Estimation of water saturation from petrophysical logs using radial basis function neural network

Amir Mollajan¹*, Hossein Memarian²

1- M.Sc at Mining Exploration Engineering, School of Mining Engineering, University College of Engineering, University of Tehran, Tehran, Iran.
2- Professor of Geo–Engineering, School of Mining Engineering, University College of Engineering, University of Tehran, Tehran, Iran.

* Corresponding Author: a.mollajan@alumni.ut.ac.ir

Abstract

Estimation of reservoir water saturation (S_w) is one of the main tasks in well logging. Many empirical equations are available, which are, more or less, based on Archie equation. The present study is an application of Radial Basis Function Neural Network (RBFNN) modeling for estimation of water saturation responses in a carbonate reservoir. Four conventional petrophysical logs (PLs) including DT, LLd, RHOB and NPHI related to four wells of an oil field located in southwest of Iran are taken as inputs and S_w measured from core analysis as output parameter of the model. To compare performance of the proposed model with empirical equations, the same database was applied. Superiority of the RBFNN model over empirical equations was examined by calculating coefficient of determination and estimated root mean squared error (RMSE) for predicted and measured S_w. For the RBFNN model, R² and RMSE are equal to 0.90 and 0.031, respectively, whereas for the best empirical equation, they are 0.81 and 0.042, respectively.

Keywords: Water saturation, Petrophysical logs, Radial Basis Function Neural Network, Iran.

1–Introduction

Water saturation is the most important petrophysical properties of a hydrocarbon reservoir that is mainly used to estimate the volume of hydrocarbon in place and determining pay zones. This parameter can be measured directly from Routine Core Analyses (RCAL) or estimated by petrophysical methods. Various equations have been developed based on petrophysical models like Archie equation for clean sand formations (1942). The presence of clay mineral in sand formations (Worthington, 1985) or irregular distribution of pore sizes in carbonate rocks (Van Golf–Rocht, 1982) causes an additional conductivity while Archie equation presupposes that the matrix of rock doesn't have electrical conductivity. This additional conductivity causes overestimation in prediction of water saturation. Diverse equations have been presented to estimate water saturation in shaly–sand formations which can be categorized into two groups:

The first group includes models that import shale volume in their equations (Hossin 1960; Simandoux 1963; Poupon and Leveaux 1971, Dual–water 1977) and suppose that the additional conductivity is related to the volume of the shale.

The second group includes models that consider the influence of clay mineral types in addition to the shale volume (Waxman and Smits 1968; and Dual–water 1984). These models that have been developed based on the concepts of cation exchange capacity and ionic double–layer, have some disadvantages. For example, the laboratory tests to obtain the input variables of these models are usually non–economic and time consuming. In addition different laboratory techniques result different answers for same core sample and it also depends on precision of
the operator. Hence, some methods have been developed to overcome this limitation like new LSU model (2002).

Beside the above mentioned equations, some other activities have been done. Balch et al. (1999) predicted the water saturation in a sandstone reservoir in Mexico using artificial intelligence and seismic attributes. Kamel and Mabrouk (2002) introduced an equation for estimating water saturation in clean formations utilizing resistivity and sonic logs. The effect of water and gas saturation on P and S wave velocity values in sandstone samples have been studied by Kitamura et al. (2006). Al–Bulushi et al. (2009) developed an artificial neural network to estimate water saturation and fluid distribution. A new predictive capillary pressure function have presented by Tillero for Better Estimation of permeability and water Saturation (2012).

However, all of these models are appropriate for shaly–sand formation. Consequently, it seems to be necessary to find a new method for carbonate reservoirs with respect to their special properties. There are just a few works on the prediction of water saturation in carbonate reservoirs (Obeida et al., 2005; Lucia, 2007; Mollajan et al., 2013). The present study focuses on Sw prediction from PLs in a carbonate reservoir using Radial Basis Function Neural Network (RBFNN).

### 3–Data set

To commence this study, four conventional logs including DT, LLd, RHOB, and NPHI related to four wells of an oil field in southwestern Iran were used as input data. As satisfactory estimation results could only be obtained through the selection of appropriate data, the water saturation measured from core analysis were used as output of the model and well test responses were also employed to verify the results. Scatter plots between selected PLs and Sw for well No.1 is shown in Fig.1.

![Figure 1] Figure 1) Scatter plot of selected PLs versus water saturation in well No. 1. (a) As resistivity decreases, S_w consistently increases. (b) DT has decreased in corresponding with increase in S_w. (c) Increase in S_w (hydrogen content) causes dropping of NPHI. (d) Increasing rate of RHOB, resulting in S_w increasing.
3–Methodology

3–1–Radial Basis Functions

Radial Basis Functions Neural Network (RBFNN) is an excellent tool for prediction or interpolation used as an alternative for MLP neural networks. A particular RBFNN consists of three layers namely: input layer, hidden layer(s) and output layer. The input layer is a buffer that presents data to the network and contains the input variables while the hidden layer is composed of a number of RBF nodes with radial Gaussian activation functions. The output layer is the following layer in the network, which presents the output response to a given input and is connected to the previous nodes in the hidden layer by linear weights (Bishop, 1995).

