Design and Simulation of MOEMS Thermal Sensor Based on a Bimetallic Mechanism

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Abstract: In this paper, a MOEMS Thermal Sensor Based on a Bimetallic Cantilever Beam was designed. The governing thermo opto-mechanical equations were derived and solved analytically. The temperature rising was expressed with respect to changes of reflected light angle. The results of beam deflection were compared well with the existing results. Copyright © 2009 IFSA.

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1. Introduction

Micro –opto-electro –mechanical system (MOEMS) are a sub-division of micro-electromechanical system, having optical functional elements [1]. MOEMS component of the program focuses on projects whose primary goal is to endow systems with the ability to alter or modulate the path of a light beam; and in some cases, to temporally or spectrally modify the light beam. The most common micro-optical elements are those that reflect, diffract or refract light. The field of modern optics has been largely concerned with the generation, manipulation, guidance, and detection of light for information processing. The operation that is relevant to MOEMS is the manipulation of light in one-, two- or three-dimensional space. There are three primary characteristics that make MOEMS an important technology development: (1) the batch process by which the systems are fabricated, (2) the size of the elements in the systems, and (3) perhaps the most distinctive, is the possibility to endow the optical elements in the system with the ability for precise and controlled motion. Movement of a micro-optical element permits the dynamic manipulation of a light beam. A micro-cantilever based sensor can detect extremely small external stimuli that include temperature changes. Deflection sensing methods can be divided in to two categories: electrical and optical. Although the electrical
method, including capacitance and piezoresistive sensing, is promising due to its compatibility with electric signal processing, it is limited due to lack of thermal isolation and Johnson noise. The most common readout techniques for cantilever motion are optical including optical lever and interferometer methods. These optical methods can detect cantilever motion with sub-angstrom resolution limited only by thermal vibrational noise.

Here a novel MOEMS thermal sensor based on bimetallic cantilever beam is considered which is easy to fabricate with acceptable sensitivity. In the proposed micro sensor the temperature rising is measured based on declivity changes due to the tip deflection of the bimetallic cantilever beam. The system of differential equations for bimetallic micro beam based on Euler-Bernoulli beam theory was derived and solved analytically and the temperature rising was expressed with respect to declivity changes.

2. Model Description

As it is shown in Fig. 1, this model is a Micro Opto Electro Mechanical System that consists of a bimetallic cantilever beam and one mirror that is jointed at the tip of the cantilever beam that reflect the light from source.

Because of different temperature expansion coefficient of selected materials thereby when the operating temperature changed in system, the micro cantilever beam deflected. When the cantilever deflected due to thermal effects the declivity of beam is changed and angle of reflected light is changed too. Therefore temperature deferential can be detected by a detector such as a digital camera. This system acts as a micro thermal sensor.

3. Mathematical Modeling

Fig. 2 shows an element of bimetallic cantilever beam. Assume a beam with length \( l \), thickness \( h \), width \( b \), expansion coefficient \( \alpha \), cross sectional area \( A \) and isotropic with Young’s modulus \( E \), whereas for lower micro beam sub indicate 1 is used and for the other one 2 is used. Suppose that \( x \) is the coordinate along the length of the beam with its origin at the left end, and \( z \) is the coordinate along the cross section with its origin at the neutral axis of cross section. \( w(x) \) is the deflection of the beam, and \( u \) is the displacement along \( x \)-axis. In this figure dash lines is whereabouts neutral axis.
For selected micro beams, $l/h$ is usually large enough to neglect the shear deformation and using of strain definition [5], the total strain at the given cross section can be written as:

$$\varepsilon_{\text{tot}} = z \frac{d^2 w}{dx^2}$$

(1)

Total strain at $x$ direction at a given cross section of the beam, is the sum of thermal and mechanical strains, thus:

$$\varepsilon_{\text{tot}} = \varepsilon_m + \varepsilon_T,$$

(2)

where:

$$\varepsilon_T = \alpha \Delta T$$

(3)

