U-notched Brazilian disc specimens made of a type of commercial graphite were used to measure experimentally the mode I notch fracture toughness of material. The experimental results were estimated by means of the mean stress and the point stress fracture criteria. An excellent agreement was found to exist between the results of the mean stress criterion and the experimental results for different notch tip radii. Also, found in this research was that the point stress criterion provides weaker estimates compared to the mean stress model except when one deals with larger values of the notch tip radius.

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1. Introduction

Graphite materials are very vulnerable to mechanical loads. In the presence of stress raisers like cracks and notches, graphite components are more seriously prone to sudden fracture due to the brittleness of the material and also because of the stress concentration. Thus, for engineering design purposes, fracture assessment of notched graphite materials under mechanical loads should be essentially performed both experimentally and theoretically.

For graphite materials weakened by a sharp crack, fracture experiments have been carried out under mixed mode I/II loading conditions by Awaji and Sato [1] on two different graphite materials using a centrally cracked disc-type specimen. The mixed mode I/II fracture toughness of graphite materials has also been evaluated by Yamauchi et al. [2,3] using disc-type specimens, namely the cracked Brazilian disc (CBD) sample and the semi circular bend (SCB) specimen subjected to three-point bending loading. Li et al. [4] have employed a single-edge crack specimen to measure mixed mode fracture in a type of polycrystalline graphite. Shi et al. [5] investigated the mode I fracture toughness of a nuclear graphite material through using a three-point bending specimen. Also, Etter et al. [6] have evaluated the fracture toughness of polycrystalline graphite under mode I loading conditions using single-edge cracked beam specimen. Dealing with theoretical works, Awaji and Sato [1] fitted a curve to the experimental results obtained from fracture tests on two different graphite materials and offered a semi-empirical equation for predicting mixed mode brittle fracture in graphite. Ayatollahi and Aliha [7] made use of a generalized maximum tangential stress (GMTS) criterion and through using T-stress, they presented very good estimates for experimental results reported in [1]. Short crack propagation in poly-granular graphite has been quantitatively studied by Mostafavi and Marrow [8] by digital image correlation. More recently, Mostafavi et al. [9] have investigated three-dimensional crack propagation in poly-granular graphite.
Several researches have been performed in the past dealing with fracture in graphite materials in the presence of notches. The stress concentration and the notch sensitivity have been studied by Bazaj and Cox [10] and Kawakami [11] on different graphite materials. However, considering the notch fracture mechanics (NFM) approach which takes into account the notch stress intensity factors (NSIFs) as the governing fracture parameters, the first study on brittle fracture of graphite materials weakened by a notch has been carried out by Ayatollahi and Torabi [12] who performed a set of pure mode I fracture toughness tests on three different V-notched graphite samples. They estimated very well the experimental results using the mean stress (MS) criterion. Ayatollahi and Torabi [13] have also provided extensive experimental data on mixed mode I/II fracture of V-notched Brazilian disc (V-BD) specimens made of a type of polycrystalline graphite. The provided test results (i.e. the notch fracture toughness and the notch bifurcation angles) have been successfully estimated by using the V-notched maximum tangential stress (V-MTS) fracture criterion [13]. Beside V-MTS, the strain energy density (SED) criterion has been successfully employed by Ayatollahi et al. [14] in order to estimate the mixed mode fracture test data reported in [13]. Another study has also been performed by Berto et al. [15] on out-of-plane fracture of V-notched graphite bars in which the test results have been well predicted by the SED criterion. It is worth mentioning that the SED approach has been frequently applied to static and fatigue failure of engineering components (see for example [16–19], etc.).

