}$</td>
<td>$1 + \sqrt{2}$</td>
<td>$\alpha_{PRTCC} = \left[ 1 + \left( 0.480 + 1.574\omega - 0.176\omega^2 \right) \left( 1 - T_e^{-1} \right) \right]^2$, $T_e \leq 1$</td>
</tr>
<tr>
<td>PRGGPR</td>
<td>0.45724</td>
<td>0.07780</td>
<td>$1 - \sqrt{2}$</td>
<td>$1 + \sqrt{2}$</td>
<td>$\alpha_{PRGGPR} = \exp\left[ 2 + 0.836T_e \right] \left( 1 - T_e^{1.345 + 0.598\omega - 0.0807\omega^2} \right)$, $T_e &gt; 1$</td>
</tr>
</tbody>
</table>

3.1. Statistical parameters

Four parameters have been defined i.e. $\text{SSE}$, $R^2$, $R^2_{\text{adj}}$ and RMSE for a comprehensive and in-depth comparison between the proposed model and real behavior of the experimental data which are described as follows [17,18].

Sum of Squares due to Error of the fit (SSE) as defined by Eq. (20) used to measure the total deviation of the predicted values ($y_{\text{cal}}$) from the model to the experimental values ($y_{\text{exp}}$). Here $\omega_i$ are weight values that in this study are assumed as unity.

$$\text{SSE} = \sum_{i=1}^{n} \omega_i (y_{i,\text{exp}} - y_{i,\text{cal}})^2$$ (20)

$R$-square (or $R$-sq., $R^2$) is the square of the correlation between the solubility values and the predicted solubility values. This parameter (Eq. (21)) measures how successful the model is in explaining the variation of the data. In other words, it is the square of the correlation between the experimental values and the predicted values:

$$R\text{-square} = \frac{\text{SSR}}{\text{SST}} = 1 - \frac{\text{SSE}}{\text{SST}}$$ (21)

where SSR means the sum of squares of the regression and SST the total sum of squares as described in Eqs. (22) and (23). Here $\text{SST} = \text{SSR} + \text{SSE}$ and $\bar{y}$ is the average of all data.

$$\text{SSR} = \sum_{i=1}^{n} \omega_i (y_{i,\text{cal}} - \bar{y})^2$$ (22)

$$\text{SST} = \sum_{i=1}^{n} \omega_i (y_{i,\text{exp}} - \bar{y})^2$$ (23)

$R$-square is defined to handle the models have many constants. In fact, when the number of models coefficients increase, $R$-square will increase but the model may not improve in a practical sense. This parameter (Eq. (24)) is the degrees of freedom adjusted
R-square. In this Equation n is number of data and m is number of coefficient in the model.

\[
\text{Adj } R^2 = 1 - \frac{\text{SSE}(n - 1)}{\text{SST}(n - m)}
\]

RMSE is the model standard error and the standard error of the regression. It is an estimation of the standard deviation of the random component in the data and can be defined as Eq. (25):

\[
\text{RMSE} = \sqrt{\frac{\text{SSE}}{n - m}}
\]

It is worthwhile to mention four rules of thumb as:

(i) A SSE value closer to zero indicates a fit that has a smaller random error component and is more useful for prediction,
(ii) An R-sq. value closer to 1 indicates that a greater proportion of variance is accounted for by the model
(iii) An Adj R-sq. value closer to 1 indicates a better fit however it can take on any value less than or equal to 1, and
(iv) A RMSE value closer to 0 indicates a fit that is more useful for prediction.

Individual Absolute Relative Deviation (IARD) and Accumulative Absolute Relative Deviation (AARD) can be defined by Eqs. (26) and (27):

\[
\text{IARD} = 100 \left( \frac{|y_{\text{cal}} - y_{\text{exp}}|}{y_{\text{exp}}} \right)
\]

\[
\text{AARD} = \frac{100}{N} \sum_{j=0}^{10} \frac{|y_{\text{cal}} - y_{\text{exp}}|}{y_{\text{exp}}} (\frac{r}{a})^j
\]

3.2. Optimization method

Here, for the first time – as far as we know – we utilized a new heuristic optimization approach i.e. Black Hole Optimization Method (BH) in such problem of study[14]. As the method is new, we provide a short but precise description of the BH Method.

This method, Black Hole (BH), is a population-based method (PBM) such as Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) Method. There are features which are common

---

**Table 2**

The main steps in the BH method.

<table>
<thead>
<tr>
<th>Loop</th>
<th>Start</th>
</tr>
</thead>
<tbody>
<tr>
<td>For each star, evaluate the objective function</td>
<td></td>
</tr>
<tr>
<td>Select the best star that has the best fitness value as the black hole</td>
<td></td>
</tr>
<tr>
<td>Change the location of each star according to ( x(t + 1) = x(t) + \text{rand} \times (x_{\text{best}} - x(t)) )</td>
<td></td>
</tr>
<tr>
<td>If a star crosses a location with lower cost than the black hole, exchange their locations</td>
<td></td>
</tr>
<tr>
<td>If a star reaches a location with cost equal to the black hole, replace it with a new star in a random location in the search space</td>
<td></td>
</tr>
<tr>
<td>If any stop criteria, such as number of iterations or a sufficiently good fitness is met, exit the loop</td>
<td></td>
</tr>
</tbody>
</table>

---

**Table 3**

Optimum model coefficients obtained for pure hydrogen using procedure described in Section 3.

<table>
<thead>
<tr>
<th>( a_i )</th>
<th>( i )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>j</td>
<td>0</td>
<td>0.6669</td>
<td>-3.661</td>
<td>0.3016</td>
<td>-1.911</td>
</tr>
<tr>
<td>1</td>
<td>-3.398</td>
<td>13.31</td>
<td>-1.897</td>
<td>6.151</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6.704</td>
<td>-5.386</td>
<td>3.943</td>
<td>-1.072</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3.11</td>
<td>-2.959</td>
<td>7.893</td>
<td>-0.3782</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-17.85</td>
<td>59.67</td>
<td>-13.62</td>
<td>25.81</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>21.5</td>
<td>3.953</td>
<td>44.63</td>
<td>21.87</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-0.7166</td>
<td>-9.654</td>
<td>-6.788</td>
<td>-6.909</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>13.19</td>
<td>-17.79</td>
<td>-17.28</td>
<td>9.912</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>29.86</td>
<td>-10.09</td>
<td>10.03</td>
<td>-19.42</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-12.97</td>
<td>6.46</td>
<td>11.56</td>
<td>-6.617</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-3.661</td>
<td>5.93</td>
<td>-13.66</td>
<td>15.44</td>
<td></td>
</tr>
</tbody>
</table>

The optimum functional coefficients expressed as a function of \( T_r \):

\[
A_1 = 0.5999 - 0.061837T_r - 0.03843T_r^2 + 0.009401T_r^3 - 0.00064157T_r^4 + 0.000015777T_r^5.
\]

\[
A_2 = -0.2883 + 0.64127T_r - 0.31037T_r^2 + 0.02512T_r^3.
\]

\[
A_3 = 1.345 - 0.5115T_r + 0.049447T_r^2 - 0.000545917T_r^3.
\]

\[
A_4 = -0.04248 + 0.17677T_r - 0.12187T_r^2 + 0.010217T_r^3.
\]
among these population-based methods. In a PBM method, some solution of the problem under study are generated randomly, which are then evaluated using fitness functions. The best evaluated solution will be used to guide all other solutions candidates to the best values. The mechanism of directing the solution to the optimal solution differs in each optimization method.

As an example [19], mutation and crossover are used in GA while in PSO the best found locations are used to move the candidate solutions around. In BH method, which were used in this study, all the candidates (stars) will be moved toward the best candidate in each iteration (the black hole) and candidates entering within the range of the black hole are replaced by newly generated candidates. An interested reader would refer to [14] for more detailed instruction on BH method. The main steps in the BH method are summarized in Table 2. It is worthwhile to mention that BH method is like PSO in nature.

4. Result and discussion

4.1. Application to hydrogen

In order to examine the application and accuracy of proposed model in prediction and correlation for pure Hydrogen’s properties
(especially compressibility here), the thermodynamic properties of Hydrogen compressibility factor were collected from available literatures[16,20]. It is required to determine the proposed model functional coefficients using available compressibility factor data. To do this, the method of calculations mentioned in Section 3 were followed and the obtained coefficients of the model are listed in Table 3. Also, these coefficients can be expressed as function of $T_r$ which is reported in Table 3 but with much higher errors. The statistical parameters value corresponding to correlation results are summarized in Table 4. The result of model for PVT data (density) prediction can be seen in Fig. 1.

![Fig. 4.](image)

**Fig. 4.** Experimental compressibility factor ($Z$) data for NIST1 at each isotherms ($T_r$).

<table>
<thead>
<tr>
<th>$R^2$</th>
<th>$R^2$-Adj</th>
<th>RMSE</th>
<th>SSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9129</td>
<td>0.9134</td>
<td>0.0112</td>
<td>0.0019</td>
</tr>
</tbody>
</table>

According to obtained results, it is clear that the model with coefficients listed in Table 3 is able to (i) correlate and predict the compressibility factor data of pure hydrogen and (ii) can be readily employed and assessed to calculate the thermodynamic properties.

![Fig. 5.](image)

**Fig. 5.** Experimental compressibility factor ($Z$) data for NIST2 at each isotherms ($T_r$).
For example, properties which have related to compressibility factor (such as fugacity coefficient) are more favorable as the model's coefficients were optimized the compressibility factor data here. Now, the applicability of model for hydrogen containing mixtures will be discussed.

4.2. Extension to mixtures

To be able to assess the applicability of the proposed model for hydrogen containing mixtures, the compressibility factor data of five hydrogen containing mixtures which are common in natural gas and petroleum industries were measured experimentally. These five systems compositions are listed in Table 5 together with pseudo critical properties. To measure these data experimentally method of Pycnometer was used. The Pycnometer data were shown in Figs. 1–6.

Using optimum obtained coefficients of model as listed in Table 3 and pseudo critical properties, the correlation results of model are as summarized in Table 6. The optimum obtained coefficient of model in the case of using the measured experimental data

![Fig. 6. Experimental compressibility factor (Z) data for RG2 at each isotherm (Tc).](image)

Table 6

<table>
<thead>
<tr>
<th>R²</th>
<th>R²-Adj</th>
<th>RMSE</th>
<th>SSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8428</td>
<td>0.8377</td>
<td>0.3961</td>
<td>0.4341</td>
</tr>
</tbody>
</table>

For hydrogen containing mixtures are listed in Table 7. The correlation results of proposed model and Cubic EOSs are summarized in Table 8. Again, these coefficients can be expressed as function of Tc which is reported in Table 8 but with higher errors. Also, Fig. 7 shows the accuracy of proposed model together with Cubic EOSs studied for GU2 at Tc = 0.7982 as an example.

According to obtained results, it is clear that the model with those coefficients listed in Table 3 – or even Table 7 – is able to (i) correlate and predict the compressibility factor data of hydrogen containing mixtures and (ii) can be utilized readily for calculation of the thermodynamic properties which are related to the compressibility data (for other properties, an assessment is welcomed).

Table 7

Optimum model coefficients obtained for hydrogen containing mixtures using procedure described in Section 3.

![Table 7](image)
accuracy of this simple model is desirable according to comparisons to some Cubic EOSs.

5. Conclusion

A thermodynamic model was developed from Virial Equation of State, here, to predict and correlate hydrogen and hydrogen containing mixtures. The proposed model's functional coefficients were determined and calculated using four well-defined statistical parameters. The minimization of errors and optimization of models' performance were done using the Black Hole Optimization Method. According to obtained results and measured experimental data, it can be concluded that the model would be used for thermodynamic property calculations of hydrogen and hydrogen containing mixtures with desirable accuracy.

6. Further development opportunity

- The wide application and desired accuracy of BWR correlation, which were developed on the basis of Virial EOS, feed the idea of new models and correlations as followed here.
- It would be worthwhile to examine the model in the way it can cover a wider range of materials.
- An attempt to compare the model to solely empirical correlations reported in literatures would be interesting.
- For faster calculations is optimization part of determination of model's functional coefficients, one may use some faster optimization algorithms. Here, we used the Black Hole Optimization method as this was new in such problems.

References


Table 8

Obtained correlation results for application of models for determination of thermodynamic properties of hydrogen containing mixtures.

<table>
<thead>
<tr>
<th>EOS</th>
<th>$R^2$</th>
<th>$R^2$-Adj</th>
<th>RMSE</th>
<th>SSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>RK</td>
<td>0.8080</td>
<td>0.8071</td>
<td>0.2809</td>
<td>0.4989</td>
</tr>
<tr>
<td>SRK</td>
<td>0.8088</td>
<td>0.8072</td>
<td>0.2541</td>
<td>0.5000</td>
</tr>
<tr>
<td>PR</td>
<td>0.8100</td>
<td>0.8003</td>
<td>0.2910</td>
<td>0.3989</td>
</tr>
<tr>
<td>RKTCC</td>
<td>0.8300</td>
<td>0.8101</td>
<td>0.2561</td>
<td>0.4079</td>
</tr>
<tr>
<td>RKNB</td>
<td>0.8099</td>
<td>0.8099</td>
<td>0.2566</td>
<td>0.4090</td>
</tr>
<tr>
<td>PRGPR</td>
<td>0.8201</td>
<td>0.8071</td>
<td>0.2311</td>
<td>0.4009</td>
</tr>
<tr>
<td>Proposed (mV-EOS)</td>
<td>0.8591</td>
<td>0.8572</td>
<td>0.2012</td>
<td>0.1886</td>
</tr>
</tbody>
</table>

Fig. 7. The accuracy of proposed model together with Cubic EOSs studied for GU2 at $P_r=0.7982$. 

![Graph](image-url)