The Figure 2 shows the typical RBF neural network architecture used in this study. In Figure 2, x1, x2..., xN represents the number of input nodes ϕ1, ϕ2, ϕ3,..., ϕM represents the basis function nodes, w0 is the weight of the bias node (optional), and w1, w2,……, wM are connection weights between hidden nodes and output node.

The net output from hidden layer (y) will be:

\[ y(x) = \sum_{j=1}^{M} w_j \phi_j(x) + w_0 \]  \hspace{1cm} (1)

As is stated before, in this study the Gaussian function is employed as commonly used basis function in Eq.1 (ϕj) and can be expressed as:

\[ \phi_j = \exp\left(- \frac{\|x - \mu_j\|^2}{2\sigma_j^2}\right) \]  \hspace{1cm} (2)

where \( \mu_j \) and \( \sigma_j \) are the center and width parameter, respectively.

In the learning process, the network is presented with a pair of patterns and network computes its own output. Afterward, the actual output is compared with target values or the desired output. So, the error can be calculated at any output in layer j as follow:

\[ E = y_j - t_j \]  \hspace{1cm} (3)

Where \( t_j \) is the desired output and \( y_j \) is the actual output. The total error function is given by:

\[ \varepsilon = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{M} [y_j(x_i) - t_{ij}]^2 \]  \hspace{1cm} (4)

3–2–Empirical equations

As mentioned earlier, water saturation is conventionally estimated using several suggested equations which are exclusively developed for shaly–sand formations. In this study, Archie equation in addition to two frequently used equations from Vsh models has been proposed as predictors for \( S_w \). Selected equations are shown in Table 1.

In these equations, shale volume \( (V_{sh}) \) and resistivity of water formation \( (R_w) \) can be obtained through the following equations, respectively:

\[ I_{GR} = \frac{GGR_{max} - GGR_{min}}{GGR_{max} - GGR_{min}} , V_{sh} = 0.33 (2^{I_{GR}} - 1) \]  \hspace{1cm} (5)

\[ R_w = \left[ \left( \frac{400000}{T_f} \right) P_w \right]^{0.88} \]  \hspace{1cm} (6)

where \( T_f \) is the formation temperature and \( P_w \) is the salinity of water formation. For Dual–water
equation (Clavier et al., 1977), parameters are defined as follow:

\[ \emptyset = \phi_{t}(1 - S_{wb}) \]  

(7)

\[ S_{we} = \frac{S_{wt} - S_{wb}}{1 - S_{wb}} \]  

(8)

It is noted that the value of \( R_{sh} \) can be calculated from resistivity logs at shaly intervals.

**Table 1) Different conventional predictors**

<table>
<thead>
<tr>
<th>Name</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Archie (1942)</td>
<td>[ C_t = \frac{C_w \rho^n}{f} ]</td>
</tr>
<tr>
<td>Hossin (1960)</td>
<td>[ \frac{1}{R_t} = \frac{S_{w}^2 V_{sh}^2}{F R_w + R_{sh}} ]</td>
</tr>
<tr>
<td>Dual Water (1977)</td>
<td>[ C_t = \frac{\phi_{m} S_{w}^{n}}{a} \left[ C_w + \frac{S_{wb}}{S_{wt}} (C_{wb} - C_w) \right] ]</td>
</tr>
</tbody>
</table>

**4– Implementation process**

In this section, the results obtained from the above mentioned methodologies are briefly presented. Minimum and maximum values of the PLs used in the models along with their symbols are shown in Table 2.

**Table 2) Description of input and output parameters in models.**

<table>
<thead>
<tr>
<th>Type of data</th>
<th>Well log</th>
<th>Symbol</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input</td>
<td>Neutron</td>
<td>NPHI</td>
<td>0.0056</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>Deep Resistivity</td>
<td>LLd</td>
<td>2.89</td>
<td>196.79</td>
</tr>
<tr>
<td></td>
<td>Density</td>
<td>RHOB</td>
<td>2.24</td>
<td>2.705</td>
</tr>
<tr>
<td></td>
<td>Sonic</td>
<td>DT</td>
<td>162.401</td>
<td>279.19</td>
</tr>
<tr>
<td>Output</td>
<td>Water Saturation</td>
<td>S_{w}</td>
<td>0.06</td>
<td>0.93</td>
</tr>
</tbody>
</table>

In this study, to validate the predictive models based on the comparing predicted and measured values, \( R^2 \) (Eq.10) is used. Also, RMSE (Eq.11) is used to compare the results of RBFNN and empirical models.

\[ R^2 = 100 \left[ \frac{\sum (x_{ipred} - x_{ipred}) (x_{imeas} - x_{imeas})}{\sum (x_{imeas} - x_{imeas})^2} \right]^2 \]  

(10)

\[ \text{RMSE}(x) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_{imeas} - x_{ipred})^2} \]  

(11)

Where \( x_{imeas} \) is the \( i^{th} \) measured element, \( x_{ipred} \) is the \( i^{th} \) estimated element, \( \bar{x}_{pred} \) and \( \bar{x}_{meas} \) are average of measured and estimated values, respectively, and \( n \) is the number of data used.

