Seismic Behavior Prediction of Flanged Unreinforced Masonry (FURM) Walls

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Seismic Behavior Prediction of Flanged Unreinforced Masonry (FURM) Walls

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In most available studies, unreinforced masonry (URM) walls are idealized as rectangular sections, while in reality walls have effective sectional shapes such as C, I, T, and L. In this article, the results of experimental and analytical assessment of flange effects on the behavior of I- and C-shaped URM walls are reported. Four clay brick walls at half scale were tested. Two specimens were designed with I- and C-shaped sections, and for comparison, two additional specimens were designed without flanges. The tests showed that under constant axial load the strength of the I-shaped wall increases, but that of the C-shaped wall decreases, because of out-of-plane distortion effects. Despite the loss of strength, both flanged walls indicated almost similar initial stiffness, deformation capacity, and mode of failure in comparison with walls without flanges. A mixed-mode analytical model is proposed to predict the lateral force displacement curve of flanged URM (FURM) walls. The proposed analytical model is based on section analysis of the walls and shows good agreement with previous experimental results.

Keywords Unreinforced Masonry Walls; Rectangular Section; Flanged Section; Experimental; In-Plane Behavior; Diagonal Tension Failure

1. Introduction

Many unreinforced masonry (URM) buildings have been constructed in earthquake-prone regions worldwide. The results of past disaster earthquakes have shown that most of existing URM buildings are susceptible to suffer heavy damages. In Iran more than 70% of residential buildings have been constructed using URM with weak shear strength. These buildings have limited strength and ductility against lateral forces, and are expected to sustain heavy damages under moderate and strong earthquakes [Jafari et al., 2005; Fallahi et al., 2003]. In reality, the behavior of masonry structures are more complex than other structures and exploring of expected behavior needs more experimental test and calibrated analytical procedures. Hence, the assessment of URM elements behavior under lateral demands has been the subject of numerous experimental studies. [Epperson and Abrams, 1989; Abrams and Shah, 1992; Magenes and Calvi, 1992; Manzouri et al., 1995; Anthoine et al., 1995; Yi., 2004; Moon., 2004; Lee et al., 2008; Calderini et al., 2009]. Based on previous investigations, the failure modes of URM walls are established on four well-known behaviors or their combination: rocking failure, toe crushing, sliding shear failure, and diagonal tension failure. These failure modes are affected by material
properties, wall aspect ratio (height/length), axial stress, boundary conditions, thickness, and bond pattern of masonry wall.

In addition to experimental researches, several analytical approaches have aimed to quantify the nonlinear in-plane response of URM walls [McDowell et al., 1956; Abrams, 1992; Magenes and Calvi, 1997; Tomazevic, 1999; Yi et al., 2005; Penelis, 2006]. For practical purposes, current guidelines and pre-standards such as FEMA-356 ASCE-41 and NZSEE 2006 provide design equations for recognizing failure modes and corresponding lateral strength and displacement capacity of URM walls.

In most previous studies, URM walls had been idealized as rectangular sections, but in reality, walls in URM buildings are integrated to each other and expected to behave as flanged walls such as C, I, L, and T. Several previous experimental researches at the structural level have highlighted the effects of transverse walls (flanges) on the in-plane response of the walls such as failure modes, maximum strength, and displacement capacity [Tomazevic et al., 1993; Abrams and Costley, 1996; Paquette and Bruneau, 2003; Moon et al., 2006].

Yi et al. [2005] conducted an experimental program on a full-scale two-stories URM building and developed an analytical model to predict the effects of flanges on the behavior of individual non rectangular L-shaped URM walls. Their investigations showed that the presence of flanges have significant effects on the four primary in-plane failure modes, and the behavior of URM walls including maximum strength, failure mode, and the lateral force-displacement responses. Their studies revealed that flange effect increases lateral strength in all failure modes except in the diagonal tension mode, which may decrease or increase lateral strength depending on the flange location. The conducted experiments by Russell and Ingham [2010] on nine walls with and without flanges also endorse the previous study results of Moon [2004] and Yi [2004]. They concluded that flanges increase the force and displacement capacity of in-plane loaded walls. It was found that for FURM walls, flexure is less likely to be a behavior mode and shear is more likely to limit the lateral strength. Russell and Ingham [2010] reported that crack orientation may be influenced by the presence of flanges in the ends of the in-plane loaded URM wall.

Despite this, few experimental programs are available regarding seismic evaluation of either C- or I-shaped walls, and, consequently, their behavior and modeling parameters have not been explicitly addressed in current codes and instructions. Findings on masonry walls significantly are affected by material properties, geometry, and design actions which are different from country to country and expanding of results need more investigations. Hence, in the current study, the seismic performances of FURM walls on both ends are investigated which have not been comprehensively addressed in literature. For better understanding of the behavior of walls and for studying weak shear strength effects on FURM walls, two half-scale specimen walls were chosen from typical prototype 3-story building and were tested in structural lab. Two specimens were designed with flanges, and for comparison, two other specimens were designed without flanges. An analytical model was proposed to predict the force-displacement response of FURM walls and calibrated against test responses. The capability of proposed model to predicting responses was examined using available experimental tests on FURM walls.

2. Effective Pier Model

Yi et al. [2005] proposed a macro model termed the effective pier model to describe the nonlinear in-plane response of individual URM walls corresponding to four primary failure modes. They assumed that wall behavior is initiated with flexure and followed by other possible modes. The possible mode is checked during the each incremental curvature.
Flanged Unreinforced Masonry Walls

Figures 1A and 1B illustrate the assumed externally applied forces and distribution of the base flexural stresses in a URM wall with flexural cracks and fix-free boundary condition. For considering the effect of discrete diagonal cracks on overall pier behavior, the smeared crack technique was assumed. Based on effective pier model’s concept the effective area of the wall remains continuous even after the development of diagonal cracks. In order to model strength degradation after diagonal tension failure, they supposed negative post yield slope equal to 10 percent of initial elastic modulus.

