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Notch Fracture Toughness Evaluation for a Brittle Graphite Material

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Reference

ABSTRACT
First, several disc-type test samples weakened by a central bean-shaped slit with two U-shaped ends, called U-notched Brazilian disc (UNBD) specimens, made of commercial graphite, were prepared. Four notch tip radii were considered in the tests. Then, a monotonic compressive load was diametrically applied to each sample along the slit, resulting in pure mode I loading. After sudden fracture, the fracture loads were recorded and converted into the corresponding values of the notch fracture toughness (NFT) $K_{IC}$. The main advantage of the UNBD specimen is that the NTF test can be easily conducted without using complicated fixtures. The well-established brittle fracture model, namely the strain energy density (SED) over a specified control volume, which embraces the notch edge, was employed to estimate the fracture loads theoretically. A good agreement was found between experimental and theoretical results. Under Mode I loading, the SED and the NFT are linked by a simple relationship providing a powerful tool for the fracture assessment of different specimens and structural components weakened by notches.

Keywords
notch fracture toughness, U-notch, brittle fracture, graphite material, U-notched Brazilian disc, mode I loading
Introduction

As is well-known, fracture toughness is a vital material property in damage tolerant design of engineering components. It can be experimentally measured for different materials by using the test procedures described in universal standards such as ASTM E399 [1] for metallic materials, etc. Depending on brittleness and ductility, fracture toughness is denoted by various symbols, e.g., $K_{IC}$ [1] for brittle and quasi-brittle materials with abrupt fracture behavior and $J_{IC}$ and CTOD [2] for ductile materials exhibiting stable ductile rupture.

For non-metallic brittle materials, however, most of the specimens and test procedures used to characterize the fracture toughness have been developing by researchers because of the lack of universal standard tests. Although most of these experimental methods have not been formally standardized, it is expected that they can be classified in universal standard forms if their validity are verified by numerous fracture tests on wide range brittle materials.

Graphite is one of the most important brittle materials utilized widely as heat shields in high-tech industries like aerospace and nuclear industries, etc. Besides its excellent resistance against very high temperature gradients due to its very low value of the coefficient of thermal expansion, it is so vulnerable to mechanical loads, which are usually present in real engineering applications together with thermal loads. Therefore, many attempts have been made in the past to characterize the mechanical properties of graphite materials, particularly the fracture toughness. A literature survey indicated that crack growth and brittle fracture have been extensively investigated by several researchers on various graphite materials, both theoretically and experimentally. Mixed mode I/II fracture toughness tests have been conducted by Awaji and Sato [3] on two different graphite materials by means of the centrally cracked Brazilian disc (CCBD) specimen. Yamauchi et al. [4,5] have also measured the fracture toughness of graphite materials by using the CCBD and the semi-circular bend (SCB) specimens under asymmetric three-point bending loading. A single-edge cracked specimen has been utilized by Li et al. [6] for experimentally measuring the mixed mode fracture toughness of a type of polycrystalline graphite. Shi et al. [7] evaluated the fracture toughness of a nuclear graphite material under pure mode I loading by using a three-point bending specimen. Additionally, Etter et al. [8] evaluated the mode I fracture toughness of polycrystalline graphite by employing a single-edge cracked beam specimen. Dealing with theoretical and experimental studies on fracture toughness of graphite materials, one can refer to Ayatollahi and Aliha [9], Mostafavi and Marrow [10,11], Mostafavi et al. [12–14], and Nakhodchi et al. [15].

