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What is This?
A methodology for determination of extended strain-based forming limit curve considering the effects of strain path and normal stress

R Hashemi1, K Abrinia2 and G Faraji2

Abstract
In this paper, an approach based on the modified Marciniak-Kuczynski (M-K) method for computation of an extended strain-based forming limit curve (FLC) is presented. An extended strain-based FLC is built based on equivalent plastic strains and material flow direction at the end of forming. This curve has some advantages in comparison with other necking criteria such as the traditional FLC and also the stress-based FLC. This new criterion is much less strain path dependent than the conventional FLC. Furthermore, the use and interpretation of this new curve is easier than the stress-based FLC. The effect of strain path on the predicted extended strain-based FLC is reexamined. For this purpose, two types of pre-straining on the sheet metal have been imposed. Moreover, the plane stress state assumption is not adopted in the current study. The verifications of the theoretical FLCs are performed by using some available published experimental data.

Keywords
Forming, sheet, simulation, strain path

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Introduction
A conventional forming limit curve (FLC) in terms of major and minor strains is applied to predict necking in sheet metal forming processes.1 This curve can be obtained experimentally and numerically.2 There are different theoretical methods for obtaining strain-based FLCs in the literature.3,4 Marciniak and Kuczynski5 presented the most well-known method (M-K method) to calculate sheet metal forming limits. There are many researches6–9 which used the M-K method to obtain the FLC. There are several factors such as the selection of yield functions, material constants, temperature and strain rate which affect the determination of the FLC.7–11 The effects of strain path on the FLCs were examined by applying different bilinear strain paths (non-proportional loading histories). The FLCs are strain path dependent and therefore are not valid for the formability evaluation of sheet metal forming processes undergoing non-proportional loading paths.11–14

Recently, the stress-based FLC12–15 has received increasing attention as an alternative to the conventional strain-based FLC. It has the advantage of not depending on deformation paths, and can be conveniently converted from traditional strain-based FLCs. As articulated by Stoughton and Zhu,12 the deformation paths are naturally embedded in the final state of stresses because of the incremental nature of plastic flow. The characterization of forming limits with final stresses does not alter any theoretical or simulative aspects of sheet metal formability. Instead, it should be interpreted as a change of observation platforms from which the deformation paths are no longer needed to be considered. Despite the attractiveness of a stress-based FLC, it has so far found limited applications in industrial practice. Several factors work against its wider acceptance. First, the classic FLC is deeply rooted in the practical world of circle grid analysis on the shop floor. Second, stresses are a more abstract concept and cannot be measured in any practical way when looking at a...
formed part. Third, the accuracy in stress prediction is less reliable than for strain, even with the tremendous advances in numerical technology for sheet metal forming over the past decade. Additionally, the use of dynamic explicit algorithms for forming simulations generates stress oscillations due to dynamic effects and switching of contact conditions. Finally, the sheet metal often experiences cyclic bending in addition to the dominant in-plane bi-axial deformation during the forming process. The resulting stress differential is much wider than the strain differential across the sheet metal thickness; this adds to the challenge of selecting the appropriate stress values for evaluating against stress-based FLCs.

The prediction of the sheet metal strain and stress limits generally assumes plane stress conditions. This hypothesis is only valid for processes with negligible out of the plane stresses such as open die stamping. So, a few works\textsuperscript{16,17} have investigated the effect of through-thickness compressive normal stress in the prediction of the FLC. It is shown that the sheet metal limit strains increase when the through-thickness compressive normal stress increases.

Since necking in some processes such as hydroforming can occur at locations where, in addition to the in-plane stresses, a through thickness compressive normal stress acts, the plane stress assumption is not proper for these processes. So, in this work, the effect of the through-thickness normal stress is considered in the prediction of the extended strain-based FLC. There are some differences between the present approach to consider the effect of a through-thickness compressive normal stress and the previous works.\textsuperscript{16,17} In Smith and his co-worker\textquotesingle s model,\textsuperscript{16} the FLC was predicted as a function of $\gamma$ which was the ratio of $\sigma_3$ to $\sigma_1$. (see Figure 1). As the value of $\sigma_1$ was different at different points on the FLC, the $\sigma_3$ value (or the normal pressure) was not the same on different points on the same FLC. To make this statement clearer, let us consider a drawing process (in hydroforming) where the normal stress acts, the plane stress assumption is not proper for these processes. So, in this work, the effect of the through-thickness normal stress is considered in the prediction of the extended strain-based FLC.