The effective pier model was modified by Yi et al. [2008] to investigate the flange effects on the behavior of an individual non-rectangular URM wall. The new model was developed to evaluate in-plane loaded response of URM walls with single flange and fix-free condition. For compacting paper length, the details of model are not presented here and only the key equations are summarized in Eqs. (1)–(9):

\[
V_f = \frac{P_b}{h} \left( \frac{L_a t}{2} - \frac{1}{6} L_e^2 t + (a_i - a_f) A_f \left( 1 - \frac{a_f}{L_e} \right) \right)
\]

\[
V_{tc} = \frac{P_b}{h} \left( \frac{L_{etc} t}{2} - \frac{1}{6} L_{etc}^2 t + (a_i - a_f) A_f \left( 1 - \frac{a_f}{L_{etc}} \right) \right)
\]

\[
P_b = \frac{1}{2} \beta f_m L_{etc} + A_f \beta f_m \left( 1 - \frac{a_f}{L_{etc}} \right)
\]

\[
V_s = \mu P_b + (A_f + L_{etc}) c
\]

\[
V_{dt} = tL \xi f_y \sqrt{1 + \frac{\sigma_{avg}}{f_y}} \left( 1 + \frac{W_f}{W} \right) \left( \frac{1}{2} \left( 1 + \frac{W_f}{W} \right) + 2 \frac{W_f}{W} \left( \frac{a_f}{L} - \frac{1}{2} \right)^2 \right) \left( \frac{1}{2} + \frac{W_f a_f}{W L} \right)^2 \text{ if } a_f \geq \frac{L}{2}
\]
\[ V_{dt} = tL\xi_f^d \sqrt{1 + \frac{\sigma_{avg}}{f'_d}} \left( 1 + \frac{w_f}{W} \right) \left( \frac{1}{6} \left( 1 + \frac{W}{W} \right) + 2\frac{W}{W} \left( \frac{a_f}{L} - \frac{1}{2} \right)^2 \right) \text{ if } a_f < \frac{L}{2} \]

\[ \Delta = \frac{V_r - P \tan (\theta)}{K} \]

\[ K = \frac{1}{\frac{\delta y h^3}{E L^3} + \frac{h}{G L^2}}. \]

If the flange is not within the effective area, (flange in tension zone) Eqs. (7), (8), and (9) are used:

\[ V_r = \frac{L}{h} P_b \left( \frac{a_i}{L} - \frac{L_e}{3L} \right) \]

\[ V_{tc} = \frac{L}{h} P_b \left( \frac{a_i}{L} - \frac{2P_b}{3f_m L_t} \right) \]

\[ V_s = \frac{\mu P_b + 3a_f \tau_0}{1 + \frac{3a_f \tau_0}{\mu P_b}}, \]

where \( V_r, V_{tc}, V_s, \) and \( V_{dt} \) are the lateral strength of the URM wall corresponding to rocking, toe crushing, sliding and diagonal tension, respectively; \( P_i \) is the applied axial force at the top of the wall; \( P_b \) is the applied axial force at the bottom of the wall; \( h \) is the height of the pier; \( L \) is the length of the pier; \( L_e \) is the lengths of the un-cracked sections at the bottom of the pier; \( a_i \) is the distance between inertia center and compression edge of wall; \( a_f \) is the distance between center of flange and compression edge of wall; \( t \) is the thickness of the web; \( A_f \) is the cross-sectional area of the flange; \( W \) is the weight of the in-plane wall; \( W_f \) is the weight of the flange; \( \mu \) is the coefficient of friction; \( \sigma_{avg} \) is the average vertical compressive stress at the inflection point level; \( \sigma_{cb} \) is the maximum vertical compressive stress at the bottom of the pier; \( \tau_0 \) is the initial cracking shear strength of the bed joint; \( L_{etc} \) is contact length at the base corresponding to toe crushing mode which is determined by Eq. (2b); \( f_m \) is the compressive strength of masonry; and \( \beta \) is a factor that accounts for the erroneous assumption of a linear stress distribution along the length of the pier (i.e., \( \beta \) should be larger than 1). To be consistent with the equivalent stress block analogy as outlined in MSJC 530-02, \( \beta \) should be taken as 1.28; \( c \) is the cohesive strength of masonry bed joints; \( f_d \) is the diagonal tensile strength of masonry; \( \xi \) is a shear stress factor that accounts for the erroneous assumption of a constant shear stress distribution. For slender piers \((L/h < 0.5)\), the horizontal shear stress distribution is a parabola, which implies \( \xi \) is equal to 1.5. For squat piers \((L/h > 2)\), the horizontal shear stress distribution approaches a constant value, which implies \( \xi \) is equal to 1.0; \( \tau_o \) is the initial cracking shear strength of the bed joint; \( \Delta \) is the lateral displacement in each step; \( \theta \) is angle between the central axis of the deformed pier and the vertical line; \( K \) is the instantaneous lateral stiffness of the pier; \( \gamma \) is a coefficient that describes the boundary conditions of the pier (\( \gamma \) is equal to 1.0 for fixed–fixed end conditions, and 0.5 for cantilever end conditions); \( E \) is the elastic modulus of masonry; and \( G \) is the shear modulus of masonry, which is taken as \( 0.4E \).
3. Proposed Analytical Model

In current research using expanded beam theory philosophy, displacement capacity of FURM walls is investigated both for ductile and brittle failure modes. The beam theory philosophy has been widely used to discuss normal and shear stress distribution through the section. Except deep beams, it involves linear distribution and ignores shear strain contribution. Furthermore, lateral displacement of members can be derived based on integration of curvature through length. The results of studies carried out by Yi et al. [2008] showed that applying beam theory concepts to predict masonry wall deformation capacity (both flanged and rectangular wall) is reasonable except diagonal tension failure mode. In current study the authors revise Yi et al. [2008] model for FURM walls with flanges in both ends and also new approach is proposed to predict shear-displacement curve of diagonal tension failure mode after cracking which has not previously been addressed.