The following sections describe the implementation process and results of each of the models.

**4–1– \( S_w \) estimation using RBFNN**

To predict the water saturation in carbonate intervals, the RBFNN model was used. In this regard, four conventional logs including DT, LLd, RHOB and NPHI were considered as inputs and \( S_w \) measured from core analysis as output of the models. In order to build the proposed model, the bellow steps have been followed:
1. Existing wells were divided into two groups; one for model construction (including wells 1 to 3) and one for examining the generalization capability of the model (well No.4).

2. Model construction data set randomly were subdivided into two data sets namely training data, with 70% of the data points and testing data with the remaining 30%.

3. To check the generalization ability of the models, data associated to the well No 4 that were not entered in model construction were used.

The trial and error method was used to establish the number of hidden neurons. Using this strategy the optimal number of hidden neurons is the one that produces the least network error. Applying this method, the 4–10–1 network structure is considered to be the desired network. The radial basis function parameters (i.e. \( \mu \) and \( \sigma \)) were obtained from normalized input data using method of random sampling. Also, the weights are computed to minimize the error function (Eq.4).

Figure 3 shows the results obtained from RBFNN model in well No.4. As seen, the determination coefficient \( R^2 \) between predicted and measured \( S_w \) values and measured RMSE for this well are 0.90 and 0.031, respectively.

4–2– \( S_w \) estimation using empirical equations

To evaluate performance of the empirical equations, the same datasets used for RBFNN model were applied. It was observed that Dual–water predictor is the most appropriate predictor among the conventional models, better fits the data and has a greater determination coefficient whereas, Archie predictor is the poorest one (Fig. 4). Also, the values of estimated error for all methods have been offered in Table 3.

According to this table, for Dual–water equation the value of determination coefficient is the highest one and its calculated RMSE is lower than other empirical equations that are considered.

Table 3) error values in employed equation

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE</th>
<th>( R^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Archie (1942)</td>
<td>0.042</td>
<td>0.78</td>
</tr>
<tr>
<td>Hossin (1960)</td>
<td>0.040</td>
<td>0.80</td>
</tr>
<tr>
<td>Dual-water (1977)</td>
<td>0.042</td>
<td>0.81</td>
</tr>
</tbody>
</table>

5–Discussion

The results of this study are shown in Figure 5 and Table 4. As can be seen, RBFNN model is closer to the measured \( S_w \), whereas estimation by conventional equations has wide variation. In addition, these empirical models are appropriate for clean sands as well as shaly–sand formations and are not efficient enough for carbonates due to their specific properties as mentioned in introduction.
According to the results, because of the very low shale volume, the results of empirical relations are very close to each other. However, the proposed model has considerably high performance in estimation of $S_w$. It is concluded that, at least in carbonate reservoirs using this method for calculating water saturation is more appropriate.

![Figure 5) Comparison of the estimated $S_w$ with measured.](image)

**Table 4) comparison between error values**

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dual-water (1977)</td>
<td>0.042</td>
<td>0.81</td>
</tr>
<tr>
<td>RBFNN model</td>
<td>0.031</td>
<td>0.90</td>
</tr>
</tbody>
</table>

6–Conclusions

This paper presents a new approach based on Radial Basis Function Neural Network (RBFNN), for equations of water saturation from PLs in a carbonate reservoir. The proposed model was constructed by using data related to three wells and its performance was examined by a well which were not incorporated in the model development. Also, to compare the results of the RBFNN model with empirical equations, the same database was applied. It was concluded that performance of the proposed model is considerably better than the empirical models. For the RBFNN model $R^2$ and RMSE were equal to 0.90 and 0.031, respectively and for the best model of empirical models (Dual–water) were 0.81 and 0.042, respectively. Also, according to the obtained results, it is obvious that there is no need to calculate the complex coefficients such as cementation factor, tortuosity factor, saturation exponent and etc.

Acknowledgments:

The authors would like to thank Dr. B. Tokhmechi and Dr.S.A. Ouadfeul for their kind and careful comments that made the manuscript improved. We also would like to express their sincere thanks to the Exploration Directorate of National Iranian Oil Company (NIOC) for monetary supporting and their assistance in providing data and information in this study.

References:


Clavier, C., Coates, G., Dumanoir, J. 1977. The theoretical and experimental bases for the “dual water” model for the interpretation of shaly


Received: 07 July 2013 / Accepted: 10 September 2013 / Published online: 17 September 2013

EDITOR–IN–CHIEF:

Dr. Vahid Ahadnejad:
Payame Noor University, Department of Geology , PO BOX 13395–3697, Tehran, Iran.
E–Mail: edchief@jtethys.org

EDITORIAL BOARD:

Dr. Jessica Kind:
ETH Zürich Institut für Geophysik, NO H11.3, Sonnegg strasse 5, 8092 Zürich, Switzerland
E–Mail: jessica.kind@erdw.ethz.ch

Prof. David Lent: