Analytical Modeling and Analysis of Axial-Flux Interior Permanent-Magnet Couplers

Sajjad Mohammadi, Student Member, IEEE, Mojtaba Mirsalim, Senior Member, IEEE, Sadegh Vaez-Zadeh, Senior Member, IEEE, and Heidar Ali Talebi, Senior Member, IEEE

Abstract—Analytical calculations are effective in preliminary design stages and analysis of electric machines. In this paper, for axial-flux interior permanent-magnet (PM) eddy-current couplers, an analytical model is developed, which is able to deal with the complex machine geometry and take into account material properties such as iron saturation and PM characteristics. The design considerations of the machine are also presented. Moreover, the studied structure shows several advantages in terms of mechanical and magnetic performances over the couplers with surface PMs. The three-dimensional (3-D) finite-element method is employed in the analyses and evaluations. Finally, an experimental prototype is built to validate the model.

Index Terms—Analytical modeling, axial flux, eddy-current coupler, finite-element method (FEM), interior permanent magnet (PM), saturation.

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>B</td>
<td>Magnetic flux density.</td>
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<tr>
<td>(B_{\text{knee}})</td>
<td>Knee flux density of saturation curve of iron.</td>
</tr>
<tr>
<td>J</td>
<td>Current density.</td>
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<tr>
<td>L</td>
<td>Length.</td>
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<tr>
<td>P</td>
<td>Permeance.</td>
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<td>R</td>
<td>Reluctance.</td>
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<tr>
<td>r, (\theta, z)</td>
<td>Cylindrical coordinates.</td>
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<tr>
<td>T</td>
<td>Torque.</td>
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<tr>
<td>(v, \omega)</td>
<td>Translational and angular speeds.</td>
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<tr>
<td>x, (y, z)</td>
<td>Rectangular coordinates.</td>
</tr>
<tr>
<td>(\theta_p, \tau_p)</td>
<td>Pole-pitch angle and pole pitch.</td>
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<td>(\phi)</td>
<td>Flux.</td>
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<th>Subscript</th>
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<tr>
<td>av</td>
<td>Average.</td>
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<tr>
<td>cs</td>
<td>Conductive sheet (CS).</td>
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Effective air gap. | lb | Leakage from back side. |
Leakage from the air-gap side. | lg | Leakage from the shaft side. |
Leakage from back side. | lsh | Leakage from the shaft side. |
Leakage from the top side. | lt | Permanent magnet (PM). |
Primary and secondary. | p, s | Primary and secondary yokes. |

I. INTRODUCTION

MAGNETIC couplers transmit torque without any mechanical contact, while damping vibrations and allowing a soft start. Also, there is no need for the high-accuracy alignment that is necessary in the conventional mechanical couplers. Magnetic couplers consist of two parts: one part is attached to the prime mover, while the other is fixed to the load. There are mainly two types: 1) eddy-current; and 2) synchronous couplers, both with radial- or axial-flux configurations. Axial-flux couplers easily enable us to transfer a torque through a separation wall with application to isolated systems, such as vacuum or high-pressure vessels, and pumps. Also, the shock-absorbing characteristic couplers can provide advantages for PM wind generators in higher torques.

Electromagnetic devices can numerically or analytically be studied. The former such as finite-element method (FEM) [1], [2], albeit precise, are time consuming, while the latter offer flexible and accurate frameworks to effectively be used in preliminary designs. Modeling of PM-assisted devices may be performed on the basis of Laplace’s and Poisson’s equations [3], [4] or by employing magnetic equivalent circuit (MEC) techniques, as done for linear [5], axial-flux [6]–[8], and radial-flux [9]–[12] machines. The former, although powerful, is complicated, incapable of effectively accounting for material properties, and having problems with complex geometries. The latter is simpler and flexible in structures but has no symmetry. It is also able to account for iron saturation and all material properties [8]–[10]. A comparison is also done in [13].

Based on the first approach, analytical models for eddy-current couplers with surface PMs [14], [15], eddy-current dampers [16], and synchronous couplers [17] have been presented, wherein besides the mentioned problems, back irons are assumed infinitely permeable and only remanence of PMs is accounted for. MEC-based models for radial-flux eddy-current couplers have also been developed in [18]–[20], wherein the mentioned issues are somewhat solved. In [21], the previous coupler is interestingly employed in a slip-synchronous PM
wind generator with a number of advantages, for which a semianalytical model is also developed whose main drawbacks are that some model parameters should numerically be approximated by FEM and, also, the current density distribution is assumed to be symmetrical.

The main contribution of this paper is to develop a new analytical framework for a new interior PM (IPM) axial-flux eddy-current coupler by combining three-dimensional (3-D) MEC approaches with Faraday’s and Ampere’s laws. In addition to flexibility and simplicity, the proposed approach is able to account for saturation and permeability of iron, and both characteristics of PMs, i.e., remanence and coercivity. A number of design considerations are also presented. The IPM coupler introduces several advantages over surface-mounted PM structures. Superiorities of the proposed model over the existing approach are also demonstrated. Three-dimensional FEM is also employed in the analysis. Finally, a prototype is built for validations.

II. PROPOSED MODEL

Fig. 1 shows the exploded view and geometrical parameters of the coupler. Therein, circumferentially polarized PMs alternating in the direction of magnetization are embedded within the primary iron, and the CS is located on the secondary, behind which a rolled back iron is also placed. The active region of the device corresponding to PMs and back irons is delimited by \( R_i \) and \( R_o \), while the CS is extended by overhangs of length \( H \) from both sides to provide a return path for the induced currents. Table I summarizes the specifications of the studied coupler. Compared to the surface-mounted PM couplers [14], [15], [18]–[20], it is worth noting that, in the IPM coupler, besides utilizing the low-cost rectangular-shaped PM, there is no centrifugal force on them, and they are mechanically protected by the frame.

![Fig. 1. Exploded view and geometrical parameters of IPM coupler.](image)

<table>
<thead>
<tr>
<th>TABLE I SPECIFICATIONS OF THE CASE-STUDY COUPLER</th>
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<tbody>
<tr>
<td>parameter</td>
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<tr>
<td>active inner radius, ( R_i )</td>
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<tr>
<td>active outer radius, ( R_o )</td>
</tr>
<tr>
<td>PM height, ( h_m )</td>
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<tr>
<td>air-gap length, ( g )</td>
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<tr>
<td>CS thickness, ( L_{cs} )</td>
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<tr>
<td>overhang length, ( H )</td>
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<tr>
<td>number of PMs, ( p )</td>
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![Fig. 2. Functional block diagram of the proposed model.](image)

![Fig. 3. (a) Flux paths. (b) MEC.](image)

The block diagram depicted in Fig. 2 illustrates how the proposed model is established. The magnetic flux density due to PMs is first determined via the implemented MEC. The induced current in the CS due to the relative speed is then calculated with Faraday’s law. Afterward, the reaction field developed by the induced currents on the original field is taken into account through Ampere’s law. Finally, the developed torque is obtained by Lorentz law.

A reduced 2-D scheme in rectangular coordinates \( (x, y, z) \) is used to simplify the calculations by linearly expanding the geometry along with the average radius \( R_{av} \). The pole pitch, effective length, and translational speed can be defined as

\[
\tau_p = R_{av} \theta_p \quad L = R_o - R_i \quad v = R_{av} \omega.
\]

A. Field Analysis

The flux paths and the MEC of one flux loop are depicted in Fig. 3. The equivalent flux source of a PM is as follows:

\[
\varphi_r = B_r L_{yp} L.
\]
where
\[
P_{\text{lh}} = \mu_0 \int_{h_m/2}^{(R_i \theta_p - h_m)/2} \frac{L_{\text{yp}} dl}{2 \alpha L} = \mu_0 L_{\text{yp}} \ln \left( \frac{R_{i \theta_p - h_m}}{h_m} \right)
\]

and
\[
P_{\text{ls}} = 0.26 \mu_0 L_{\text{yp}}
\]

where
\[
\alpha = \arccos \left( \frac{R_i \theta_p - h_m}{4 R_i} \right).
\]

For the leakage permeance of the top side, we have
\[
P_{\text{lt}} = P_{\text{lt1}} + P_{\text{lt2}}
\]

where
\[
P_{\text{lt1}} = \mu_0 \int_{h_m/2}^{R_{i \theta_p - h_m}} \frac{L_{\text{yp}} dl}{2 \beta L} = \mu_0 L_{\text{yp}} \ln \left( \frac{R_{i \theta_p - h_m}}{h_m} \right)
\]

and
\[
P_{\text{lt2}} = 0.26 \mu_0 L_{\text{yp}}
\]

where
\[
\beta = \pi - \arccos \left( \frac{R_i \theta_p - h_m}{4 R_i} \right).
\]

According to Fig. 4(a), by averaging the areas of the surfaces from which flux lines emanate and at which flux lines end, the iron reluctance of the primary part is calculated as follows:
\[
R_{\text{yp}} = \frac{0.5(\tau_p - h_m)}{\mu_0 \mu_{ip}(A_{p1} + A_{p2})/2}
\]

where
\[
A_{p1} = L_{\text{yp}} L, \quad A_{p2} = L(\tau_p - h_m)/2.
\]

The flux density in the secondary iron yoke is higher behind the PM regions. Thus, the reluctances are calculated in three parts, as shown in Fig. 4(a) and by the following:
\[
R_{\text{ys}} = R_{\text{ys1}} + 2 R_{\text{ys2}}
\]

where
\[
R_{\text{ys1}} = \frac{h_m}{\mu_0 \mu_{is1} A_{s1}} \quad R_{\text{ys2}} = \frac{0.5(\tau_p - h_m)}{\mu_0 \mu_{is2}(A_{s1} + A_{s2})/2}
\]

where
\[
A_{s1} = L_{\text{ys1}} L \quad A_{s2} = L(\tau_p - h_m)/2
\]

where \(\mu_{ip}, \mu_{is1},\) and \(\mu_{is2}\) are the relative permeabilities of the associated iron components. Finally, according to Fig. 3(b), fluxes flowing into the circuit can be calculated as follows:

\[
\varphi_g = \frac{R_{m || R_{\text{ig}} || R_{\text{ib}} || R_{\text{ls}} || R_{\text{sh}} || R_{\text{yp}}}}{R_{m || R_{\text{ig}} || R_{\text{ib}} || R_{\text{sh}} || R_{\text{yp}}}} \times 2 \varphi_r
\]

\[
\varphi_m = R_{m || R_{\text{ig}} || R_{\text{ib}} || R_{\text{sh}} || R_{\text{yp}} || R_{\text{sh}} || R_{\text{sh}} || 2 R_{\text{ge}} + 2 R_{\text{yp}} + R_{\text{ys}}}
\]

\[
\varphi_{yp} = \varphi_{ys} = \varphi_g/2.
\]

As illustrated in Fig. 5, an iterative procedure is used to calculate the permeability of the saturable elements. It starts by assigning an initial value to the relative permeability of iron
reluctances to determine the reluctance network and calculate the fluxes. Next, the magnetic flux densities within saturable reluctances are all calculated as follows:

\[
B_{ys1} = \varphi_{ys}/A_{s1}, \quad B_{ys2} = \varphi_{ys}/[(A_{s1} + A_{s2})/2].
\]

(26)

Then, according to the \(B-H\) curve of the utilized steel, new permeabilities can be updated as follows [10]:

\[
\mu_i^{(k)} = B_y^{(k-1)}/\mu_0H_y^{(k-1)}, \quad \mu_i^{(k)} = \left[\mu_i^{(k)}\right]^{d}\left[\mu_i^{(k-1)}\right]^{1-d}
\]

(27)

where \(k\) refers to the iteration number and \(d\) is a damping constant set to 0.1 herein. The process continues until the following criterion is satisfied for all permeabilities:

\[
\left|\left[\mu_i^{(k)} - \mu_i^{(k-1)}\right]/\mu_i^{(k)}\right| \leq \varepsilon.
\]

(28)

The axial component of the flux density in the air gap, as well as the CS, can be expressed as follows:

\[
B_m(x) = \begin{cases} 
B_{\text{max}} = \frac{\varphi_y}{(\tau_p-h_m)/L}, & -\frac{\tau_p-h_m}{2} \leq x \leq \frac{\tau_p-h_m}{2} \\
0, & \text{elsewhere}.
\end{cases}
\]

(29)

**B. Induced Current**

Having the flux density calculated, the induced current density in the CS in the \(y\)-direction can be calculated as

\[
J(x) = \sigma \bar{v} \times \bar{B} = \sigma vB_z(x) = R_{av}\sigma \omega B_z(x).
\]

(30)

**C. Reaction Field**

The sketch shown in Fig. 6(a) shows the fluxes due to PMs, the induced currents, and the resulting reaction fluxes. The total flux density in the air gap and CS is as follows:

\[
B_z(x) = B_m(x) + B_{cs}(x)
\]

(31)

where \(B_{cs}(x)\) denotes the reaction flux density produced by the induced currents in the CS whose associated flux lines are depicted in Fig. 6(b) and can be found by applying Ampere’s law to path \(C\) given in Fig. 6(c), as in the following:

\[
\int_C H \cdot dl = \int_0^L \int J(x) dz dx
\]

(32)

where the term after the equality denotes the total current enclosed in path \(C\). To avoid extra calculations, MMF drops across the iron parts are neglected, and the recoil permeability of PMs is assumed unity. Thus, substitution of (30) into (32) and simplifying as in the Appendix yield

\[
B_{cs}(x) = m \int_{x_1}^{x_2} [B_m(x) + B_{cs}(x)] dx
\]

(33)

\[
m = \mu_0 \sigma vL_{cs}/2\sigma_e.
\]

(34)

Differentiating from (33) with respect to \(x\) yields

\[
dB_{cs}(x)/dx - mB_{cs}(x) = mB_{m}(x)
\]

(35)

whose general solution is derived as follows:

\[
B_{cs}(x) = \begin{cases} 
B_{cs1} = k_1e^{mx}, & -\frac{\tau_p}{2} \leq x \leq \frac{\tau_p-h_m}{2} \\
B_{cs2} = k_2e^{mx} - B_{\text{max}}, & \frac{\tau_p-h_m}{2} \leq x \leq \frac{\tau_p-h_m}{2} \\
B_{cs3} = k_3e^{mx}, & \frac{\tau_p-h_m}{2} \leq x \leq \tau_p
\end{cases}
\]

(36)

Given in the Appendix, constants \(k_1, k_2,\) and \(k_3\) are determined using the following conditions:

\[
B_{cs2}(x_0) = 0
\]

(37)

\[
B_{cs1}(x = -[\tau_p-h_m]/2) = B_{cs2}(x = -[\tau_p-h_m]/2)
\]

(38)

\[
B_{cs2}(x = [\tau_p-h_m]/2) = B_{cs3}(x = [\tau_p-h_m]/2)
\]

(39)

where (37) is the main boundary condition denoting a particular point where the total currents enclosed in the intervals \([-\tau_p/2, x_0]\) and \([x_0, \tau_p/2]\) are the same, while (38) and (39)
state the continuity of $B_{cs}(x)$. Given further details in the Appendix, $x_0$ is determined through the following:

$$\int_{-\tau_p/2}^{\tau_p/2} \int_0^{L_{cs}} J(x) dz \, dx = \int_{x_0}^{x_0} \int_0^{L_{cs}} J(x) dz \, dx. \quad (40)$$

Finally, the reaction flux density is calculated as follows:

$$B_{cs1}(x) = B_{max} \left( \frac{\cosh[m h_{m}/2]}{\cosh[m \tau_p/2]} - e^{m(\tau_p - h_{m}/2)} \right) e^{m x} \quad (41)$$

$$B_{cs2}(x) = B_{max} \left( \frac{\cosh[m h_{m}/2]}{\cosh[m \tau_p/2]} - 1 \right) \quad (42)$$

$$B_{cs3}(x) = B_{max} \left( \frac{\cosh[m h_{m}/2]}{\cosh[m \tau_p/2]} - e^{m(\tau_p - h_{m}/2)} \right) e^{m x} \quad (43)$$

Having $B_{cs}(x)$ determined, $B_z(x)$ is calculated by (31).

D. Developed Torque

The developed torque can be determined using the total ohmic losses dissipated in the CS [14], [15], [18], given by

$$T = P/\omega = (L/\sigma) \int |J(x)|^2 dx \, dz. \quad (44)$$

Substituting (30) into (44) yields

$$T = \sigma v_p L_{cs} R_{av} B_{max}^2 \left\{ 2 \cosh^2[m h_{m}/2] \times \tanh[m \tau_p/2] - \sinh[m h_{m}] \right\}. \quad (45)$$

E. Accounting for Actual Current Paths

The 3-D corrections of the equivalent 2-D geometry are carried out herein. The CS is linearized as shown in Fig. 7, in which the real current paths flowing within the central region and overhang regions are depicted. The sinusoidally assumed traveling flux density wave in the central region is as follows:

$$B_z(x) = B_1 \cos \left( \left( \frac{p}{2} \right) x - vt \right) / R_{av}. \quad (46)$$

Thus, the induced current density is calculated as follows:

$$J_y(x) = R_{av} \sigma \omega B_1 \cos \left( \left( \frac{p}{2} \right) x - vt \right) / R_{av}. \quad (47)$$

The dissipated power loss in the CS can thus be calculated as

$$P_0 = \int_{-R_{av} \pi}^{R_{av} \pi} |J_y(x)|^2 dx. \quad (48)$$

Finally, the flux density wave in the central region is as follows:

$$F_{2D} \left( \frac{x}{y} \right) = \int_{-\tau_p}^{\tau_p} \int_0^{L_{cs}} J(x) dz \, dx = \int_{x_0}^{x_0} \int_0^{L_{cs}} J(x) dz \, dx. \quad (40)$$

Fig. 7. Actual current paths in the CS.

Fig. 8. $B-H$ characteristic of the utilized iron.

The current density distribution is determined by [22]

$$\partial J_x/\partial y - \partial J_y/\partial x = - \sigma \partial B_z/\partial t \quad (49)$$

$$\partial J_x/\partial y + \partial J_y/\partial x = 0. \quad (50)$$

The power loss is obtained as

$$P = \int_{-R_{av} \pi}^{R_{av} \pi} \int_{-L/2}^{L/2} |J_y(x,y)|^2 dy \, dx = K_s P_0 \quad (51)$$

where $J_{y1}$ is calculated by solving (49) and (50) as in [22]

$$K_s = 1 - \tanh(pL/4R_{av})/(pL/4R_{av})(1 + \lambda) \quad (52)$$

where $\lambda$ is the overhang coefficient defined by

$$\lambda = \tanh(pL/4R_{av}) \tanh(\alpha_L pL/4R_{av}) \quad (53)$$

where $\alpha_L = 2H/L$. The developed torque can then be modified through the correction of the CS conductivity [18] as follows:

$$\sigma' = K_s \sigma. \quad (54)$$

F. Design Considerations

The maximum current density in the CS should be limited [18] to prevent excessive temperature rise. Given the details in the Appendix, the average current density in CS is calculated as

$$J_{av} = (2\sigma v B_{max}/m \tau_p) \times \left( \cosh[m h_{m}/2] \times \tanh[m \tau_p/2] - \sinh[m h_{m}] \right). \quad (55)$$

The following relationship should also be satisfied to limit the flux flowing into the secondary yoke and avoid saturation:

$$L_{ys} \geq \varphi_{ys}/L_{B_{knee}}. \quad (56)$$

Moreover, it is essential to limit the ratio of the field intensity inside the PMs $H_{m}'$ to its coercivity $H_c$, in order to avoid irreversible demagnetizations [20]. Also

$$H_{m}' / H_c = 1 - B_{m}' / B_r. \quad (57)$$

III. FEM AND EXPERIMENTAL EVALUATION

The $B-H$ curve of the utilized iron is shown in Fig. 8. Fig. 9 illustrates the magnetic field at stationary condition, and the reaction field and the total magnetic field in the air gap at a speed of 400 r/min. A meshed model of the employed 3-D
Fig. 9. (a) Air-gap flux density at a speed of 0 r/min and reaction field at a speed of 400 r/min. (b) Total air-gap flux density at a speed of 400 r/min.

Fig. 10. Three-dimensional FEM. (a) Meshed model. (b) Flux density distribution on the surface of the iron and the PM parts. (c) and (d) Flux density distribution on the surface of the CS at speeds of 0 and 400 r/min, respectively. (e) and (f) Current density distribution and vectors in the CS.

FEM is presented in Fig. 10(a). As shown in Fig. 10(b), the flux density within the secondary iron behind the interpolar region corresponding to $R_{ys1}$ is 1.5 T, i.e., the knee point of the $B-H$ curve, as designed. The magnetic flux density distributions on the surface of the CS at speeds of 0 and 400 r/min are shown in Fig. 10(c) and (d), respectively. The actual current density distribution and vectors in the CS are provided in Fig. 10(e) and (f).

Fig. 11 presents the prototyped device and the experimental setup including a dc motor (TERCO; 0.9 kW; 2820 r/min; 160 V and 7.5 A for rotor; 190 V and 0.3 A for excitation; made in Sweden) as the prime mover and a torque sensor (TorqStar; Model No. 1942H502B07090A; made in USA). The characteristics of the average current density in the CS, together with the developed torque versus relative speed, are depicted in Fig. 12, whereby the operating region of the device, i.e., the limitation of the maximum torque of 2 N·m via the maximum current density of 50 A/mm$^2$ can be determined. Also, the ratio of $H_{m}/H_{c}$ has a satisfactory value of 0.28 that provides a very good demagnetization withstand capability. A close agreement between the analytically calculated, experimental, and FEM results can also be observed. Finally, the proposed model is compared to an approach similar to the existing subdomain-based model [15], in which the permeability of iron is assumed infinity and the relative recoil permeability of the PMs is considered as unity. As illustrated in Fig. 13, in the cases of either employing a smaller yoke thickness or utilizing PMs with lower coercivities, the inaccuracy problems become more considerable in the existing model.

Finally, the developed torque of the IPM coupler is compared to the one with surface PMs and having the same dimensions and PM volume, as shown in Fig. 14(a). Using the 3-D FEM whose meshed model is depicted in Fig. 14(b), at a relative speed of 500 r/min, the corresponding developed torques are 1.5 and 1.9 N·m, respectively. Although the IPM structure
offers several advantages, it is seen that its torque is 25% lower than the one with surface PMs. In other words, to transfer the same torque, the relative speed (speed drop from primary to secondary) in the IPM structure will be somewhat higher. It is actually due to the inherent leakage fluxes in the IPM structures, particularly from the back of the primary part. However, since the demagnetization effects due to reaction fluxes and temperature rise do not appear in the IPM structure, it can be more efficient in the long time. However, by using a double-sided conductive part, this issue could be overcome. In that case, two conductive parts that are fixed together will be placed on both sides of the PM part.

IV. CONCLUSION

In this paper, by combining the MEC techniques and Faraday’s and Ampere’s laws, an analytical model has been established for IPM axial-flux eddy-current couplers, which shows the ability to easily deal with complex geometries having no symmetry, such as the studied coupler. Originated from the intrinsic flexibility of the implemented MEC, a fast, yet accurate design framework has been provided that accounts for all 3-D parameters and material properties, including permeability and saturation of the iron parts, and PM specifications. The 3-D FEM has also been employed in the analyses as well as verification of the model. A prototype was finally constructed to validate the results. Superiorities of the proposed approach over the existing model is revealed. The IPM coupler shows a number of magnetic and mechanical advantages over the couplers with surface-mounted PMs. All taken together offer the designers an effective framework that could be utilized in the preliminary design stages.

It is worth noting that, for high-torque couplers, a thermal analysis may be employed as an effective tool.

APPENDIX

Expanding (32) yields

$$2g_eB_{cs}/\mu_0 = \int_{x_1}^{x_2} \frac{\sigma v(B_m + B_{cs})}{L_{cs}} dx.$$  \hspace{1cm} (A.1)

Constants of $B_{cs}$ and $x_0$ are obtained as follows:

$$k_1 = B_{\text{max}} \left( e^{-m x_0} - e^{(\tau_p - h_m)/2} \right)$$  \hspace{1cm} (A.2)

$$k_2 = B_{\text{max}} e^{-m x_0}$$  \hspace{1cm} (A.3)

$$k_3 = B_{\text{max}} \left( e^{-m x_0} - e^{(\tau_p - h_m)/2} \right)$$  \hspace{1cm} (A.4)

$$x_0 = - \frac{1}{m} \ln \left( \frac{\cosh \left( \frac{m h_m}{2} \right)}{\cosh \left( \frac{m \tau_p}{2} \right)} \right).$$  \hspace{1cm} (A.5)

Equation (55) is obtained by substituting (30) into the following:

$$J_{av} = \int_{0}^{-\tau_p/2} J(x) dx / L_{cs} \tau_p.$$  \hspace{1cm} (A.6)

REFERENCES


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