Shear stress distribution in rectangular, L-, I-, and C-shaped walls adopted from beam theory are illustrated in Fig. 2. With applying an in-plane shear force V parallel to web, shear stress distribution in L-shaped wall is different from I and C-shaped and maximum shear stress is occurred in different location. In symmetric flanged sections, the shear stress varies linearly along the length of the flange from $\tau = 0$ at the ends of the flange, to a maximum at the web-flange intersection. For continuity in shear flow, the shear stress at the end of the web is equal to the maximum shear stress in the flange. The maximum shear stress at the center of the web is considerably increased regarding to flange length. The presence of flanges and increasing in shear stresses will raise the potential of shear cracks and forming of diagonal tension failure mode.

The main assumptions of proposed model are adopted from Yi et al. [2005, 2008] studies and modified accordingly for both displacement and force control behaviors. Figures 3A and 3B illustrate the general assumptions of proposed model for a typical cantilever I-shaped URM wall. $V$ is the applied shear force; $P_t$ and $P_b$ are the compressive axial forces applied at the top and the bottom of the wall, respectively; $M_b$ is the applied moment at the bottom of the wall; $W$ is the self weight of web; $W_f$ is the self weight of the flange; and $h$ is the height of the wall.

A step-by-step procedure is carried out to investigate possible failure mechanisms. At first, the procedure is initiated by flexure at base and continued by checking all possible other modes. In each step, an incremental curvature is imposed and relevant internal stress and failure criteria are checked to identify the governed failure mode and relevant displacement response. In the following sections, the details of identifying four primary failure modes, mixed mode, and performing of load-displacement curve are presented.

![FIGURE 2 Shear stress distribution through URM walls with and without flange for a shear force applied parallel to the web.](image-url)
FIGURE 3 (A) Applied external forces on I-section URMW with fix-free end conditions; (B) base flexural strain and curvature of I-section URM wall.

3.1. Rocking

Rocking failure mode usually occurs in walls with high aspect ratio (height to length) and low axial load ratios. In rocking mode, flexural cracks initiate at the base and develop through the height but not more than the few first rows. As the displacement increases the wall deforms and rotates about the compressive toe. Once the loading is reversed, the flexural cracks attempt to close and the wall behaves as an un-cracked wall until the flexural cracks be open in opposite side. The deformation capacity of a rocking control wall is limited by toe crushing or a local diagonal shear crack at higher displacement demands. Figure 3B shows the behavior of wall at the base and provided assumptions in rocking mode. Using linear strain distribution and relationship between stress-strain and equilibrium equation at the section of an FURM wall, the effective length is calculated.

In this regard and for simplicity, the tensile strength of masonry normal to bed-joint is assumed to be zero ($f_t = 0$). Therefore, the normal flexural strain $\varepsilon_x$ in section and effective length $L_e$ have linear relationship per each step as indicated in Eq. (10):

$$\varepsilon_x = L_e \tan (\phi_x), \tag{10}$$

In order to solve unknown strain, the section’s equilibrium equations of force and moment are used. The vertical force and the flexural moment in each step are expressed in terms of the internal vertical stresses as:

$$P_b = \frac{1}{2} L e t_w \varepsilon_x E + A_f \varepsilon_x E \left( 1 - \frac{t_f}{2L_e} \right) \tag{11}$$

$$M_b = \frac{1}{12} L^2 e t_w \varepsilon_x E + \frac{1}{2} L e t_w \varepsilon_x E \left( \frac{L}{2} - \frac{L_e}{2} \right) + A_f \varepsilon_x E \left( 1 - \frac{t_f}{2L_e} \right) \left( \frac{L}{2} - \frac{t_f}{2} \right). \tag{12}$$
Solving Eqs. (11) and (12) simultaneously and substituting in \( M_b = V \times h \), the lateral strength of the wall in terms of vertical force and the effective length at the bottom of the wall is calculated:

\[
V_r = \frac{P_b}{h} \left( \frac{t}{2} L_{et} t + \frac{1}{2} L_{et}^2 t + (L - t_f) A_f \left( 1 - \frac{t_f}{2L_{et}} \right) \right) \left( Le t + 2A_f \left( 1 - \frac{t_f}{2L_{et}} \right) \right),
\]

where \( V_r \) is the lateral shear of the URM wall corresponding to rocking mode; \( P_b \) is the applied axial force at the bottom of the wall including to weight of flanges, web, and external applied force; \( t \) is the thickness of the web; and \( t_f \) is the thickness of the flange. The other parameters were defined in the previous parts.

### 3.2. Toe-Crushing

As noted by Yi et al. [2005] toe crushing is the upper limit for the rocking behavior. Crushing occurs when the maximum compressive stress at the compression toe reaches to maximum compressive strength at the masonry toe:

\[
\sigma_{n,\text{max}} = \beta f_m,
\]

where, \( \sigma_{n,\text{max}} \) is the maximum compressive stress; \( f_m \) is the compressive strength of masonry; and \( \beta \) is a factor accounting for the nonlinear vertical stress distribution, with a value equal to 1.3 according to MSJC 530-02. Using Eqs. (11), (13), and (14) and solving for \( P_b \) and \( L_{etc} \), the lateral shear corresponding to toe crushing mode as presented in Eq. (16) is achieved:

\[
P_b = \beta f_m \left( \frac{1}{2} L_{etc} t_w + A_f \left( 1 - \frac{t_f}{2L_{etc}} \right) \right)
\]

\[
V_{tc} = \frac{\beta f_m}{2h} \left[ \frac{1}{6} L_{etc}^2 t + \frac{1}{2} L_{etc} t_w (L - L_{etc}) + A_f (L - t_f) \left( 1 - \frac{t_f}{2L_{etc}} \right) \right].
\]

In the I- or C-shaped walls, with increasing displacement demand, one flange is expected to be in tension, while the other flange is in compression. The presence of compression flange helps to balance the amount of compressive stress at the toe of the wall. This effect delays the kick of toe crushing failure mode which is known as a brittle mode. It is noted that in C- and I-shaped walls one flange is already in compression and the other is in tension, hence the behavior of wall will be symmetric in pull and push direction; however, it differs in L- and T- shaped walls, which means Eqs. (7)–(9) should be used.

### 3.3. Bed-Joint Sliding

The results of the experimental researches conducted on URM walls have proved that bed-joint sliding failure can occur after developing of a long flexural crack in horizontal bed joints. This procedure is accelerated with decreasing in applied vertical force and mortar shear strength. Bed-joint sliding occurs when:

\[
\tau_{\text{avg}} = \tau_u,
\]
where \( \tau_{\text{avg}} \) is the average shear stress calculated on the effective section of the bed joint and \( \tau_u \) is the maximum bed joint shear strength. For calculations related to the bed joint sliding limit state, the shear stress is assumed to be plastic and uniformly distributed. This approach is widely adopted in design and assessment of masonry structures. The lateral shear of sliding mode \( (V_s) \) can be calculated as:

\[
V_s = \mu P_b + (A_f + L e) c,
\]

where \( \mu \) is the coefficient of friction and \( c \) is the cohesive strength of masonry bed joints.

### 3.4. Diagonal Tension

A diagonal tension failure is assumed to occur when the maximum principal tensile stress at the reference point (usually calculated in the center of the wall) is equal to the diagonal tensile strength of the masonry (see Fig. 4A). In this case:

\[
\sigma_1 = f_d'
\]

where \( \sigma_1 \) is the maximum principal tensile stress and \( f_d' \) is the diagonal tensile strength of masonry and is calculated using the following stress transformation equation:

\[
\sigma_1 = \sqrt{\left(\frac{\sigma_{\text{avg}}}{2}\right)^2 + \left(\frac{\tau_d}{\xi}\right)^2} - \frac{\sigma_{\text{avg}}}{2}.
\]

\( \tau_d \) is the maximum shear stress at the reference point calculated based on beam theory concept is modified for uneven distribution based on aspect ratio:

\[
\tau_{d,\text{max}} = \frac{3V b L_i^2 - b L_i^2 + t L_i^2}{2t b L_i^3 - b L_i^3 + t L_i^3}.
\]

**FIGURE 4** (A) Shear strength of wall and diagonal tensile strength of masonry; (B) Assumed zig-zag crack pattern; (C) normal stress distribution through crack path.
The maximum diagonal tension strength of a URM wall can be calculated by substituting Eq. (21) into Eq. (20), and solving for lateral shear. The result is shown in Eq. (22):

$$V_{dt} = \frac{2t}{3\xi f_d} \sqrt{1 + \frac{\sigma_{avg} bL^3 - bL_1^3 + t_w L_1^3}{\frac{f_d}{f_d^3} bL^2 - bL_1^2 + t_w L_1^2}}$$

All parameters were previously defined except $b$ that is the width of flange as shown in Fig. 4A. It is important to note that the authors’ test observations in cantilever walls with diagonal tension failure mode showed that, the inclined hairline cracks usually initiate at vicinity of toes at web-flange intersection; but do not develop in subsequent loading history. This behavior is deduced from combination of flexural and shear stresses at web-flange intersection. Therefore in current research, center of wall was assumed for calculating of shear failure criteria and flexural stresses effects were ignored.

4. Mixed Mode Failure: Sliding after Diagonal Tension (Diagonal Sliding)

The authors’ experimental observations indicated that after formation of diagonal tension cracks, the sliding between masonry units and mortar is occurred along crack path through bed joints and head joints. This behavior is highly anticipated when weak mortar shear strength had been provided in wall construction. This behavior can be considered semi-displacement-controlled action and therefore limited displacement capacity is expected beyond diagonal shear failure. In contrast where high quality mortar and bricks are used, the inclined cracks are passed through both bricks and mortar, and there will be no further displacement capacity beyond the point where cracking occurs. This behavior is classified as a brittle mode and considered force-controlled action. In the case where mortar is weaker than the bricks, cracking due to diagonal tension (diagonal sliding, ds) will pass through the mortar and a stair-stepped failure mechanism will occur. This phenomenon has also been reported by other researchers [Magenes and Calvi, 1997; Calderini et al., 2009, 2010; Cattari and Lagomarsino, 2009; Russell et al., 2011]. In current research, this failure mode is called mixed-mode. Although the discussed concept seems simple but it is not always easy to distinguish the occurrence of a specific type of mechanism, as many interactions may occur between them; hence the discussed failure mode is not explicitly addressed in current design and rehabilitation codes and instructions, e.g., FEMA-356, ASCE-41, and NZSEE 2006. In many existing URM buildings around the world as well as Iran, the quality of mortar is usually weaker than the quality of bricks, and a zigzag cracking pattern is likely, as illustrated in Fig. 4B. In Fig. 4C, the sliding mechanism after diagonal failure and assumed normal stress distributions are illustrated. By neglecting the flange effects, the following Mohr-Coulomb formulation suggested by Magenes and Calvi [1997] is employed:

$$\tau = \frac{c + \mu \sigma_n}{1 + \alpha_v}$$

where $\tau$ is the shear stress capacity; $c$ is the shear bond strength; $\sigma_n$ is the applied normal stress; and $\alpha_v$ is the shear span ratio ($h/L$) which was introduced to take into account the non-uniform distribution of stresses along a section. It is noted that the stress distribution on cross section both locally and globally are uneven and considering both of them in analytical solutions are complex, so for simplicity the locally uneven stress distribution on each unit (Fig. 5C) is ignored and only the globally uneven distribution is considered.
FIGURE 5  Reduction in effective contact area after sliding and normal stress distribution, (A), (B), and (C) are three different steps.