Dealing with interesting and recent studies, it is worth mentioning a paper on a porous polycrystalline graphite under hypervelocity impact (HVI) and laser impact, both leading to strong debris ejection and cratering [16]. A failure criterion relying on Weibull theory is used to relate material tensile strength to deformation rate and damage. These constitutive relations have been implemented in an Eulerian hydro-code in order to compute numerical simulations and confront them with experiments. A simple fitting procedure of the unknown Weibull parameters based on HVI results has been proposed, founding a good agreement with experimental observations of crater shapes and dimensions, as well as debris velocity.
A much applied branch of fracture mechanics is the notch fracture mechanics (NFM), in which fracture phenomenon is studied in engineering components and structures containing notches. Notches are inevitably introduced into the components for specific requirements such as to join the elements of machines and structures, etc. They, especially V, U, and end-hole shaped ones, concentrate stresses around them and make the notched component prone to crack initiation. If material is brittle or quasi-brittle, like most of industrial graphite materials, crack growth will take place rapidly and hence, sudden fracture is expected to occur. Thus, the fracture toughness of notched brittle materials should be evaluated by means of experimental and/or theoretical methods. This characteristic of material is known as the notch fracture toughness (NFT). Several researches have been performed in the past dealing with fracture in graphite materials in the presence of notches. Bazaj and Cox [17] and Kawakami [18] were the pioneering and first researchers who investigated the notch sensitivity on various graphite materials providing results for different geometries of the notch. Considering NFM approaches, an investigation on brittle fracture of notched graphite materials has been performed also by Ayatollahi and Torabi [19], both experimentally and theoretically. They conducted mode I fracture toughness tests on various V-notched graphite specimens and estimated the experimental results by using the mean stress (MS) criterion very well. They also provided extensive experimental data on mixed mode I/II fracture of V-notched Brazilian disc (V-BD) specimens made of a type of industrial polycrystalline graphite. The experimental results were successfully predicted by means of the V-notched maximum tangential stress (V-MTS) fracture model [20].

A strain energy density (SED) approach, as first proposed in Ref. [21], has also been successfully used in Refs. [22–24] with the aim to predict a large bulk of static mixed mode fracture test results from different materials. Dealing with graphite specimens weakened by blunt notches of different shape and subjected to mixed mode I/II loading, the SED criterion has been recently employed to assess the fracture load of notches characterized by different shapes [25–27].

A recent study has been devoted to the out-of-plane fracture of V-notched graphite bars, in which the test results have been well predicted by the SED criterion [28]. Brittle failure of isostatic polycrystalline graphite under pure compression loading has been also investigated experimentally by using prismatic specimens weakened by sharp and rounded-tip V-notches [29]. The SED criterion has been employed for the fracture assessment of notched graphite components under compression, as an extension of what the present authors have proposed in previous papers dealing with the cases of in-plane tension-shear loading and torsion loading in notched graphite specimens.

Torabi [30,31] re-analyzed the test results reported in Ref. [26] using the point stress (PS) and the mean stress (MS) criteria in pure mode I and the U-notched maximum tangential stress (UMTS) criterion in mixed mode I/II loading conditions, respectively. More recently, Torabi et al. [32] published a paper in which the mode I brittle fracture in disc-type graphite specimens containing central bean-shaped slit with two U-shaped ends, called U-notched Brazilian disc (UNBD), has been successfully predicted by means of the MS and the PS criteria.

In the present research, the UNBD specimens have been utilized for conducting fracture tests under pure mode I loading conditions. The fracture loads have been
converted to the corresponding values of the critical notch stress intensity factors (NSIFs) and the experimental NFT values have been obtained for different notch tip radii. The experimentally measured fracture loads have been theoretically predicted by means of the well-established brittle fracture model, namely SED over a specified critical volume which embraces the notch border. A very good agreement has been found between experimental and theoretical results.

Experimental Program

MATERIAL

The material used in NFT tests was a type of commercial poly-granular graphite with the properties presented in Table 1. This type of graphite is traditionally utilized in aeronautical applications. The same material has been studied in Refs. [9,25], where NFT tests have been carried out on laboratory-scale V-notched specimens. The method of producing the material was the cold isostatic pressing (CIP), which provides homogeneous structure and isotropic properties. The porosity of the material was about 9% providing brittle behavior.

SPECIMEN

The test sample employed for conducting fracture tests was a disc-type plate weakened by a central bean-shaped slit with two U-shaped ends, so-called in literature as the UNBD specimen. The UNBD specimen is represented in Fig. 1 schematically.

In Fig. 1, $\beta$ is the angle between the loading direction and the notch bisector line. The disc diameter, the overall slit length and the applied compressive load are denoted by $D$, $d$, and $P$, respectively. When $\beta = 0$, the U-shaped ends are subjected to pure mode I loading conditions.

The diameter, the overall slit length (i.e. the tip-to-tip distance) and the thickness of the UNBD specimens were 60, 18, and 10 mm, respectively. The notch tip radii ($\rho$) were 0.5, 1, 2, and 4 mm. To prepare the specimens, first a graphite block was provided from a manufacturing company. Then, three slices of 10 mm thick were cut from the block by using a cutter blade. The geometry of each specimen was sent to a high-precision 2D CNC water jet cutting machine and finally, the UNBD specimens were fabricated. The specimens were polished by using a fine abrasive paper before performing NFT tests. Figure 2 displays the UNBD graphite specimens before experiments.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
</table>

Properties of the tested graphite material [12,13].

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elastic modulus, E (GPa)</td>
<td>8.05</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td>0.2</td>
</tr>
<tr>
<td>Ultimate tensile strength (MPa)</td>
<td>27.5</td>
</tr>
<tr>
<td>Plane-strain fracture toughness (MPa m$^{-0.5}$)</td>
<td>1.0</td>
</tr>
<tr>
<td>Bulk density (kg/m$^3$)</td>
<td>1710</td>
</tr>
<tr>
<td>Mean grain size ($\mu$m)</td>
<td>320</td>
</tr>
<tr>
<td>Porosity (%)</td>
<td>9</td>
</tr>
</tbody>
</table>
For each notch tip radius, three tests were performed under displacement-control conditions with a loading rate of 0.1 mm/min and the fracture load of each sample was recorded. A total number of 12 test results were provided under pure mode I loading conditions. Figures 3 and 4 represent the graphite specimens during and after fracture tests, respectively.

Table 2 presents the experimentally obtained fracture loads for UNBD graphite samples.

The load-displacement curves recorded during the tests were absolutely linear up to final breakage and fracture occurred suddenly. Thus, using brittle fracture models on the basis of linear elastic fracture mechanics (LEFM) is permissible. Figure 5 represents a sample load-displacement curve for a UNBD graphite specimen.

COMPUTATION OF THE NFT VALUES
The stress field around notches is usually presented in terms of the notch stress intensity factors (NSIFs) [33–35]. The critical NSIFs are known as the NFT values.
which are corresponding to the fracture loads. While fracture load depends directly on the material properties, the notch geometry and the overall geometry of notched component, the NFT’s do not depend on the overall geometry of notched member. This makes NFTs very appropriate parameters to characterize the fracture resistance of notches wherever applied. In order to achieve the experimental NFT values for the UNBD graphite specimens under mode I loading conditions, some relationships taken from Ref. [36] have been considered. In Ref. [36], the notch shape factors (NSFs) for UNBD specimen for wide range of notch and specimen geometries have

FIG. 3
The UNBD specimen during mode I fracture test.

FIG. 4
The UNBD specimens after fracture.
The experimentally obtained fracture loads for UNBD graphite specimens.

<table>
<thead>
<tr>
<th>( \rho ) (mm)</th>
<th>( P_1 ) (N)</th>
<th>( P_2 ) (N)</th>
<th>( P_3 ) (N)</th>
<th>( P_{\text{mean}} ) (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>4164</td>
<td>4267</td>
<td>4040</td>
<td>4157</td>
</tr>
<tr>
<td>1</td>
<td>4410</td>
<td>4561</td>
<td>4830</td>
<td>4600</td>
</tr>
<tr>
<td>2</td>
<td>4228</td>
<td>4448</td>
<td>4382</td>
<td>4352</td>
</tr>
<tr>
<td>4</td>
<td>4000</td>
<td>4188</td>
<td>3835</td>
<td>4007</td>
</tr>
</tbody>
</table>

Where \( Y_{I\rho} \), \( K_{I\rho} \), \( d \), \( D \), \( t \), and \( P \) are the mode I NSF, the mode I NSIF, the overall length of the central slit, the disc diameter, the specimen thickness, and the applied load, respectively. In Eq 1, \( K_{I\rho} \) and \( P \) depend on each other. The values of the NSFs, which are non-dimensional geometrical factors, have been presented in Ref. [36] for different notch lengths and various notch tip radii. For any UNBD specimen, one should first read the NSF value from Ref. [36]. If \( K_{I\rho} \) or \( P \) is known, the other parameter can be simply computed. For example, for a known value of the fracture load \( P_c \) (see Table 2), \( K_{I\rho,c} \) (i.e. the NFT) can be determined by using Eq. 2.

\[
K_{I\rho,c} = \sqrt{\frac{2d}{\pi} \frac{P_c}{Dt} Y_{I\rho}}
\]

See Table 3. The mean experimental values of the NFTs \( K_{I\rho,c} \) for the notched graphite specimens.
In the next section, the well-known brittle fracture model, namely the strain energy density (SED) over a specified critical volume around the notch border, is introduced and used to predict the fracture loads of the UNBD graphite specimens. The results of the SED criterion are then compared in the forthcoming sections with the experimental results.