Recently, a valuable work has been done by Berto et al. [20] on pure mode I and mixed mode I/II brittle fracture of U-notched graphite plates. They provided 60 new test results under mixed mode and pure mode I loading conditions by using tensile specimens centrally notched [20]. The SED criterion has also been employed to predict well the experimental results. The test results reported in [20] have been theoretically re-analyzed by Torabi [21,22] by means of the point stress (PS) and the mean stress (MS) criteria in pure mode I and by the U-notched maximum tangential stress (UMTS) model in mixed mode I/II loading conditions, respectively. More recently, Lazzarin et al. [23] have investigated both experimentally and theoretically the mixed mode I/II brittle fracture of rectangular graphite specimens containing central key-hole notches. Like their previous works, the SED failure criterion has been successfully utilized to estimate the experimental results [23].

Except Ref. [20], no reference has been found by the authors in open literature dealing with fracture in U-notched graphite components either experimentally or theoretically.

In this research, first, twelve U-notched disc-type graphite specimens were fabricated. Then, the specimens were subjected to pure mode I loading conditions and the corresponding monotonic fracture loads were recorded. Finally, the mean stress (MS) and the point stress (PS) failure criteria, presented in the past by Ayatollahi and Torabi [12,24] for mode I brittle fracture of rounded-tip V-notches, are reformulated for U-notched elements and used to predict the test results of graphite samples theoretically. A comparison between the theoretical and experimental results showed that while the MS criterion is an appropriate fracture criterion for different notch tip radii, the PS criterion can be an efficient theory when one deals with larger values of the notch tip radius.

2. Experiments

2.1. Material

The material used in fracture tests was a type of commercial polycrystalline graphite with the properties presented in Table 1. This type of graphite is usually used in some aerospace applications and it has been previously used in Refs. [12,13] for performing fracture tests on V-notched specimens in laboratory scale. The method of manufacturing was the cold isostatic pressing (CIP) which provides homogeneous structure and isotropic properties. Its porosity was about 9% providing brittle behavior.

2.2. Specimen

The specimen used to perform experiments was a disc-type plate containing a central bean-shaped slit with two U-shaped ends, so-called in literature as the U-notched Brazilian disc (UNBD) specimen [25]. Fig. 1 represents the UNBD specimen schematically.

<table>
<thead>
<tr>
<th>Table 1</th>
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<td>Properties of the tested graphite material [12,13].</td>
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<tr>
<td>Material property</td>
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<tr>
<td>Elastic modulus, $E$ (GPa)</td>
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<td>Poisson’s ratio, $\nu$</td>
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<td>Ultimate tensile strength (MPa)</td>
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<td>Plane-strain fracture toughness (MPa m$^{0.5}$)</td>
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<td>Bulk density (kg/m$^3$)</td>
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<tr>
<td>Mean grain size ($\mu$m)</td>
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<td>Porosity (%)</td>
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In Fig. 1, $\beta$ is the angle between the loading direction and the notch bisector line. The disc diameter and the applied compressive load are denoted by $D$ and $P$, respectively. When the direction of the applied load $P$ is along the notch bisector line (i.e. $\beta = 0$), the round borders of the slit are subjected to pure mode I loading conditions.

The diameter, the overall slit length $d$ (i.e. the tip-to-tip distance) and the thickness of the UNBD specimens were 60 mm, 18 mm and 10 mm, respectively. The notch tip radii ($\rho$) were 0.5, 1, 2 and 4 mm. To prepare the specimens, first, a graphite block was provided from a manufacturing company. The producer stated that the block has been carefully checked by non-destructive testing (NDT) to see if it is free of any pre-existing defect or flaw and the health of the graphite block was confirmed. Then, three slices of 10 mm thick were cut from the block by using a cutter blade. The geometry of each specimen was given to a high-precision 2-D CNC water jet cutting machine and finally, the UNBD specimens were fabricated. Before conducting fracture tests, the specimens were polished by using a fine abrasive paper. For each notch tip radius, three tests were performed under displacement-control conditions with a loading rate of 0.1 mm/min and the fracture load of each sample was recorded. A total number of 12 test results were provided under pure mode I loading conditions. Fig. 2 represents the graphite specimen under mode I fracture test.

Table 2 presents the experimentally obtained fracture loads for UNBD graphite samples.