By increasing displacement demand ($\delta$) the contact area decreases and consequently strength loss occurs. Assuming a stair- stepped crack pattern as illustrated in Fig. 5 (three different steps), the displacement capacity of a wall is depended on opening of gap. This means that two sections of a wall rigidly move far from each other and effective contact area significantly reduces, as shown in Fig. 5. Regarding the given $S$ and $\delta$, and $A_e$ is the effective contact surface and calculated by Eq. (24) in each step:

$$A_e = L_e t - n\delta_2 t,$$

(24)

where $n$ is number of bricks over the crack path; $t$ is the thickness of the wall and $\delta_2$ is the top displacement of the wall. $L_e$ is the neutral axis depth which is derived from the rocking procedure (described in the aforementioned sections) in each step. The amount of $n$ can be obtained approximately from Eq. (25):

$$n \approx \frac{L_e}{(S/2)},$$

(25)

where $S/2$ is the brick contact length that assumed half of a brick length and illustrated in Fig. 5A. Using Eqs. (23), (24), and (25), the displacement dependent lateral shear of wall is predicted (Eq. (26)).

$$V_{ds} = \frac{c t L_e \left(1 - 2\delta_2 / S \right) + \mu \left(P_t + W/2 \right)}{1 + \alpha_v}.$$  

(26)

Equation (26) indicates that shear-sliding capacity is affected by bond strength, friction coefficients, wall aspect ratio, and axial load. From theoretical point of view with increasing displacement demands, cohesion coefficient decreases and initial friction coefficient tend to a lower value. Therefore, it is expected with constant axial load the shear strength after sliding varies between an upper bound ($V_{dt}$) and a lower bound ($\mu(P_t + W/2)$), however, the tests observations haven’t shown such strength degradation. Therefore, the cohesion coefficient was not eliminated from calculation.

Total displacement capacity is obtained by the following equation in each step:

$$\delta = \delta_1 + \delta_2,$$

(27)
where $\delta_1$ is the displacement at diagonal crack formation and $\delta_2$ is the displacement of the wall after shear failure in each step. It should be noted that, diagonal cracks can provide different local complex failure modes and calculation of displacement through all contributed local modes are not practical. Hence, to avoid the complexity and for simplicity in solution, the effect of rotation of different parts of wall and consequently changing in stress distribution in local contact areas and resistant of force location is neglected and the displacement amplitude through zigzag crack path is considered equal to the displacement in the sliding mode at the base of wall.

5. Calculation of Displacement and Numerical Solution

Based on the proposed model presented in the preceding sections, a numerical solution technique was provided for predicting the nonlinear in-plane force-displacement response of FURM walls. The solution procedure involves an iterative scheme and step-by-step solution using incrementally control curvature at the base of wall at each step. The proposed procedure is started with a zero value of the control curvature at the base and is continued with increasing curvature incrementally. In each step depends on effective length ($L_e$), that is varied between $L_e = L$ to $L_e = 0$, the relevant lateral displacement and strength are determined. The procedure is begun with flexure and continued until detecting of first new failure mode, once the new failure mode was detected, the rest of response is followed using new behavior. The lateral displacement of the wall is determined using Eq. (28), which is adopted from Eq. (5):

$$\Delta = \frac{V_r}{K}.$$  
(28)

The parameters were previously defined. Due to negligible effects of the term $P\tan(\theta)$ in Eq. (5), it is omitted in current research. This decision is consistent with our experimental observations. A flowchart with details of the numerical solution algorithm for analyzing in-plane force-displacement response of flanged masonry walls is presented in Fig. 6. The solution procedure involves an iterative scheme.

6. Experimental Program

6.1. Wall Specifications and Material Properties

For a better understanding of FURM walls behavior, an experimental program was conducted by the authors. The selected specimens were based on the characteristics of one to three-story residential masonry buildings in Iran. The condition of the most existing masonry buildings in Iran are categorized as poor based on FEMA-356 recommendations. Therefore, material properties, axial load ratios, and geometry of specimens were defined based on the average values of the existing URM building. The scale of specimens and units was one half and the clay bricks dimensions were $100\,\text{mm} \times 30\,\text{mm} \times 50\,\text{mm}$. The bed joints were $6\,\text{mm}$ thick and the maximum sand aggregate size was $4\,\text{mm}$. The head joint was not perfectly filled, in consistent with previous practice. Two specimens were designed with I- and C-shaped flanges, and for comparison, two additional specimens were designed without flanges. The walls were tested in cantilever form, i.e., fixed at the base and free at the top. Details of the walls geometry are listed in Table 1 and are shown in Fig. 7.

I-shaped Wall had flanges at both ends with a length of $700\,\text{mm}$ on each side; hence, the total flange width was $1560\,\text{mm}$. The C-shaped wall had flanges at both ends one side,
and the flanges had a length of 690 mm. The effective width of the flanges was determined according to EURO CODE 6 [UC6], MSJC [2008], and Standards New Zealand [2006] recommendations. In order to apply uniform lateral load and shear stress on web and flanges, a 70 mm thin concrete slab was designed at top of the flanged walls.

The properties of mortar, brick, and masonry brickwork were determined by sample tests. The average and standard deviation of test results and adopted testing code are listed in Table 2. Mortar mix design was 1:6.5 (cement/sand by volume) and experimentally designed to simulate weak masonry shear strength in existing URM structures.
Flanged Unreinforced Masonry Walls

FIGURE 7 Details of specimens.