Fracture Criterion Based on the Strain Energy Density Averaged Over a Control Volume

In order to estimate the fracture load of notched graphite components, designers need a suitable fracture criterion based on the mechanical behavior of material around the notch tip. A strain-energy-density based criterion is described in this section by which the fracture loads obtained from the experiments can be estimated very well.

Dealing with cracked components, the strain energy density factor $S$ [37] was defined by Sih as the product of the strain energy density by a critical distance from the point of singularity. Failure was thought of as controlled by a critical value $S_c$, whereas the direction of crack propagation was determined by imposing a minimum condition on $S$.

Different from Sih’s criterion, which is a point-wise criterion, the averaged strain energy density criterion (SED) as presented in Refs. [21–23] states that brittle failure occurs when the mean value of the strain energy density over a given control volume is equal to a critical value $W_c$. This critical value varies from material to material but not on the notch geometry and sharpness. The control volume, reminiscent of Neuber’s concept of elementary structural volume [38], is thought of as dependent on the ultimate tensile strength and the fracture toughness $K_Ic$ in the case of brittle or quasi-brittle materials subjected to static loads.

Such a method was formalized and applied first to sharp, zero radius, V-notches under mode I and mixed I/II loading [21] and later extended to blunt U and V-notches [22,23].

When dealing with cracks, the critical volume is a circle of radius $R_c$ centered at the tip (Fig. 6(a)). Under plane strain conditions, the critical length, $R_c$, can be evaluated according to the following expression [39]:

$$ R_c = \frac{(1 + \nu)(5 - 8\nu)}{4\pi} \left( \frac{K_{ic}}{\sigma_l} \right)^2 $$

where:
- $K_{ic}$ is the fracture toughness,
- $\nu$ is the Poisson’s ratio, and

<table>
<thead>
<tr>
<th>$\rho$ (mm)</th>
<th>0.5</th>
<th>1.0</th>
<th>2.0</th>
<th>4.0</th>
</tr>
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<tbody>
<tr>
<td>$K_{ip,c}$ (MPa $\sqrt{m}$)</td>
<td>1.333</td>
<td>1.644</td>
<td>1.825</td>
<td>2.107</td>
</tr>
<tr>
<td>$\sigma_{lip}$ (MPa)</td>
<td>67.3</td>
<td>58.7</td>
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<td>46.1</td>
<td>37.6</td>
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\( \sigma_t \) = the ultimate tensile strength of a plain specimen that obeys a linear elastic behavior.

For a sharp V-notch, the critical volume becomes a circular sector of radius \( R_c \) centered at the notch tip (Fig. 6(b)) while for a blunt V-notch under mode I loading, the volume assumes the crescent shape shown in Fig. 6(c), where \( R_c \) is the depth measured along the notch bisector line. The outer radius of the crescent shape is equal to \( R_c + r_0 \), where \( r_0 \) is the distance between the notch tip and the origin of the local coordinate system (Fig. 6). Such a distance depends on the V-notch opening angle \( 2\alpha \), according to the expression \( r_0 = \rho (\pi - 2\alpha)/(2\pi - 2\alpha) \) [35].

Under mixed mode loading, the critical volume is no longer centered on the notch tip, but rather on the point where the principal stress reaches its maximum value along the edge of the notch. It was assumed that the crescent shape volume rotates rigidly under mixed mode, with no change in shape and size. This is the governing idea of the "equivalent local mode I" approach, as proposed and applied to U and V-notches [23,24].

When the area embraces the semicircular edge of the notch (and not its rectilinear flanks), the mean value of SED can be expressed in the following form [22]:

\[
\overline{W}_1 = F(2\alpha) \times H \left( 2\alpha, \frac{R_c}{\rho} \right) \times \frac{\sigma_{tip}^2}{E} 
\]

where \( F(2\alpha) \) depends on previously defined parameters

\[
F(2\alpha) = \left( \frac{q - 1}{q} \right)^{2(1 - \lambda_1)} \left[ \frac{\sqrt{2\pi}}{1 + \omega_1} \right]^2 
\]

### Table 4
Parameters for the stress distributions.

<table>
<thead>
<tr>
<th>( 2\alpha ) (rad)</th>
<th>( q )</th>
<th>( \lambda_1 )</th>
<th>( \omega_1 )</th>
<th>( F(2\alpha) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.0000</td>
<td>0.5</td>
<td>1</td>
<td>0.7830</td>
</tr>
<tr>
<td>\pi/6</td>
<td>1.8333</td>
<td>0.5014</td>
<td>1.034</td>
<td>0.6917</td>
</tr>
<tr>
<td>\pi/4</td>
<td>1.7500</td>
<td>0.5050</td>
<td>1.014</td>
<td>0.6692</td>
</tr>
<tr>
<td>3\pi/4</td>
<td>1.2500</td>
<td>0.6736</td>
<td>0.432</td>
<td>1.0717</td>
</tr>
</tbody>
</table>
and is reported in the last column of Table 4. H is summarized for U-notches in Table 5 as a function of the ratio \( R_c/q \) and for different values of the Poisson’s ratio.

