A new approach for investigation of damage zone properties in orthotropic materials

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\begin{abstract}
Fracture phenomenon in orthotropic materials, generally associates with region called, “damaged zone” in crack tip vicinity. In quasi-brittle materials, this area is known as fracture process zone (FPZ). This area contains a multitude of microcracks, which cause difficulties in analytical process of the region. Also, energy waste in damaged zone can affect the material fracture properties. The characteristics of damaged zone should be considered to figure out the residual strength of composite materials. It also can help to predict the value or even the direction of crack growth of orthotropic materials. So far, several efforts have been made to determine the mechanical properties of this region, but none of them (due to the immense complexity of this region) can express the behavior of this region properly. Moreover, previous approach has not been verified by new experimental and numerical results, yet. In the present study, a new approach “damaged zone simulation (DZS)” is proposed based on the experimental and numerical data, for investigating the orthotropic damaged zone properties. Comparison with existing analytical data shows the capabilities of the presented approach.
\end{abstract}

\section{Introduction}
Composites as orthotropic materials are used in various kinds of industries like aerospace. In most cases, initiation and propagation of microcracks in composite causes the structural failures. One of the preliminary considerations in these kinds of materials is estimation of fracture residual strength. As it is well known, fracture residual strength associates with mechanical behavior of damaged zone (Damaged zone is an area that appears in crack tip vicinity). In order to figure out the residual strength of such materials, it should be concentrated on the characteristics of damage zone.

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Fibrous materials have a damage zone around the crack tip and their fracture is associated with this area. It is shown that damage zone (at crack tip vicinity) has multitude of microcrack in base of cohesive zone in which by enlargement of these defects, failure process is happened (Fig. 1).

**Fig. 1. Damage zone at crack tip vicinity**

With respect to multitude of microcracks and cohesive basis in damaged zone, this area has complex behavior. Depending on the situation, the damaged zone can affect the rate of crack propagation and even maybe enhance it. Then, by recognizing this crucial region, fracture behavior in orthotropic materials could be estimated. Although this region has an important role in fracture of composite material, but it has been rarely modeled efficiently by numerical and experimental methods, yet. On the other hand, the considerable amount of papers has been published on behavior simulation of cracked solid with a focus on damaged zone in some of them. In 1983, Poe introduced a strain criterion for fracture of fibrous composite laminates. In that research, normal strain was reported for circumferentially notched tensile specimens (CNT) of especially orthotropic composite materials (Poe, 1983). Harris and Morris (1984) reported the fracture behavior of various thicknesses and fiber angles made of T300/5208 carbon/epoxy, laminated Graphite/Epoxy composites. Compact tension specimen (CT) and three-point-bending specimens (TPB) were used for thicker laminates and CNT specimens in different range of thicknesses.

In previous references, local damaged zone has not been considered. In 1991 a fictitious crack model proposed for investigation of damaged zone in different types of materials by Hillerborg et al. (1991). Veselý and Frantík (2014) proposed a new method for investigated of fracture in quasi-brittle materials by considering the effect of fracture process zone. According to wood properties which has various mechanical properties in different directions (depending on cutting process), it could be considered as orthotropic materials (Gohari & Fakoor, 2010). However, some non-linear elastic

<table>
<thead>
<tr>
<th>SYMBOLES</th>
<th>Young’s moduli in the i direction</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_i$, $i=L, R, T$</td>
<td>Elastic moduli of damage zone,</td>
</tr>
<tr>
<td>$\bar{E}, \bar{K}, \bar{\nu}, \bar{G}$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness</td>
</tr>
<tr>
<td>$E_L, E_R, E_T$</td>
<td>Elasticity modulus in the $L,R,T$ direction</td>
</tr>
<tr>
<td>$E_D, G_D, K_D$</td>
<td>Elasticity, Shear, Bulk Modulus of Damaged Zone</td>
</tr>
</tbody>
</table>

![Damage Zone Diagram](image-url)
The fracture propagation approach can be applied to woods and composites if it is accepted that fracture propagation occurs parallel to the grain (Eq. 1). In this equation, interaction of $K_I$ and $K_{II}$ has been investigated. He applied some experiments on scotch ply 1002 and Balsa wood which produced the interaction equation below (Wu, 1967):

$$\frac{K_I}{K_{Ic}} + \left(\frac{K_{II}}{K_{IIc}}\right)^2 = 1$$

Woo and Chow (1979) found that similar equation can also be applied for other woods. This relationship is very similar to the maximum tensile stress criterion of isotropic materials. Recently, a new analytical mixed mode fracture criterion has been presented on reinforcement microcrack damage models for wooden specimens that takes into account the fracture process zone (Gohari and Fakoor, 2010).

2. Methods and materials

Orthotropic materials have commonly three principal axes shown by (1, 2, and 3) symbols. These commonly symbols, can be changed depending on materials e.g. for wood specimens, the symbols identified as L, R, T which represented as radial, tangential and longitudinal directions, respectively (Fig. 2).

![Fig. 2. Orthotropic principal coordinate system](image)

![Fig. 3. Geometry for the region around the crack tip considered in the analysis of fracture in the RL system](image)

Accordingly, there are six principal systems of crack propagation, introduced by a pair of letters, i.e. TL, RL, LR, TR, RT and LT (Gohari & Fakoor, 2010). The first one stands for the direction of the normal to the crack plane and the second one indicates the direction of crack propagation. Out of systems of crack propagation (i.e., along wood fibers), the RL and TL systems constitute the most frequent ones, because of the particular design of timber structures. Let us consider an infinite, linear-elastic plate with a sharp notch of a wedge angle $2\beta$ and straight edges. We shall use a system of polar coordinates ($r, \theta$) centered at the tip of a crack, as it is shown in Fig. 3. The 2D stress field in the notch-tip neighborhood was defined by Sih (Sih et al, 1965).
\[ \sigma_{ij}(t) = \frac{K_i f_{ij}(v)}{\sqrt{2\pi r}} + \frac{K_i g_{ij}(v)}{\sqrt{2\pi r}} (i,j = x,y) \]  

(2)

where \( f_{ij} \) and \( g_{ij} \) are the functions varied by \( v \) angle. Therefore, a general brittle fracture criterion for structures with sharp notches can be formulated as follows (Irwin, 1957):

\[ K_{I,II,III} \geq K_{IC,IIIC}(\beta) \]  

(3)

where \( K_{IC}, K_{IIc}, K_{IIIc} \) are critical values of the stress intensity factors depending on the notch angle (\( \beta \)). Therefore crack will propagate from a tip of notch when the actual value of the intensity factor \( K_i, K_{II}, K_{III} \) reaches a critical value. If the crack opening mode (Mode I) is considered, criterion for damaged zone adjacent to the notch-tip can be written as follows (Novozhilov, 1969):

\[ \max \left( \int_0^{d_0} \sigma_\theta \, dr \right) \geq d_0 \sigma_c \]  

(4)

where \( \sigma_\theta, \, r, \, d_0 \) and \( \sigma_c \) denote stress component normal to the crack direction, distance measured from a singular point in the stress-gradient direction, length of damage zone corresponding to microstructure and breaking stress, respectively. By substituting the equations for the stress field in the notch-tip neighborhood, (2) and (3) into (4), following relationship can be easily obtained (Irwin, 1957):

\[ d_0 \leq \left( \frac{2}{\alpha_c} \right) \left( \frac{k_{IC}}{\sigma_c} \right)^2 \]  

(5)

where, \( \sigma_c \) is obtained from the western white pine tensile test (Fig. 4).

**Fig. 4.** Experimental stress-elongation curve for western white pine wood

As it mentioned, western white pine wood was chosen as a case study. Fracture properties of western white pine wood species have been presented in Table 1. According to the experimental data, the breaking stress (\( \sigma_c \)) and the critical stress intensity factor (\( K_{IC} \)) in RL direction obtained as 1.4 MPa and 260 kPa/\( \sqrt{m} \), respectively. By substituting \( \sigma_c \) and \( K_{IC} \) in (5) the length of damage zone (\( d_0 \)) calculated nearly 20 mm.

**Table 1.** Fracture Properties of western white pine Wood

<table>
<thead>
<tr>
<th>western white pine direction</th>
<th>TL</th>
<th>RL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{IC} ) (kPa/( \sqrt{m} ))</td>
<td>250</td>
<td>260</td>
</tr>
</tbody>
</table>
The objective of this study is to present a new approach for investigation of mechanical properties of crack tip local damage zone, based on numerical and experimental analysis. Moreover, the relationship between mechanical behavior of damage and undamaged zone was considered. For accurate investigation, three main methods i.e. analytical, experimental and numerical methods were employed. For numerical considerations first, a constant circle region by 20 mm diameter, considered as damaged zone (according to experimental results). Then, 1 to 200 defects created as microcrack within the area and the correlation between rate of elasticity modulus and number of microcrack were studied. For experimental consideration, western white pine wood prepared (with special configuration) and mentioned defects were created in damaged zone. Then, by increasing the amount of defects, the specimens were loaded and mechanical behavior of damaged zone was obtained. Eventually, the comparison between FEM\(^1\) and experimental results were done. The results indicated the compatibilities of new approach in comparison with previous analytical data. Moreover, a new fracture model established for FEM and experimental tensile tests similar to the Brazilian disk specimen (Fig. 5). Indeed, despite the conventional Brazilian disk specimen which is loaded by diametral compression (Ayatollahi & Aliha, 2007; Aliha & Ayatollahi, 2008; Aliha et al., 2012, Aliha, 2013), the suggested specimen is a circular disk subjected to diametral tension. The results also denoted the capabilities of the presented approach.

2. Model description and material properties

As is well known, western white pine has convenience mechanical properties (it is too similar to Balsa wood). Thus, its application is very common in various industries, e.g. civil and aerospace engineering. The elastic and fracture properties of this specimen have been indicated in Table 2.

**Table 2.** Elastic properties of wood species applied in this present research

<table>
<thead>
<tr>
<th>Species</th>
<th>(E_r) (GPa)</th>
<th>(E_T) (GPa)</th>
<th>(E_L) (GPa)</th>
<th>(G_{LR}) (GPa)</th>
<th>(G_{LT}) (GPa)</th>
<th>(G_{RT}) (GPa)</th>
<th>(\nu_{LR})</th>
<th>(\nu_{LT})</th>
<th>(\nu_{RT})</th>
</tr>
</thead>
<tbody>
<tr>
<td>western white pine</td>
<td>0.530</td>
<td>0.258</td>
<td>6.8</td>
<td>0.353</td>
<td>0.326</td>
<td>0.034</td>
<td>0.329</td>
<td>0.344</td>
<td>0.334</td>
</tr>
</tbody>
</table>

As it mentioned, due to lack of standard a new test specimen, a fracture model was proposed for tensile test of damaged wood in this research. Although, Chow and Woo (1979) introduced Double Cantilever Beam (DCB) for pure mode I. however, in their method, there were no control on the number and direction of microcracks in damaged zone. Therefore, in present research, SIF increased by artificially increasing the amount of defects (microcrack) in damage zone, gradually.

![Fig. 5. A new test specimen for tensile test of orthotropic materials](image)

\(^1\) Finite element method
2.1. Analytical Method

In the case of microcracks, a new analytical mixed mode fracture criterion based on a reinforcement microcrack damage model, has been presented for wood by Gohari and Fakoor (2010). Also previously, by considering the overall energy balance of the cracked body, effective elastic properties had been proposed (Budiansky, 1976). These effective elastic properties have been indicated in Eq. (6).

\[
\begin{align*}
\overline{G} &= 1 - \frac{8}{45} (10 - 7\bar{\nu})\varepsilon \\
\overline{E} &= 1 - \frac{16}{45} (1 + \bar{\nu})(5 - 4\bar{\nu})\varepsilon \\
\overline{K} &= 1 - \frac{16}{9} \left( \frac{1}{1 - \bar{\nu}^2} \right) \varepsilon 
\end{align*}
\]  \hspace{1cm} (6)

where, \(E, G\) and \(K\) are elasticity, shear and bulk modulus, respectively and also, \(\bar{\nu}\) and \(\nu\) are damaged and undamaged Poisson’s ratio, respectively. In addition, \(\varepsilon\) defines as the crack density parameter and it can be expressed as a function of Poisson’s ratio (Budiansky, 1976):

\[
\bar{\nu} = \frac{45 - 8\varepsilon + 16\varepsilon \nu}{16\varepsilon(1 + 8\nu)} + \frac{45(1 - 0.35\varepsilon + 0.032\varepsilon^2 + 1.14\varepsilon^2\nu + 10.24\nu^2\varepsilon^2 - 5.7\varepsilon\nu^2)^{1/2}}{16\varepsilon(1 + 8\nu)}
\]  \hspace{1cm} (7)

It can be found that \(\overline{E}, \overline{G}\) and \(\overline{K}\) are diminishing functions of \(\varepsilon\). In addition, they tend to be zero at critical crack density e.g. 0.56 for scots pine wood (Gohari & Fakoor, 2010).