TABLE 2 Material properties

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mortar compressive strength ($f_{mc}$)</th>
<th>Brick compressive strength ($f_{bc}$)</th>
<th>Masonry compressive strength ($f_m$)</th>
<th>Masonry Elastic Modulus in Compression ($E_m$)</th>
<th>Masonry shear Strength bed-joint</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength (MPa)</td>
<td>8.1 (0.47)</td>
<td>8.7 (0.59)</td>
<td>3.24 (0.2)</td>
<td>486</td>
<td>0.125 (0.03)</td>
</tr>
<tr>
<td>Method of Test</td>
<td>ASTM C109M-02</td>
<td>ASTM C67-03a</td>
<td>ASTM C1314</td>
<td>ASTM C1314</td>
<td>UBC</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>21-6</td>
</tr>
</tbody>
</table>
Five masonry prisms were built at the time of the construction of the URM walls and tested under uniaxial compressive loading. The stress-strain curves for the five prisms are shown in Fig. 8. The average prism strength was 3.24 MPa with standard deviation of 0.2 MPa. The modulus of elasticity was measured as the secant modulus at 0.75% of the compressive strength. Figure 9 shows the configuration of the masonry prism compressive strength tests as well as the typical failure mode.

The masonry shear strength was experimentally determined using average bed-joint shear strength, an in-place shear test procedure which is suggested in FEMA-356. Based on eight sample test results, the average value was 0.125 MPa with standard deviation of 0.03 MPa.

6.2. Test Setup

The typical test setup is shown in Figs. 10 and 11. The tested walls were constructed on a reinforced concrete foundation which had been anchored to strong floor by steel bolts. The
**FIGURE 10** Test setup (main view).

**FIGURE 11** Test setup (side view).
walls were loaded laterally by means of a horizontal hydraulic actuator reacting against reaction frame. The lateral loads were transferred through stiff plates that were fixed to the underneath of the diaphragm. In reference walls (URMW-1 and URMW-2) a stiff concrete beam was made on top of the wall for lateral loading, which showed good performance during the test.

The reference walls (URMW-1 and URMW-2) were restrained against out-of-plane movements by means of an inclined steel strut at mid length such that there was not any restraint for in-plane movement of specimens. The walls were also restrained at both ends by using guide vertical elements. The flanged walls were restrained against out-of-plane deformation on the concrete roof level by the same means used for reference walls. Horizontal and vertical applied loads were measured using the internal load cell of each actuator. To measure displacements at different locations, 14 linear variable displacement transducers (LVDT) for each specimen were employed. All specimens were tested under constant axial force corresponding to axial stress of prototype three-story building. The axial load on the I-shaped wall and URMW-1 were 44.8 kN. The axial load on C-shaped Wall and URMW-2 were 21.2 kN. These axial loads caused an axial compressive stress equal to 0.1MPa in reference walls and the same values were used for flanged walls on web. Axial forces were held constant during the test using a vertical actuator in a force-controlled mode. Lateral load was applied in a cyclic reversal pattern under displacement control mode and was applied on centerline of the web. The walls were cantilever, i.e., fixed at the base and free at the top. The loading protocol of tests is shown in Fig. 12. This pattern is adopted from past experimental tests [Porter, 1987; Harris and Sabnis, 1999]. Figures 13 and 14 show the final tests setup of flanged walls.

7. Test Observations

The crack pattern and hysteretic response of I-shaped wall at the end of the test (drift of 1.11%) are shown in Fig. 15. The specimen suffered considerable damage at drift of 0.37% where horizontal flexure cracks appeared in the web panel. With increasing displacement amplitude, the overall displacement capacity was obtained from the bed joint sliding over the height and continued up to the end of the test. In addition, inclined cracks formed at the web-flange intersection. At a drift of 0.91%, the flanges suffered damage in the form of vertical cracks parallel to the web, and the flexural cracks extended throughout the flange height. The prevailing observed failure mode was bed-joint sliding.
In the C-shaped wall, the flexural cracks initiated and formed diagonally. The specimen showed a distortional behavior through the height while it had been restrained against out-of-plane deformation at slab level. Due to the distance between the shear center and stiffness center, distortional behavior was anticipated. The distortion was initiated and amplified with increasing in displacement demands and caused considerable stiffness and strength degradation (particularly in-cycle) in response in respect of I-shaped wall specimen. Out-of-plane deformation of the web enforced additional rotations to flanges and therefore some wide and deep cracks developed in the web-flange intersection. At a drift of 0.74% the vertical crack in the vicinity of web-flange intersection separated flanges from
FIGURE 15 Crack pattern and load displacement curve of I-shaped wall at Drift 1.11%.

FIGURE 16 Crack pattern and load displacement curve of C-shaped wall at Drift 0.926%.

FIGURE 17 Crack pattern and load displacement curve of URMW-1 at Drift 1.11%.

FIGURE 18 Crack pattern and load displacement curve of URMW-2 at Drift 0.926%.
TABLE 3 Test results

<table>
<thead>
<tr>
<th></th>
<th>$V_{max}$ kN</th>
<th>Drift$<em>{v</em>{max}}$ %</th>
<th>$V_{crack}$ kN</th>
<th>Drift$_{crack}$ %</th>
<th>Drift$_u$ %</th>
<th>Failure mode.</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>I-shape</td>
<td>47.9</td>
<td>0.185</td>
<td>46</td>
<td>0.13</td>
<td>0.74</td>
<td>Sliding</td>
<td></td>
</tr>
<tr>
<td>C-shape</td>
<td>12.4</td>
<td>0.37</td>
<td>10</td>
<td>0.10</td>
<td>0.96</td>
<td>Diagonal Tension</td>
<td></td>
</tr>
<tr>
<td>URMW-1</td>
<td>41.8</td>
<td>0.37</td>
<td>28</td>
<td>0.09</td>
<td>1.11</td>
<td>Sliding</td>
<td></td>
</tr>
<tr>
<td>URMW-2</td>
<td>18.3</td>
<td>0.22</td>
<td>14.9</td>
<td>0.13</td>
<td>0.96</td>
<td>Diagonal Tension</td>
<td></td>
</tr>
</tbody>
</table>

The amount of damages, particularly out-of-plane deformation, was so significant that the potential of axial failure was concerned. Thus for preventing axial failure the test was stopped at drift of 0.96%. The crack pattern and hysteretic behavior of the C-shaped specimen are illustrated in Fig. 16.