With:

$$\Delta T = T - T_0,$$

(4)

where $\varepsilon_T$ and $\varepsilon_m$ are the thermal and the mechanical strains, respectively, $\Delta T$ is the temperature rising which is to be measured respect to the initial temperature $T_0$. Substituting Eqs. (1) And (3) into Eq. (2), the following equation can be obtained:

$$\varepsilon_m = z \frac{d^2 w}{dx^2} - \alpha \Delta T$$

(5)

Using of Hook’s low and Eq. (5), the relationship between the stress and the strain based on Euler-Bernoulli beam theory can be expressed as below:

$$\sigma = Ez \frac{d^2 w}{dx^2} - E \alpha \Delta T$$

(6)

The axial force respect to the equilibrium condition along the $x$-axis is given as:
\[ \int_{A_1} \sigma \, dA + \int_{A_2} \sigma \, dA = 0 \quad (7) \]

Substituting the Eq. (6) into Eq. (7):

\[ \int_{-d}^{d} b_1 (E_1 z \frac{d^2 w}{dx^2} - E_i \alpha_i \Delta T) \, dz + \int_{-d}^{h-d} b_2 (E_2 z \frac{d^2 w}{dx^2} - E_2 \alpha_2 \Delta T) \, dz = 0, \quad (8) \]

where \( d \) is distance of neutral axis from contact surface of two materials. By integrating, the Eq. (8) can be reduced to:

\[ \frac{d^2 w}{dx^2} \left( -h_1 b_1 - n h_2 b_2 \right) + \frac{nb_2 h_2^2 - b_1 h_1^2}{2} - \Delta T (\alpha_i b_1 h_i + n \alpha_2 b_2 h_2) = 0 \quad (9) \]

where: \( n = \frac{E_2}{E_i} \)

The bending moment \( M(x) \) at a given section is:

\[ \int \sigma \, z \, dA = M(x) \quad (10) \]

Substituting Eq. (6) into Eq. (10):

\[ \int_{A_1} b_1 (E_1 z \frac{d^2 w}{dx^2} - E_i \alpha_i z \Delta T) \, dz + \int_{A_2} (nE_1 z \frac{d^2 w}{dx^2} - nE_i \alpha_2 z \Delta T) \, dz = M(x) \quad (11) \]

Then the final expression which indicates the relationship between deflections of bimetallic cantilever at a given temperature rising can be written as follow:

\[ w(x) = \frac{3nb_2 b_2 h_2 (h_1 + h_2) (\alpha_1 - \alpha_2) \Delta T}{b_1^2 h_1^4 + n^2 b_2^2 h_2^4 + nb_2 b_1 h_1 h_2 (6h_1 h_2 + 4h_1^2 + 4h_2^2)} \, x^2 \quad (12) \]

Finally, using of Eq. (12) and deriving from it, the expression to calculate the declivity of the cantilever beam with respect to temperature rising is derived as follow:

\[ \vartheta(x) = \frac{6nb_2 b_1 h_2 (h_1 + h_2) (\alpha_1 - \alpha_2) \Delta T}{b_1^2 h_1^4 + n^2 b_2^2 h_2^4 + nb_2 b_1 h_1 h_2 (6h_1 h_2 + 4h_1^2 + 4h_2^2)} \, x \quad (13) \]

With using a digital camera \( \vartheta(l) \), the tip declivity of the cantilever beam, detected. So with rewriting Eq. 13, temperature rising can derive as:

\[ \Delta T = \frac{[b_1^2 h_1^4 + n^2 b_2^2 h_2^4 + nb_2 b_1 h_1 h_2 (6h_1 h_2 + 4h_1^2 + 4h_2^2)] \vartheta(l)}{6nb_2 b_1 h_2 (h_1 + h_2) (\alpha_1 - \alpha_2) l} \quad (14) \]
4. Calculated Results

To have more sensitivity and large deflection, two materials with high difference in their thermal expansion coefficients are considered (Gold and Silicon). The geometrical and material properties are listed in Table 1 as:

Table 1. Geometrical and material properties of the MOEMS thermal sensor.

<table>
<thead>
<tr>
<th>Design variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_1$</td>
<td>$2.6 \times 10^{-6} \text{ k}^{-1}$</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>$14.3 \times 10^{-6} \text{ k}^{-1}$</td>
</tr>
<tr>
<td>$E_1$</td>
<td>$122 \times 10^9 \text{ Nm}^{-2}$</td>
</tr>
<tr>
<td>$E_2$</td>
<td>$80 \times 10^9 \text{ Nm}^{-2}$</td>
</tr>
<tr>
<td>$b_1$</td>
<td>$100 \mu\text{m}$</td>
</tr>
<tr>
<td>$b_2$</td>
<td>$80 \mu\text{m}$</td>
</tr>
<tr>
<td>$h_1$</td>
<td>$4 \mu\text{m}$</td>
</tr>
<tr>
<td>$h_2$</td>
<td>$1.8 \mu\text{m}$</td>
</tr>
<tr>
<td>$L$</td>
<td>$500 \mu\text{m}$</td>
</tr>
</tbody>
</table>

To demonstrate the feasibility of the proposed model and obtained results, it is tried to compare the calculated results of micro beam deflection with the results predicted of Ref. [6] (Table 2).

Table 2. Comparison of calculated tip deflection of proposed model with results of Ref. [6] for a bimetallic cantilever microbeam.

<table>
<thead>
<tr>
<th>Temperature rising $^\circ \text{C}$</th>
<th>Tip deflection ($\mu\text{m}$)</th>
<th>Tip deflection in Ref. [6] ($\mu\text{m}$)</th>
<th>$\Delta%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.6970</td>
<td>2.6971</td>
<td>0.004</td>
</tr>
<tr>
<td>30</td>
<td>8.0911</td>
<td>8.0913</td>
<td>0.003</td>
</tr>
<tr>
<td>50</td>
<td>13.4852</td>
<td>13.4855</td>
<td>0.002</td>
</tr>
<tr>
<td>70</td>
<td>18.8793</td>
<td>18.8797</td>
<td>0.002</td>
</tr>
<tr>
<td>100</td>
<td>26.9704</td>
<td>26.9710</td>
<td>0.002</td>
</tr>
</tbody>
</table>

where:

$$\Delta(\%) = \frac{ABS\{\text{Obtained Results} - \text{results of Ref. [6]}\}}{\text{results of Ref. [6]}} \times 100 \quad (15)$$

Due to the Eq. (12), as the temperature increases, the tip deflection of the micro beam increases too. Fig. 3 shows the tip deflection of bimetallic cantilever beam versus temperature rising. Fig. 4. Shows the deflection of the bimetallic cantilever at temperature rising $100^\circ \text{C}$ and Fig. 5 shows declivity changes at this temperature rising. As it is shown in Fig. 6, the declivity of the tip of beam changes linearly with respect to the temperature rising. Due to the Eq. (14), this linear formula can be used to express the declivity value with respect to value of the temperature rising. Thus, the declivity of beam changes, which causes the angle of the reflected light changes. The change in angle of the reflected
light of the proposed model due to the temperature rising can be easily found by a detector such as a digital camera. The angle changes of reflected light value can be used to evaluate the temperature rising of the desire environment.

**Fig. 3.** Tip deflection versus the temperature rising.

**Fig. 4.** Cantilever deflection (temperature rising 100°C).

**Fig. 5.** Declivity changes of cantilever (temperature rising 100°C).
Fig. 6. Declivity of tip of the beam versus the temperature rising.

5. Conclusions

In this work was presented the design of a novel MOEMS Thermal Sensor used to measure the temperature rising due to the changes of a light angle. The governing thermo-mechanical equation of the model was derived and solved analytically. The obtained results were in good agreement with the other existing results. The calculated results showed that by increasing the temperature, the declivity of cantilever increasing linearly. Bimetallic micro beams can provide sensitive structure for the fabrication of temperature sensors that cover the temperature in wide range based on these structures.

References