The load–displacement curves recorded during the tests were completely linear up to final fracture and fracture took place suddenly. Therefore, using brittle fracture models on the basis of linear elastic fracture mechanics (LEFM) is permissible. Fig. 3 represents a sample load–displacement curve for a UNBD graphite specimen.
In the next section, two well-known failure concepts, namely the mean stress (MS) and the point stress (PS) are applied to the mode I stress field around a U-notch and two closed-form expression are derived for mode I notch fracture toughness. The results of the MS and the PS criteria are then compared in the forthcoming sections with the experimental results.

3. Brittle fracture criteria

Fig. 4a displays a typical blunt V-notch and its polar coordinate system. The origin of the coordinate is located at the distance \( r_0 \) behind the notch tip on the notch bisector line. Since a U-notch can be geometrically considered as a rounded-tip V-notch with zero notch angle, blunt V-notches are entirely shown in this manuscript instead of U-notches with the aim to stress that the formulations of U-notches can be fundamentally obtained from those of blunt V-notches.

The mean stress (MS) and the point stress (PS) failure criteria suggested in the past for mode I brittle fracture of rounded-tip V-notches \([12,24]\) are similarly formulated for U-notches with the aim to obtain theoretically a closed-form expression for mode I fracture toughness of U-notched domains.

3.1. Mean stress (MS) criterion

Fracture takes place in accordance with the MS concept when the mean value of the tangential stress over a specified critical distance attains a critical value. Creager and Paris \([26]\) derived the elastic stress field equations for blunt cracks. Lazzarin and Tovo have also evaluated the elastic stress distributions in the vicinity of cracks and notches \([27]\). The tangential stress along a U-notch bisector line (\( \theta = 0 \)) can be approximately written in terms of \( r \) as \([26,27]\):

\[
\sigma_{\theta\theta}(r, 0) = \frac{K_{U\theta}}{\sqrt{2\pi r}} \left[ 1 + \frac{\rho^2}{2r^2} \right]
\]

where \( K_{U\theta} \) is the mode I notch stress intensity factor (NSIF). Fig. 4b shows a schematic rounded-tip V-notch with critical distances associated with the MS criterion.

In Fig. 4b, \( d_c \) and \( d_c' \) are the critical distances measured from the notch tip and from the origin of coordinate system, respectively \([12]\). Taking into consideration the requirements of the MS criterion, brittle fracture occurs for a U-notch (i.e. a V-notch with zero notch angle) when

\[
\frac{1}{d_c} \int_{r_0}^{d_c} \sigma_{\theta\theta}(r, 0) \, dr = (\sigma_{\theta\theta})_c
\]
Eq. (2) has been suggested in the past by Seweryn [28] who referred to a work by Novozhilov [29]. This equation has been utilized in Ref. [12] to assess the static failure in rounded-tip V-notched graphite specimens. Note that $r_0 = \rho/2$ for a U-notch where $\rho$ is the notch tip radius. Substituting Eq. (1) into Eq. (2) gives

$$\frac{1}{d_c} \int_{r_0}^{r_c} \frac{K_{Ic}^{U\rho}}{\sqrt{2\pi r}} \left[ 1 + \frac{\rho}{2r} \right] dr = (\sigma_{oo})_c$$  \hspace{1cm} (3)

After doing the integrations in Eq. (3) and by taking into consideration that at the onset of fracture, the notch stress intensity factor and the tangential stress must be equal to the notch fracture toughness $(K_{Ic}^{U\rho})$ and $(\sigma_{oo})_c$, one can write

$$\frac{K_{Ic}^{U\rho}}{\sqrt{2\pi}} \left( 2\sqrt{d_c} - \frac{\rho}{\sqrt{d_c}} \right) = d_c(\sigma_{oo})_c$$  \hspace{1cm} (4)
The mode I notch fracture toughness can be obtained from Eq. (4) as

$$K_U^{I} = \frac{\sqrt{2\pi(\sigma_w)_{c}d_{c}}}{2\sqrt{d_{c}^{2} - \rho^{2}}}$$

(5)

The critical parameter ($\sigma_w$) is assumed to be a material property and is commonly considered equal to the ultimate tensile strength $\sigma_u$ for brittle and quasi-brittle materials [12,13], [30–36].

The critical distance $d_{c}$ is considered to be equal to [37]:

$$d_{c} = \frac{2}{\pi} \left[ \frac{K_{c}}{(\sigma_w)_{c}} \right]^{2}$$

(6)

where $K_{c}$ is the plane-strain fracture toughness of material. Eq. (6) has also been employed in Ref. [12] for evaluating mode I brittle fracture of V-notched graphite samples. According to Fig. 4b, we have for U-notches

$$d_{c} = d_{c} + r_{0} = d_{c} + \frac{\rho}{2}$$

(7)

If the plane-strain fracture toughness ($K_{c}$) and the ultimate tensile strength ($\sigma_u$) of the material are known, the critical distances $d_{c}$ and $d_{c}'$ can be simply computed from Eqs. (6) and (7), respectively. By using the data in Table 1, $d_{c}$ can be computed for the tested graphite material to be equal to 0.84 mm. By substituting critical distances and other known parameters into Eq. (5), one can directly calculate the mode I notch fracture toughness for a U-notched component using the MS criterion. It is worth mentioning that the concept of critical distance has been frequently used in the past by several researchers both in static and fatigue loading conditions (see for instance [38–40], etc.).

3.2. Point stress (PS) criterion

The point stress (PS) criterion implies that brittle fracture takes place when the tangential stress at a specified critical distance attains its critical value. Fig. 4c displays schematically a rounded-tip V-notch with critical distances associated with the PS criterion. The parameters $r_{c}$ and $r_{c}'$ are critical distances measured from the notch tip and from the origin of the coordinate system, respectively. By taking into account the requirements of the PS criterion, one can write

$$\sigma_{w}(r_{c}', 0) = (\sigma_w)_{c}$$

$$K_{c}^{I} = K_{c}^{II}$$

(8)

Substituting Eq. (1) into Eq. (8) gives

$$(\sigma_w)_{c} = \frac{K_{c}^{II}}{\sqrt{2\pi r_{c}'} \left[ 1 + \frac{\rho}{2r_{c}'} \right]}$$

(9)

From Fig. 4c

$$r_{c}' = r_{0} + r_{c} = \frac{\rho}{2} + r_{c}$$

(10)

Substituting Eq. (10) into Eq. (9) gives

$$(\sigma_w)_{c} = \frac{K_{c}^{II}}{\sqrt{\pi(\rho + 2r_{c})} \left[ 1 + \frac{\rho}{\rho + 2r_{c}} \right]}$$

(11)

Finally, Eq. (11) can be rewritten as

$$K_{c}^{II} = \frac{\sqrt{\pi(\rho + 2r_{c})}(\sigma_w)_{c}}{\left[ 1 + \frac{\rho}{\rho + 2r_{c}} \right]}$$

(12)

Eq. (12) is a closed-form expression for estimating the mode I notch fracture toughness according to the PS criterion. The critical distance $r_{c}$ can be taken from the literature as [12,24,32]

$$r_{c} = \frac{1}{\pi} \left[ \frac{K_{c}^{II}}{(\sigma_w)_{c}} \right]^{2}$$

(13)

Eq. (13) has previously been utilized together with T-stress to assess brittle fracture in different engineering materials like rocks containing a sharp crack under mixed mode loading conditions. In this area, one can refer to Ref. [41] in which the generalized maximum tangential stress (GMTS) criterion has been utilized to predict the fracture trajectory in a limestone rock under mixed mode loading. By substituting Eq. (13) into Eq. (12) one can simply compute the mode I notch fracture toughness according to the PS criterion. By using the data provided in Table 1, $r_{c}$ is calculated for the present graphite material from Eq. (13) to be equal to 0.21 mm.
4. Results and discussion