The crack pattern and hysteretic behavior of the reference wall (URMW-1) is illustrated in Fig. 17 at the end of the test at drift of 1.11%. The first cracks initiated at drift of 0.18% at the bottom of the wall, about 300 mm above the foundation. This crack expanded in the horizontal direction and continued approximately to 60% of the wall length. At a drift of 0.37%, sliding mechanism was observed and with increasing drift the sliding behavior was developed. Except a semi-diagonal wide crack that formed at the end of the test, the dominated failure mode was bed-joint sliding. The test results are listed in Table 3. The comparison between the reference wall and the I-shaped wall indicates that the flanged wall presents 15% increasing in strength, whereas the reference wall undergoes significantly severer damage relative to the flanged wall at a drift of 1.11%. The location of damage in the reference wall concentrates in the bed joints at the bottom of the wall, but in the flanged wall, the damage distributes over the height of the wall.

In the reference wall corresponding to C-shaped wall, diagonal cracks initiated at a drift of 0.18%, and extended toward the entire height at a drift of 0.74%. Specimen’s damages at the end of test and lateral load behavior are presented in Fig. 18. Comparison between URMW-2 and C-shaped walls indicates that due to distortional behavior, the flanged wall suffers more than 32% basic strength degradation in comparison with reference wall. However, both walls indicated almost similar initial stiffness, deformation capacity, and mode of failure. Table 3 shows some important test results including to: cracking and ultimate lateral load; drift ratio at first cracking, at ultimate, and at peak strength; and also observed failure modes. Ultimate drift is defined corresponding to point which the lateral strength degrades to 80% of maximum load.

8. Investigation of the Flange Effects by the Proposed Analytical Model

The proposed model was used to simulate the force-displacement response of the four specimens reported in this paper. A nonlinear pushover program was assembled to numerically investigate the response of walls.

In the I-shaped wall, the observed failure mode was classified as flexural cracking degrading to sliding at drift of 0.185%. The compressive strength of masonry, the initial elastic modulus, and masonry shear strength were taken from material test results as listed in Table 2. Based on FEMA-356 recommendations, the values of masonry shear strength can be replaced with diagonal tension strength where the material condition is poor. The values of material shear strength were assumed instead of diagonal tension strength as listed in Table 2. Based on the values of compressive strength and shear strength of masonry, the
condition of masonry is classified between “fair” and “poor,” according to ASCE-41 and FEMA-356. Due to lack of additional information regarding to C and \( \mu \) in ASCE-41 and FEMA-356, the values of initial bed-joint strength (c) and the coefficient of friction (\( \mu \)) were taken from NZSEE 2006 by interpolating between soft and firm condition. The assumed values are 0.15 MPa and 0.55, respectively. \( \beta \) and shear factor \( \zeta \) were assumed to be 1.28 and 1.0, respectively, since the aspect ratio (\( h/L \)) of this wall was 0.5 [Yi et al., 2005, 2008].

The proposed model predicted the flexural crack at a drift of 0.075 % and diagonal tension failure mode with lateral force of 48.6 kN which is followed by diagonal sliding. The push-over curves of predicted response and test are illustrated in Fig. 19. The comparison of test and analytical curves indicate that the proposed model predictions are satisfactory for stiffness, strength, and displacement capacity, but the predicted failure mode is different with the test result.

In the C-shaped wall the experimental behavior was classified as flexural cracking followed by diagonal tension mode with a peak load of 12.4 kN. The key parameters of material were the same as values of I-shaped wall except the shear factor \( \zeta \) that was assumed to be 1.3 since the aspect ratio (\( h/L \)) of specimen was 0.71. The proposed model predicted the flexural crack at drift of 0.16% followed by diagonal tension failure with a lateral force of 16.3 kN and then diagonal sliding. The results of the test and analytical model are presented in Fig. 20. As illustrated in Fig. 20, the initial stiffness and displacement capacities as well as strength degradation are predicted with reasonable accuracy. The peak strength of the predicted model is slightly higher than test results. The main reason for these differences is the simplicity of the assumed procedure and neglecting of distortion effects.

In URMW-1, the failure observed mode was flexural cracking degrading to slide at drift of 0.36%. The maximum strength for the specimen was 41.8 kN. The effective pier model predicted the flexural crack at a drift of 0.54% and a lateral force of 42.6 kN. The model also predicted bed-joint sliding at the base of the wall. The predicted model and experimental results are presented in Fig. 21. In this figure, the vertical axis is base shear and the horizontal axis is drift in percent. A good agreement is observed between predicted and test result responses.
FIGURE 20 C-shaped wall back bone curve; test and analytical predictions.

FIGURE 21 URMW-1 back bone curve; experimental and predicted response.

FIGURE 22 URMW-2 back bone curve; test and predicted response.
In URMW-2, the specimen’s behavior was classified as flexural cracking degrading to diagonal tension cracking with a peak load of 18.3 kN. The proposed model predicted flexural cracking degrading to rocking and toe crushing with strength of 17.9 kN. The results of both the experiment and proposed model are shown in Fig. 22. For both reference specimens the same aforementioned modeling parameters were taken.

The comparison between test results and analytical responses show that with reasonable degree of precision, the proposed model can predict lateral load-displacement curve of masonry walls.

9. Comparison with Other Experimental Results

This section describes the comparison between the proposed model described in this paper and the model developed by Yi et al. [2008]. Based on authors’ searching, few experimental tests have been conducted on FURM walls; therefore, only three accessible flanged wall specimens that were tested by Russell et al. [2010] are used for comparison purposes. The reported results showed that the bed-joint sliding followed by diagonal tension cracking was the predominated mode i.e. mixed mode. Table 4 presents the taken important material properties and specifications for the tested specimens. The initial bed-joint strength and coefficient of friction assumed 0.1 MPa and 0.7, respectively. They also defined that the masonry tensile strength of the wall is 0.67 times of initial bed-joint strength. These assumptions were based on the field testing of existing buildings which had been done by Derakhshan et al. [2010]. The comparison between experimental strength and observed failure modes with those predicted from Yi et al. [2008] model and proposed model are listed in Table 5. The results of Table 5 indicate that the proposed model with reasonable accuracy predicts strength and failure modes particularly for those the flange is in tension, e.g., specimen A7t. The obtained lateral force-displacement curves based on proposed model and Yi et al. [2008] model are plotted together with the experimental measured lateral force-displacement curves in Figs. 23–25. It is apparent that the lateral

### Table 4 Wall specifications

<table>
<thead>
<tr>
<th>Wall shape</th>
<th>( P_t ) (kN)</th>
<th>( L ) (mm)</th>
<th>( h ) (mm)</th>
<th>( L_f ) (mm)</th>
<th>( b ) (mm)</th>
<th>( t_w ) (mm)</th>
<th>( t_f ) (mm)</th>
<th>( f_m ) (MPa)</th>
<th>( c ) (MPa)</th>
<th>( \mu ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A6</td>
<td>73</td>
<td>4000</td>
<td>2000</td>
<td>3540</td>
<td>2160</td>
<td>230</td>
<td>230</td>
<td>9.1</td>
<td>0.1</td>
<td>0.7</td>
</tr>
<tr>
<td>A7</td>
<td>76</td>
<td>4000</td>
<td>2000</td>
<td>3540</td>
<td>2160</td>
<td>230</td>
<td>230</td>
<td>11.9</td>
<td>0.1</td>
<td>0.7</td>
</tr>
<tr>
<td>A8</td>
<td>71</td>
<td>4000</td>
<td>2000</td>
<td>3540</td>
<td>1200</td>
<td>230</td>
<td>230</td>
<td>9.1</td>
<td>0.1</td>
<td>0.7</td>
</tr>
</tbody>
</table>

### Table 5 Comparison of results between Yi et al. [2008] model and proposed model

<table>
<thead>
<tr>
<th>Wall</th>
<th>( V_{\text{Y,KN}} )</th>
<th>( V_{\text{P,KN}} )</th>
<th>( V_{\text{Y,KN}}/V_{\text{P,KN}} )</th>
<th>Exp. failure mode</th>
<th>Predicted failure mode, Yi et al. [2008]</th>
<th>Predicted failure mode, Proposed model</th>
</tr>
</thead>
<tbody>
<tr>
<td>A6</td>
<td>66.8</td>
<td>59.1</td>
<td>1.13</td>
<td>0.97</td>
<td>dt/ds</td>
<td>dt/ds</td>
</tr>
<tr>
<td>A7t</td>
<td>75</td>
<td>66.2</td>
<td>77.5</td>
<td>1.13</td>
<td>dt/ds</td>
<td>dt/ds</td>
</tr>
<tr>
<td>A7c</td>
<td>59.3</td>
<td>138.5</td>
<td>77.5</td>
<td>0.43</td>
<td>dt/ds</td>
<td>r</td>
</tr>
<tr>
<td>A8</td>
<td>66.9</td>
<td>62.4</td>
<td>68.9</td>
<td>1.07</td>
<td>dt/ds</td>
<td>dt/ds</td>
</tr>
</tbody>
</table>

\( V_{\text{Y,KN}} = V \) according to Yi et al. [2008], \( V_{\text{P,KN}} = V \) according to proposed model, A7t = flange in tension; A7c = flange in compression.
The model proposed by Yi et al. [2008] is not provided post peak response of diagonal tension failure. They proposed a negative stiffness as 10% of initial stiffness based on their judgments. However, the authors’ proposed model can predict both force-controlled and displacement controlled behaviors, i.e., all of possible failure modes. The insignificant shortcoming in the proposed model is that the initial strength derived from the model is greater than tests values. The authors did not have access to the details of damages and testing procedure to discover the reason. However, based on the authors’ experiences the distortion and out-of-plane behaviors may affect peak strength values. The simplified assumptions involved in the proposed model are another reason for overestimating peak strength. Generally, it seems that an acceptable level of accuracy can be derived from the proposed model.
10. Conclusion

The influence of flanges (return walls) on the in-plane lateral behavior of unreinforced masonry (URM) walls with weak shear strength was studied. The study included a cyclic loading test program on four wall specimens with aspect ratios of 0.5 and 0.7, two with I- and C-shaped flanges, and two without flanges. An analytical model was developed to predict the response of FURM walls.

Regarding the current research scope, the experimental results showed an increase in strength of the I-shaped wall with less damage relative to the reference rectangular wall. A significant loss of strength was observed in C-shaped wall that originated from out-of-plane distortion effects in comparison with I-shaped wall and reference wall. Both FURM walls showed almost similar initial stiffness, deformation capacity, and mode of failure in comparison with their reference walls.

Using developed analytical model, load-displacement response of I- and C-shaped URM walls were investigated. Proposed model was validated using the experimental test results of FURM walls. In this regard the values of mechanical properties, including compressive strength, bed-joint strength and modulus of elasticity derived from complementary tests and the others properly adopted from FEMA-356 and NZSEE 2006 suggestions. The results indicated that for a diagonal tension controlled wall, once a stair-stepped crack is opened up, sliding can be expected to occur along the bed joints and deformation can be expected in this manner. The comparison between tests results (5 FURM walls) with proposed model showed that the proposed model can predicts lateral force-displacement response with acceptable level of accuracy.

It is important to note that regarding the few tests conducted on FURM walls, the developed results on experiments and analytical approach should be considered primary and needed more investigation.

Funding

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