By simply using the definition of the mode I NSIF for blunt V-notches, a simple relationship between \( \sigma_{\text{tip}} \) and \( K_{1q} \) can be obtained as follows:

\[
K_{1q} = \sqrt{2\pi} \frac{\sigma_{\text{tip}}}{1 + q} \left( \frac{q - 1}{q} \right)^{1 - \zeta_1} = \sqrt{F(2\pi)} \sigma_{\text{tip}} p^{1 - \zeta_1}
\]  

(6)

Then, it is possible to rewrite Eq 6 in a more compact form

\[
\bar{W}_1 = H \left( 2\pi, \frac{R_c}{\rho} \right) \times \frac{K_{1q}^2}{E} \times \frac{1}{R_c^{2(1 - \zeta_1)}}
\]

(7)

Equation 7 can be used to evaluate the SED under mode I loading once \( K_{1q} \) is known.

Alternatively, avoiding any simplified assumption, the SED values can be directly derived from finite element (FE) models. The advantage of the direct evaluation of the SED from a FE model is that the value of this parameter is mesh-independent as described in Refs. [40,41]. A very coarse mesh can be adopted for the SED evaluation contrary to the mesh required to evaluate the notch stress intensity factors or other stress based parameters.

**SED Approach in Fracture Analysis of the Tested Graphite Specimens**

The fracture criterion described in the previous section is employed here to estimate the fracture loads obtained from the experiments conducted on the graphite specimens. In order to determine the SED values, first a finite element model of the graphite specimens was generated. A typical mesh used in the numerical analyzes is shown in Fig. 7(a). The averaged SED criterion states that failure occurs when the mean value of the strain energy density over a control volume, \( \bar{W} \), is equal to a critical value \( W_c \), which depends on the material but not on notch geometry [21,22].

### Table 5

H values for U-notched specimens.

<table>
<thead>
<tr>
<th>( R_c/\rho )</th>
<th>( \nu = 0.1 )</th>
<th>( \nu = 0.15 )</th>
<th>( \nu = 0.2 )</th>
<th>( \nu = 0.25 )</th>
<th>( \nu = 0.3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0005</td>
<td>0.6294</td>
<td>0.6215</td>
<td>0.6104</td>
<td>0.5960</td>
<td>0.5785</td>
</tr>
<tr>
<td>0.001</td>
<td>0.6286</td>
<td>0.6207</td>
<td>0.6095</td>
<td>0.5952</td>
<td>0.5777</td>
</tr>
<tr>
<td>0.005</td>
<td>0.6225</td>
<td>0.6145</td>
<td>0.6033</td>
<td>0.5889</td>
<td>0.5714</td>
</tr>
<tr>
<td>0.01</td>
<td>0.6149</td>
<td>0.6068</td>
<td>0.5956</td>
<td>0.5813</td>
<td>0.5638</td>
</tr>
<tr>
<td>0.05</td>
<td>0.5599</td>
<td>0.5515</td>
<td>0.5401</td>
<td>0.5258</td>
<td>0.5086</td>
</tr>
<tr>
<td>0.1</td>
<td>0.5028</td>
<td>0.4942</td>
<td>0.4828</td>
<td>0.4687</td>
<td>0.4518</td>
</tr>
<tr>
<td>0.3</td>
<td>0.3528</td>
<td>0.3445</td>
<td>0.3341</td>
<td>0.3216</td>
<td>0.3069</td>
</tr>
<tr>
<td>0.5</td>
<td>0.2672</td>
<td>0.2599</td>
<td>0.2508</td>
<td>0.2401</td>
<td>0.2276</td>
</tr>
<tr>
<td>1</td>
<td>0.1590</td>
<td>0.1537</td>
<td>0.1473</td>
<td>0.1399</td>
<td>0.1314</td>
</tr>
</tbody>
</table>

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This critical value can be determined from the ultimate tensile strength $\sigma_t$ according to Beltrami’s expression

$$W_c = \frac{\sigma_t^2}{2E} \tag{8}$$

In parallel, the control volume definition via the control radius $R_c$ needs the knowledge of the fracture toughness $K_{lc}$ and the Poisson’s ratio $\nu$; see Eq 3. The critical load that is sustainable by a notched component can be estimated by imposing $W$ equal to the critical value $W_c$. This value is considered here constant under mode I, mode II, and in plane mixed-mode conditions. This assumption has been extensively verified for a number of different brittle and quasi-brittle materials [21–24].