2.2. Numerical Method

Previously, crack density parameter was obtained about 0.56 by investigating the fracture process zone of scots pine wood. It can be applied for all types of wood, analytically (Gohari & Fakoor, 2010). On the other hand, at this critical crack density within, damaged zone lose its properties and consequently, failure is happened. Take in to account of microcrack location, random distribution of microcrack can be assumed. Moreover, with the respect of microcrack modeling complexity, a simplification has been employed, in a way that, damage zone modeled as a circle (d=1mm). Eventually, fracture process zone would be independent of microcracks location and their directions. In present research, the material is simulated as a lamina constituted by a linear isotropic elastic matrix under far field stress, and containing random distribution of microcracks. Also, a powerful FEM software, (i.e. ABAQUS) was used for numerical analysis. In damaged zone, geometrical configuration of microcrack and process zone can be calculated by Eq. (8).

\[
\begin{align*}
d & = 1mm, t = 8mm, \\
V_{\text{microcrack}} & = \pi \cdot \frac{d^2}{4} \cdot t = \pi \cdot \frac{1^2}{4} \cdot 8 = 6.28 \\
d & = 20mm, t = 8mm, \\
V_{\text{damaged zone}} & = \pi \cdot \frac{d^2}{4} \cdot t = \pi \cdot \frac{20^2}{4} \cdot 8 = 2512
\end{align*}
\]  \hspace{1cm} (8)

where, \(t\) and \(d\) stands for thickness and diameter, respectively. Hereby, three numerical stages, were considered including, preprocess, process, and post process (Barbero, 2007). At the first stage, material property, loading and meshing was studied. At the second and third one, problem was solved and the results were verified. In addition, Tri-mesh structure CPS3\(^2\) was considered for meshing, as two different mesh sizes accounted for damaged zone.

\(^2\) 3-node linear triangle
By defining some path from center of damaged zone in to its boundary (Fig.7), the maximum and minimum amount of strain and stress, obtained (post process). The paths could be perpendicular or parallel to the direction of applied loads and fibers. Consequently, average of strain and stress was evaluated and the effective modulus of process zone (i.e. elasticity, bulk, and shear modulus) was defined as the course of crack density parameters (Table 3).

### Table 3. Mechanical behavior of damage zone in the course of crack density parameters (numerically)

<table>
<thead>
<tr>
<th>N</th>
<th>$\bar{E}$ (GPa)</th>
<th>$E_L$ (GPa)</th>
<th>$E_T$ (GPa)</th>
<th>$E_R$ (GPa)</th>
<th>$\frac{\bar{E}}{E_L} \times 10$</th>
<th>$\frac{\bar{E}}{E_T}$</th>
<th>$\frac{\bar{E}}{E_R}$</th>
<th>$V$ (mm$^3$)</th>
<th>$V'$ (mm$^3$)</th>
<th>$\varepsilon = V \ast N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.73</td>
<td>6.800</td>
<td>0.258</td>
<td>0.53</td>
<td>1</td>
<td>2.92</td>
<td>1.3</td>
<td>6.28</td>
<td>2512</td>
<td>0.0025</td>
</tr>
<tr>
<td>5</td>
<td>0.62</td>
<td>6.800</td>
<td>0.258</td>
<td>0.53</td>
<td>0.911</td>
<td>2.48</td>
<td>1.14</td>
<td>31.4</td>
<td>2512</td>
<td>0.0125</td>
</tr>
<tr>
<td>15</td>
<td>0.367</td>
<td>6.800</td>
<td>0.258</td>
<td>0.53</td>
<td>0.52</td>
<td>1.44</td>
<td>0.67</td>
<td>94.2</td>
<td>2512</td>
<td>0.0375</td>
</tr>
<tr>
<td>50</td>
<td>0.022</td>
<td>6.800</td>
<td>0.258</td>
<td>0.53</td>
<td>0.072</td>
<td>0.0916</td>
<td>0.0432</td>
<td>314</td>
<td>2512</td>
<td>0.0125</td>
</tr>
<tr>
<td>100</td>
<td>0.0042</td>
<td>6.800</td>
<td>0.258</td>
<td>0.53</td>
<td>0.0066</td>
<td>0.0168</td>
<td>0.0079</td>
<td>628</td>
<td>2512</td>
<td>0.25</td>
</tr>
<tr>
<td>167</td>
<td>0.003</td>
<td>6.800</td>
<td>0.258</td>
<td>0.53</td>
<td>0.0048</td>
<td>0.0134</td>
<td>0.0063</td>
<td>1048</td>
<td>2512</td>
<td>0.451</td>
</tr>
<tr>
<td>187</td>
<td>0.0030</td>
<td>6.800</td>
<td>0.258</td>
<td>0.53</td>
<td>0.0044</td>
<td>0.0128</td>
<td>0.006</td>
<td>1174</td>
<td>2512</td>
<td>0.5 (fail)</td>
</tr>
</tbody>
</table>

In Table 3, $V$ and $V'$ stand for undamaged, damaged zone volume, respectively. The obtained numerical results showed that, by increasing the number of microcracks, mechanical properties of damaged zone are decreased (see Figs. 8-10).
2.3. Experimental Tests

Western white pine was considered for experimental analysis. Microcracks were modeled as circular (D=1mm). For creation of microcracks, a precision drill/grinder (FBS 240/E Proxxon) was used (Fig.11).