Brittle fracture test results are trivially obtained from the test machine in the form of maximum loads associated with sudden fracture. On the other hand, most of the fracture models are derived in terms of the governing parameters like the stress intensity factors, etc. because of some advantages. To compare the experimental and theoretical results, we need essentially to convert the form of one of the results to another (i.e. both in the form of fracture load or critical NSIF). Because, theoretical predictions in the form of critical NSIFs depends only on the notch geometry and the material properties (i.e. it does not depend on the overall geometry of notched component, but fracture load depends), it was decided to present both results in the form of the notch fracture toughness $K_{IC}^{U\rho}$. For this purpose, a useful paper published by Torabi and Jafarinezhad [42] was considered. In Ref. [42], they have computed the notch shape factors (NSFs) for UNBD specimen for wide range of notch and specimen geometries. Such factors can be simply used herein to calculate the critical NSIF corresponding to any fracture load without requiring performing finite element (FE) analysis. Details of the conversion fracture load-to-notch fracture toughness can be found in [42]. However, a brief description of the procedure to convert one of these parameters to another one is presented herein. As presented in Ref. [42], the mode I NSF in UNBD specimen is

$$Y_{l\rho} = K_{l\rho}^{U\rho} \sqrt{\frac{\pi}{2d}} \frac{D_t}{P}$$

where $Y_{l\rho}$, $K_{l\rho}^{U\rho}$, $d$, $D$, $t$ and $P$ are the NSF, the NSIF, the overall length of the central slit, the disc diameter, the specimen thickness and the applied load, respectively. In Eq. (14), $K_{l\rho}^{U\rho}$ and $P$ depend on each other. The values of the NSFs, which are non-dimensional geometrical factors, have been presented in Ref. [42] for different notch lengths and various notch tip radii. For any UNBD specimen, one should first read the NSF value from Ref. [42]. If $K_{l\rho}^{U\rho}$ or $P$ is known, the other parameter can be simply computed. For example, for a known value of the fracture load ($P_c$), $K_{l\rho}^{U\rho}$ can be determined by using Eq. (15). Trivially, the fracture load corresponding to any known NFT can also be simply computed.

$$K_{l\rho}^{U\rho} = \sqrt{\frac{2d}{\pi}} \frac{P_c}{D_t} Y_{l\rho}$$

Fig. 5 shows the theoretical results of the MS and PS criteria together with the experimental results of the UNBD graphite specimens. The discrepancies between the theoretical and the mean experimental results are also presented in Table 3.

As can be seen in Fig. 5, $K_{l\rho}^{U\rho}$ slightly increases in a monotonic manner, as the notch tip radius enhances. The slight dependence of the mode I notch fracture toughness on the notch tip radius (i.e. the low notch sensitivity) suggests also slight dependence of the fracture load. Despite the scatter in the experimental results (rising and falling behavior of the fracture loads), this can be seen in Table 2 where the fracture loads of the UNBD graphite specimens are close enough to each other. It is necessary to highlight that unlike the fracture load that depends also on the overall geometry of the U-notched member, $K_{l\rho}^{U\rho}$ depends only on the material properties and the notch tip radius. Therefore, completely similar U-notches in two
geometrically different components made of a same material (i.e. equal values of $K_{U,I}^q$) may result in significantly different fracture loads.

The unexpected rising and falling behavior in the experimental results suggests that the number of the tests performed were not enough to capture the scatter in the data. While the authors believe on the basis of their experience that the scatter is natural, with this number of experimental data, the attribution of the obtained results to the natural scatter might be questionable. Thus, more experiments are needed to capture the scatter in the test data.