Different from the traditional FLC, the new FLC was constructed based on effective strains and material flow direction at the end of forming (e.g. an extended strain-based FLC). This criterion combined the advantages of FLSC for its path-independence and the conventional strain-based FLC for its easy interpretation. In this study, the modified M-K model for the computation of the extended strain-based FLC is developed. Two types of pre-straining on the sheet metal have been imposed to reexamine the effect of stress path on the predicted extended strain-based FLC. The verifications of the computed results are done by using some available published experimental data.

**Extended strain-based FLC**

As mentioned earlier, the FLCs are strain path dependent. So this curve cannot be applied to analyze sheet metal-forming process taken under non-linear strain paths. Therefore, an extended strain-based FLC has been represented and this curve is much less sensitive to strain path changes than the conventional FLC.

The extended strain-based FLC is constructed based on effective strains (equivalent strains) at the onset of localized necking and material flow direction at the end of sheet metal forming.

The equivalent strain is calculated using Hill\textsuperscript{\textcopyright}'s yield criterion\textsuperscript{15} as follows

$$\bar{\varepsilon} = \sqrt{1 + \frac{r}{2} \left( \frac{\varepsilon_{22}^2}{1 + r} + \varepsilon_{11}^2 \right) + \frac{2r}{1 + r} \left( \varepsilon_{22}^2 \varepsilon_{11}^2 \right)}$$  \hspace{1cm} (1)

The current strain path $\rho$ is specified as the ratio of incremental minor strain to the major strain and is expressed as follows

$$\rho = \frac{\text{de}_2}{\text{de}_1} \quad \hspace{1cm} (2)$$

By using a yield function and associated flow rule, the strain ratio $\rho$ can be related to the stress ratio $\sigma_2/\sigma_1$.

As with traditional FLC, the extended strain-based FLC divides the condition for the onset of necking, under which supposed to be invulnerable from failure. However, this extended strain-based FLC is much more strain path independent than the traditional FLC and applicable to any forming paths under non-proportional loadings.

The extended strain-based FLC can be used to specify part quality in the press shop by measuring the principal surface strains in areas of concern. Then the strain ratios and the equivalent strain at determined locations can be calculated (e.g. from equations (1) and (2)). Since parts are manufactured through multi-stage forming processes (e.g. under non-linear strain histories), it would be necessary to manufacture parts at different intermediary steps throughout the
forming process. To build the multi-linear strain path for the specified location, the strains in the same material locations at each successive step are measured. Then all the measured effective strains display on the extended FLC. The forming process will be safe if all the measured effective strains are located under the extended strain-based FLC.

Furthermore, the extended strain-based FLC can be implemented into finite element numerical simulations to analyze and design the sheet metal forming process. Since finite element software such as ABAQUS or ANSYS can compute the strains incrementally in each element, and therefore, the strain ratio and the equivalent strain in each element can be derived at every increment of deformation. Ultimately, the equivalent strains and corresponding strain ratios for the entire strain path of each element can be determined, and the deformation process can be analyzed by comparing the equivalent strains vs. strain ratios for the final strain increment with the extended strain-based FLC.

Since necking in some processes such as hydro-forming can occur at locations where in addition to the in-plane stresses a through thickness compressive normal stress acts, the plane stress assumption is not proper for these processes. Thus, in this work, the existence of through-thickness normal stress is considered in the prediction of the extended strain-based FLC. Moreover, the effect of strain path on the predicted extended strain-based FLC is examined in this study. For this purpose, two types of pre-straining on the sheet metal have been loaded.

**Theoretical analysis**

In this study, determination of the sheet metal forming limits is based on the M-K model with some modifications on the stress states for consideration of the normal stress effects. The yield function has been used in a three-dimensional form and a modified energy equation has been used in the groove zone. The basic assumption of the M-K model is considering a narrow groove in the sheet metal surface (e.g. a geometrical inhomogeneity). It is assumed that there is a defect zone and material thickness of this area is slightly smaller than a safe zone. The zone with a nominal thickness is denoted by “a”, and the defect zone is denoted by “b”. This geometrical inhomogeneity leads to the plastic instability (localized necking) in the sheet. A schematic representation of the M-K model is illustrated in Figure 2. To model the defect zone, a fractional ratio is defined which represents the thickness ratio $f = \frac{t_b}{t_a}$, where “t” presents material thickness. As a result of loading, the defect area grows and gradually develops into a neck. Ultimately, necking is initiated from the defect zone.

To start the analysis, a small value for $dC_2$ (0.0001) is applied. Computation is initially performed on the safe zone. Then, by using the obtained information
from the safe zone, computation of the groove region is performed. It is assumed that the normal stress is a principal stress in two regions.

When $d\varepsilon_\alpha$ is determined, the equivalent strain $\bar{\varepsilon}_\alpha$ will be calculated (e.g. $\varepsilon_{new} = \varepsilon_{old} + d\varepsilon$). The equivalent stress can be obtained by using the hardening rule ($\bar{\sigma}_Y = k(\bar{\varepsilon})$). Then, using $\alpha = \sigma_2/\sigma_1$, the equivalent stress obtained by hardening law and specific yield functions (e.g. Hill’s 1948 yield function) $\sigma_1^{\alpha}$ and $\sigma_2^{\alpha}$ are calculated. Now, using the flow rule and incompressibility condition, strain increments in the safe zone are obtained

$$d\varepsilon_i = d\varepsilon_{\alpha} \frac{\Delta \bar{\varepsilon}_i}{\Delta \bar{\varepsilon}_1}, \quad i = 1, 2 \tag{3}$$

$$d\varepsilon_3 = -d\varepsilon_1 - d\varepsilon_2 \tag{4}$$

where $d\varepsilon_{\alpha}$ is defined as $d\bar{\varepsilon}$.

At this stage, by using the rotation matrix, stress and strain increment tensors at groove system of coordinates are obtained.

Using strain increments, strain values can be updated.

$$\varepsilon_{ij|new} = \varepsilon_{ij|old} + d\varepsilon_{ij} \tag{5}$$

It is seen that all the stress and strain components in the safe region are computed. Now, these components must be computed in the groove region.

Calculating of stress and strain components at the groove zone is not as simple as the safe region. Unknown parameters of this region are stress components ($\sigma_{11}^{gb}, \sigma_{12}^{gb}, \sigma_{22}^{gb}$) and strain increments ($d\varepsilon_{11}^{gb}, d\varepsilon_{12}^{gb}, d\varepsilon_{22}^{gb}$). Flow rule relates the strain increments to $\alpha_{11}^{gb}, \alpha_{12}^{gb}, \alpha_{22}^{gb}$ and $\Delta \varepsilon_{\alpha}$. Thus, the unknown parameters of the groove zone are reduced to $\sigma_{11}^{gb}, \sigma_{12}^{gb}, \sigma_{22}^{gb}$ and $\Delta \varepsilon_{\alpha}$. To obtain these unknown parameters, four equations are required. One of them is the strain compatibility equation ($d\varepsilon_{11}^{gb} = d\varepsilon_{11}^{gb}$). Two other equations are the force equilibrium equations in the groove bound. An energy equation is used as the fourth equation. In the energy equation, the component of the normal stress, $\sigma_3$, is involved.

$$\frac{d\varepsilon_{11}^{gb}}{d\sigma_{11}^{gb}} - 1 = 0 \tag{6}$$

where $f$ changes according to

$$f = \exp(e_i^2 - e_f^2) \tag{7}$$

To solve this set of non-linear equations, a numerical method is required. In this study, Newton-Raphson method is used. In this numerical method, $d\varepsilon_{11}^{gb}$ and $d\varepsilon_{12}^{gb}$ are computed in each step and compared with each other; if $d\varepsilon_{11}^{gb}/d\varepsilon_{12}^{gb} > 10$ the necking occurs and the limiting strains ($e_i^2$, $e_f^2$) are saved. This numerical process is repeated for different groove angles and stress ratios until the whole limit strains are obtained. The logic flow diagram of the numerical work is shown in Figure 3.

A significant problem to the theoretical methods such as the M-K theory is that the computed limit strains are so susceptible to the inhomogeneity coefficient, $f$. Therefore, several methods have been suggested to calibrate the computed FLC. In this study, to reduce this sensitivity, $f$ is related to the surface roughness and grain size of the sheet. Thus, $f$ is given as follow

$$f = f_0 \exp(e_i^2 - e_f^2) \tag{8}$$

$$f_0 = \frac{d_0^2 - 2(R + kd_0^{0.5}e_f^2)}{d_0^2} \tag{9}$$

In these relations, $d_0$ is the initial grain size, $k$ is the material constant, and $R$ is the initial surface roughness of the sheet.

To obtain sheet metal forming limits under non-proportional loadings, two types of pre-straining have been loaded on the sheet metal prior to obtaining its FLC. A bilinear strain path is made up of two stages. At each stage, the loading is linear and described by the following relation

$$\begin{align*}
\rho &= \rho_1 \quad \text{if} \quad e_1^0 \leq e_1^s \\
\rho &= \rho_2 \quad \text{if} \quad e_1^s > e_1^s
\end{align*} \tag{10}$$

where $\rho_1$ and $\rho_2$ specify the strain ratios that are applied, and $e_1^s$ is the pre-strain value.
Inclusion of the normal stress

For any arbitrary through-thickness compressive normal stress (e.g. $\sigma_3$) and for any arbitrary stress ratio of $\alpha$ (e.g. $\alpha = \sigma_2 / \sigma_1$), the components of in-plane principal stresses $\sigma_1$ and $\sigma_2$ are described as follow:

$$\sigma_i = -B + \sqrt{(B^2 - 4AC)}$$

where

$$A = \alpha^2 + 1 + R(1 - \alpha)^2$$
$$B = -2\sigma_1(\alpha + 1)$$
$$C = 2\sigma_3^2 - \frac{2}{3} * (2 + R) * \sigma_2^2$$

Table 1. Mechanical properties.

<table>
<thead>
<tr>
<th>Material</th>
<th>$K$ (MPa)</th>
<th>$n$</th>
<th>$m$</th>
<th>$\varepsilon_0$</th>
<th>$r$</th>
<th>$t$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST127</td>
<td>238</td>
<td>0.3</td>
<td>0.01</td>
<td>0.01</td>
<td></td>
<td>1.21</td>
</tr>
<tr>
<td>ST14</td>
<td>505</td>
<td>0.26</td>
<td></td>
<td></td>
<td></td>
<td>1.20</td>
</tr>
<tr>
<td>AA6011</td>
<td>254.9</td>
<td>0.265</td>
<td></td>
<td></td>
<td></td>
<td>0.574</td>
</tr>
<tr>
<td>STKM-11A</td>
<td>1500</td>
<td>0.15</td>
<td></td>
<td></td>
<td></td>
<td>2.14</td>
</tr>
</tbody>
</table>

Figure 3. Logic flow diagram of analysis.
Model validations

At first, verifications of theoretical results have been done by comparing results of the present work with some published experimental data for ST12 low carbon steel, ST14 low carbon steel, Al6111-T4, and STKM-11A steel alloy under the plane stress conditions and when the through-thickness compressive normal stress has been considered. Moreover, the computed FLCs under non-linear strain paths have been validated. Table 1 presents the mechanical properties of these materials and Table 2 presents material constants used in the inhomogeneity coefficient function, $f_0$. Equation (13) is used to calibrate the FLCs of ST12 low carbon steel and the other materials used in this study; the imperfection factor $f_0$ is used as a factor for calibration. Then the effects of strain path on the extended FLC are investigated theoretically.

To evaluate theoretical results obtained based on the modified M-K method in this study, several experimental data are selected. Figure 4(a) illustrates a comparison between the theoretical FLC and the experimental data obtained under proportional loading and under plane stress state ($\sigma_3 = 0$).

![FLCs under proportional loading and plane stress condition: (a) ST12 low carbon steel and (b) ST14 low carbon steel.](image)

Table 2. Material constants using in the relation 13.

<table>
<thead>
<tr>
<th>Material</th>
<th>$k$</th>
<th>$d_0$ (mm)</th>
<th>$R$ (mm)</th>
<th>$t_0$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ST12</td>
<td>0.104</td>
<td>0.025</td>
<td>0.006</td>
<td>2.5</td>
</tr>
</tbody>
</table>
The predicted FLC is in reasonable agreement with the experimental data for as-received ST12 low carbon steel. Figure 4(b) illustrates the computed and experimentally determined FLCs for the ST14 low carbon steel obtained under proportional loading and under plane stress state. This figure illustrates a suitable agreement between FLC prediction based on the modified M-K method and experimental FLC determined without pre-straining and for the plane stress conditions. To determine the FLCs experimentally, the out-of-plane Nakazima test was performed. This test involves two stages: fixing the sample using circular die rings as the blank-holder, and stretching the sample using a hemispherical punch. A hydraulic press was employed to provide the required force. In Hashemi et al., to obtain the right-hand side of the FLC and generate all required strain states, different sample geometries and lubricants were used.

The theoretical and experimental FLCs of ST12 low carbon steel determined after pre-straining in uniaxial and biaxial tension parallel to the prior rolling direction are shown in Figures 4 and 5. Figure 5(a) and (b) illustrates the computed and experimental FLCs after tensile pre-strain levels of $\varepsilon_{1s}^{0} = 0.1$ and $\varepsilon_{1s}^{0} = 0.15$, respectively. Figure 6 shows the computed and experimental FLCs after biaxial pre-strain levels of $\varepsilon_{1s}^{0} = 0.08$. The predicted FLCs for ST12 low carbon steel under non-proportional loadings (e.g., Figures 5 and 6) show almost good correlation with experiments.

To verify the computed FLCs with the existence of a through-thickness compressive normal stress ($\sigma_3 \neq 0$), some published experimental data are chosen. Figure 7(a) shows the computed and experimental FLCs for AA6011 aluminum tubes. In Hwang et al., the corresponding experimental process was tube hydroforming with up to $-12$ MPa.

**Figure 5.** (a) FLCs after uniaxial tensile pre-strain of $\varepsilon_{1s}^{0} = 0.1$ and (b) FLCs after uniaxial tensile pre-strain of $\varepsilon_{1s}^{0} = 0.15$.

**Figure 6.** FLCs after equi-biaxial pre-strain of $\varepsilon_{1s}^{0} = 0.08$. 
internal pressure (which corresponds to \(\sigma_3 = -6\) MPa).

In the present study, the constitutive model uses Hill’s quadratic anisotropic yield criterion. However, it is well known that Hill’s quadratic yield criterion is not applicable to aluminum alloys, and Karafillis-Boye yield function\(^{25}\) or one of Barlat’s yield criteria (e.g. Yld\(^{26}\)) would be more appropriate. Nevertheless, the present results (see Figure 7(a)) show a reasonable agreement with experimental data.\(^{23}\)

Figure 7(b) illustrates the computed and experimental FLCs for STKM-11A steel alloy with the existence of a through-thickness compressive normal stress of \(\sigma_3 = -56\) MPa. Kim et al.\(^{24}\) carried out a series of bulge tests and the tubes were supported between a lower and an upper die. In the present numerical study, since the outer surface is supported by the die, the same value of internal pressure \(-56\) MPa has been applied \((\sigma_3 = -56\) MPa\). In Kim et al.\(^{24}\) there were no experimental data for the right-hand side of the FLC.

However, the difference between theoretical results and experiment may be due to errors in measuring the strains; because the experimental data used to evaluate the present approach were determined by the conventional circle grid analysis method. Therefore, the errors originating from this method inevitably led to some variability in the results.

To show the influence of strain path on the extended FLC, based on the present modified M-K method, a series of FLCs with and without pre-strains are generated (e.g. see Figures 4–6). ST12 low carbon steel is selected in this example. As illustrated in Figure 4(a), a traditional FLC with linear strain path is determined theoretically. Then, as illustrated in Figure 5(a) (b), two FLCs are computed under 0.10 and 0.15 uniaxial pre-tensions. The limit strains shift to the left-hand side and increase as the pre-strain level increases. A FLC is also computed in the same way under 0.08 equi-biaxial pre-strain as illustrated in Figure 6. In this mode of pre-straining, limit strains change to right-hand side and decrease as pre-strain levels increase. This trend is consistent with the experimental testing data.\(^{7,22,27}\)

As expected, Figures 5(a) and (b) and 6 illustrate the strain path dependent nature of the traditional

**Figure 7.** (a) FLCs for AA6011 with the existence of a through-thickness compressive normal stress \((\sigma_3 = -6\) MPa\) and (b) FLCs for STKM-11A with the existence of a through-thickness compressive normal stress \((\sigma_3 = -56\) MPa\).
Figure 8. (a) The theoretical FLCs for different loading paths and (b) The theoretical extended strain-based FLCs for different loading paths.

Table 3. The comparison between FLC, FLSC, and extended FLC.

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Forming Limit Curve (FLC)</td>
<td>• Easy interpretation</td>
<td>• Sensitive to strain path changes</td>
</tr>
<tr>
<td></td>
<td>• FLC widely used to assess localized necking in sheet metal forming process due to its straight forwardness and convenience of measuring deformation strains either physically in forming tests or simultaneously in finite element modeling</td>
<td></td>
</tr>
<tr>
<td>Forming Limit Stress Curve (FLSC)</td>
<td>• FLSC has the advantage of not depending on strain paths</td>
<td>• Limited applications in industrial practice, since stresses are a more abstract concept and cannot be measured in any practical way when looking at a formed part and the accuracy in stress prediction is less reliable than for strain, even with the tremendous advances in numerical technology for sheet metal forming over the past decade.</td>
</tr>
<tr>
<td></td>
<td>• FLSC can be conveniently converted from traditional strain-based FLC</td>
<td></td>
</tr>
<tr>
<td>Extended strain-based Forming</td>
<td>• EFLC has the advantage of not depending on strain paths</td>
<td>–</td>
</tr>
<tr>
<td>Limit Curve (EFLC)</td>
<td>• The use and interpretation of EFLC is easier than FLSC</td>
<td></td>
</tr>
</tbody>
</table>
FLCs. So, recently, a new strain-based FLC has been introduced, and this new criterion is strain path independent, unlike the traditional FLCs.

Now, the strain path independency of an extended strain-based FLC is reexamined by converting the traditional FLCs in Figure 8(a) to the extended FLC. As shown in Figure 8(b), all the FLCs at different loading paths are degenerated to one single curve. So the present results confirm that the extended strain-based FLC is strain path independent, unlike the traditional FLC.

The comparison between FLC, forming limit stress curve, and extended strain-based FLC is shown in Table 3. As one can see, this new curve (e.g. extended FLC) has some advantages in comparison with other necking criteria such as the traditional FLC and also the stress-based FLC.

Conclusions

In this article, the strain path independency of the extended FLC was reexamined. The obtained results showed that the extended FLC is much more strain path independent than the conventional FLC. From this point of view, the extended FLC is similar to the stress-based FLC. But the stress-based FLC has some practical limitations in the industry. Thus, the extended FLC is a better criterion because this curve is almost strain path-independent similar to the FLSC, and it is easy to interpret and use in industrial practice. Moreover, the plane stress state assumption is not adopted in the current study.

Funding

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References


**Appendix**

**Notation**

- $f_0$: initial imperfection factor
- $f$: imperfection factor or fractional ratio
- $d_0$: initial grain size
- $K$: strength coefficient
- $m$: strain rate sensitivity exponent
- $n$: strain hardening exponent
- $r$: normal anisotropy coefficient
- $R_0$: initial surface roughness
- $t$: material thickness
- $\alpha$: in-plane principal stresses ratio ($\sigma_2/\sigma_1$)
- $\dot{\varepsilon}$: rate of effective plastic strain
- $\dot{\varepsilon}_p$: effective plastic strain
- $\varepsilon_0$: pre-strain value
- $d\varepsilon$: strain increment tensor
- $d\theta_1, d\theta_2, d\theta_3$: strain increments in the material coordinates
- $d\theta_{hit}, d\theta_{hit}, d\theta_{hit}$: strain increments in the groove coordinates
- $d\lambda$: plastic multiplier
- $\theta$: groove angle between the groove coordinates and the material coordinates
- $\gamma$: ratio of $\sigma_1$ to $\sigma_1$
- $\rho$: ratio of incremental minor strain to the major strain
- $\sigma_1, \sigma_2, \sigma_3$: principal stress components
- $\sigma_{hit}, \sigma_{hit}, \sigma_{hit}$: in-plane stress components in the groove coordinates
- $\bar{\sigma}_y$: effective stress obtained from hardening law
- $\tilde{\sigma}_y$: effective stress obtained from yield function