![Fig. 11. Precision drill/grinder FBS 240/E Proxxon used in this study](image)

![Fig. 12. Experimental model by increasing the number of microcrack in FPZ](image)

However, in order to measure the mechanical properties and critical crack density, first 1 to 200 microcracks were created in previous condition, respectively (Fig.12). Then each specimen (1-200), was loaded by rate of 1 mm/min using a tension test machine shown in Fig.13. Eventually, extremes of strain and stress were obtained for each test sample. Thus, effective mechanical properties of damaged zone were evaluated in course of increasing the number of microcracks. For each ascending in microcrack, effective mechanical property of damaged zone versus microcrack density was calculated using experimental data. In this method, after polishing and drilling of specimen, the damage zone was observed using a microscope (10x to 100x magnification) as it is shown in Fig.14 (a-d). The results obviously indicated that although every defect has nearly smooth surface, (due to the very small discordant and defect) it could be behave as microcrack.

![Fig. 13. Universal test machine used in present study](image)

![Fig. 14. Defect in damage zone, (a), (b), (c) and (d) were investigated by 10x, 20x, 50x and 100x, respectively](image)

Therefore, by increasing the crack density, it is expected that mechanical behavior of damaged zone should be decreased but not similar to the numerical method (i.e. the defects cannot be propagate easily). In present research, deviations, inferred from natural tiny cracks (that distributed in wood), circularly artificial defects (that causes microcrack cannot propagated, properly), calibration of test fixture, tiny magnitude of discordant in Nano scale and other reasons. On the other hand, these deviations may effect on the propagation of crack and cause dilation in failure, means rate of propagation is low.
3. Results and discussion

In this section, a comparison between numerical, experimental and analytical data is done. Since $K$, $E$ and $G$ (i.e. bulk, elasticity and shear modulus, respectively) represent the mechanical properties; the comparison was done among these results (Figs.15-17).

![Effective elasticity modulus versus crack density](image1)

**Fig. 15.** Effective elasticity modulus versus crack density

![Effective shear modulus versus crack density](image2)

**Fig. 16.** Effective shear modulus versus crack density

![Effective bulk modulus versus crack density](image3)

**Fig. 17.** Effective bulk modulus versus crack density

The presented method for every microcrack density has been compared with existing analytical and experimental criteria for fracture in wood, which indicates a deviation in results. As it mentioned, there are several factors that can affect the mechanical properties. The deviation increases with increasing the microcracks density that inferred from several factors e.g. microcracks geometry (which in numerical analysis for simplifying was modeled as a circle), types of elements at crack tip vicinity (which here was modeled by triangular elements), type of shape function and partition, natural tiny crack in wood and etc. Despite of this digression, present approach, can accurately simulate the orthotropic fracture process zone. It means that if the method applies for orthotropic materials with low natural tiny crack, the results would be improved. Eventually, a polynomial function in fiber direction was calculated that correlates the elasticity modulus in the course of microcrack density parameters (Eq. 9).

\[
\frac{ED}{EL} = A \cdot exp(B \cdot e) + C \cdot exp(D \cdot e) \tag{9}
\]

where $A$, $B$, $C$ and $D$ depends on boundary condition (loading and geometry). It has to be mentioned that, the polynomial equation was obtained from curve fitting of FEM data.

![Polynomial function of Effective elasticity modulus versus crack density Parameters](image4)

**Fig.18.** Polynomial function of Effective elasticity modulus versus crack density Parameters

4. Conclusion

In present research, a new approach referred to “damaged zone simulation (DZS)” was proposed for investigation of mechanical properties of the crack tip local damage zone based on numerical and experimental tests. This method consists of fracture analysis of orthotropic materials (wood was tested as a case study) with arbitrarily oriented microcrack. Take in to account that microcracks, were distributed in process zone, randomly. For simplification the defects were modeled as circle. Three
mechanical properties $K$, $G$ and $E$ (bulk, shear and elasticity modulus) were evaluated and compared with existing analytical and experimental data. Moreover, the results indicated that if the microcrack density parameter is approximately about 0.50, the failure would be happened in wood. It means that, critical microcrack density from numerical method obtained nearly 0.50 whereas the value of 0.56 was obtained from the analytical method. Eventually, comparison between numerical and experimental results with available analytical data showed the capabilities of the presented simulation.

References