It can be seen in Table 3 that the maximum discrepancies between the experimental and theoretical results are obtained for the notch tip radius equal to 1 mm. However, the values of the maximum discrepancies are significantly different for the MS and PS criteria. The variation of the accuracy of the PS criterion versus the notch tip radius is large (between 99.2% and 76.4%) while that of the MS criterion is relatively small (between 90.5% and 98.7%) and the variation is smooth. From the view point of engineering design, a large variation in the accuracy of the fracture criterion is a major disadvantage. Therefore, the PS model seems to be an inappropriate failure criterion for mode I fracture of U-notched graphite components. Also, The MS model with the mean accuracy of about 95% seems to be a suitable fracture criterion for the assessment of mode I fracture in U-notched graphite elements. Other failure models like the cohesive zone model (CZM) [43] would most probably provide successful estimates to the experimental results on graphite specimens since like for MS model; CZM also utilizes exclusively the ultimate tensile strength and the fracture toughness of material in its predictions. This can be examined by the researchers in future works.

A major advantage of the MS criterion in engineering design applications is its closed-form expression $K_{U,I}^q = K_{U,I}^q$ similar to $K_I = K_{Ic}$ for sharp cracks. This makes an interesting correlation between a U-notch and a sharp crack and lets engineers estimate rapidly and conveniently the load-bearing capacity of U-notched graphite components. As we are aware, the criterion $K_I = K_{Ic}$ is for the onset of crack propagation in brittle materials under static and monotonic loading conditions. Since the material is brittle, the speed of crack growth is very high and hence, the onset of crack extension is in fact, the onset of sudden fracture. However, $K_{U,I}^q = K_{U,I}^q$ is a criterion for the instance of crack initiation from the tip of a U-notch. Again due to the brittleness, crack initiation is simultaneous with the final fracture.

In order to determine theoretically by the criterion $K_{U,I}^q = K_{U,I}^q$ whether or not a U-notched graphite component fractures under pure mode I loading conditions, one should first apply a unit load to the notched body and calculate the tensile stress at the notch tip ($r = \rho/2$). Then, the computed value should be substituted into Eq. (1) in order to compute the corresponding value of $K_{U,I}^q$. Finally, the load is gradually increased until $K_{U,I}^q$ reaches $K_{U,I}^q$ (see Eqs. (5) and (12)). The obtained load is in fact, the fracture load of the U-notched component.

The major disadvantage of the Brazilian disc (BD) specimen is that it is tested under compressive loading which may result in buckling before brittle fracture. Although selecting the specimen dimensions in accordance with the ones suggested in the literature guarantees the success of the test, buckling analysis is essential before brittle fracture test for those BD specimens having arbitrary dimensions to estimate the minimum buckling load. In this research, buckling analysis were performed before testing and it was found that the minimum buckling loads for the graphite specimens were more than twice the brittle fracture loads. To conduct buckling analysis, four UNBD specimens with different notch tip radii were modeled in a finite element software and a unit diametral compressive load was applied to the models. Then buckling analysis was performed for each model based on the eigenvalue method and from which the buckling mode shapes and buckling loads were determined. The buckling loads for the four models were almost identical and the minimum was about 10.2 kN. Before conducting the experiments, by initially assuming that the PS and MS models are successful, the theoretical NFTs were first computed by using Eqs. (5) and (12) and then converted to the corresponding fracture loads. The magnitudes of the estimated fracture loads were obtained to be between 3.8 and 5 kN. These results made us confident about happening brittle fracture before buckling.

5. Conclusions

The mode I fracture loads of twelve U-notched disc-type specimens made of polycrystalline graphite were theoretically estimated by means of the two newly formulated mean stress (MS) and point stress (PS) fracture models. It was found that while the MS criterion is an appropriate model for various notch tip radii, the PS criterion can be an appropriate theory when larger values of the notch tip radius are considered. The crack-like closed-form failure model $K_{U,I}^q = K_{U,I}^q$ for mode I fracture makes it possible to estimate the tensile load-bearing capacity of U-notched graphite components more rapidly and conveniently compared with other failure criteria.

Acknowledgement

This research was supported by the Iran National Science Foundation (INSF) under the Contract No. 90006227.

References