As mentioned earlier, the properties of the graphite material used in the present investigation are: $\sigma_t = 27.5$ MPa, $K_{lc} = 1$ MPa√m, Poisson’s ratio $\nu = 0.2$. As a result, the critical SED for the tested graphite is $W_c = 0.0469$ MJ/m$^2$ whereas the radius of
the control volume is $R_c = 0.429$ mm considering realistic plane strain conditions [25].

The SED occurring inside the control volume embracing the edges of U-notches has been calculated numerically by using the FE code ANSYS. For each geometry, two models were created, one with a fine mesh (Fig. 7(a)), and the other with a coarse mesh (Fig. 7(b)). Both models require an accurate definition of the control volume where the strain energy density should be averaged (see Fig. 7). All of the analyses have been carried out by using eight-node elements under the hypothesis of plane strain conditions.

Figure 7 refers to the case $\rho = 0.5$ mm. The strain energy density contour lines inside the control volume are shown in Fig. 7(c). Note that the SED is symmetric with respect to the notch bisector line.

Table 6 summarizes the outlines of the experimental, numerical, and theoretical findings for the tested graphite specimens with four different notch tip radii ($\rho = 0.5, 1, 2, 4$ mm) investigated in the present research and re-analyzed by means of SED. In particular, Table 6 summarizes the averaged experimental loads to failure ($P_{\text{exp}}$) for every notch radius $\rho$ compared with the theoretical values ($P_{\text{th}}$) based on the SED evaluation. Table 6 also gives the SED value as obtained from the FE models of the graphite specimens characterized by a fine and by a coarse mesh. The SED results have been obtained by applying to the model the averaged value of the critical load.

The last column of Table 6 reports the relative deviation between the experimental critical loads and the theoretical values assessed by means of SED. As visible from the table, the agreement between the experimental results obtained for the notched graphite specimens and the theoretical predictions based on a constant value of the local strain energy is satisfactory with the relative deviation ranging from –7.1 to 6.8 %.

The results are given also in graphical form in Fig. 8, where the experimental values of the critical loads (open dots) have been compared with the theoretical predictions based on the constancy of the SED in the control volume (solid line). The plots are given for the notched graphite specimens as a function of the notch radius $\rho$. The trend of the theoretically predicted loads is in good agreement with the experimental ones.

A synthesis in terms of the square root value of the local energy averaged over the control volume (of radius $R_c$), normalized with respect to the critical energy of
the material as a function of the notch tip radius is shown in Fig. 9. The plotted parameter is proportional to the fracture load. The new data are plotted together with previous test data under pure mode I and mixed mode I/II loading from the same material, as taken from Ref. [25].

The aim is to investigate the influence of the notch tip radius on the fracture assessment based on SED. From Fig. 9, it is clear that the scatter of the data is very limited and almost independent of the notch radius. All the values fall inside a scatter ranging 0.80–1.20 with the majority of the new data inside 0.90–1.10. The synthesis also confirms the choice of the control volume, which seems to be suitable to characterize the material behavior under pure mode I and in-plane mixed mode loading. The scatter of the experimental data presented here is in good agreement with the recent database in terms of SED reported in Refs. [23,24].

FIG. 8 Fracture assessment based on SED.

FIG. 9 Synthesis of brittle failure data from graphite specimens. Present results compared with data taken from Ref [25].
Conclusions

Brittle fracture in U-notched polycrystalline graphite specimens was investigated both experimentally and theoretically under mode I loading. Fracture tests were conducted on Brazilian disk specimens containing rounded-tip U-notches characterized by different notch tip radii. The SED criterion was applied in order to estimate the fracture loads of notched graphite components. It was shown that the proposed method is suitable for the tested polycrystalline graphite, being the experimental results in good agreements with the results estimated by the SED approach. From the sound agreement between the theoretical and experimental results, it can be straightforward deduced that for the polycrystalline graphite, the critical energy and the radius of the control volume are both constant material properties independently of the notch geometry and shape.

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