Efficiency analysis of contactless electrical power transmission systems

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A B S T R A C T

In this paper, efficiency analysis and design of contactless electric power transfer systems working based on an inductive method are elaborated. Large leakage inductances of the systems are compensated to achieve resonance conditions for maximum efficiency. Considering the circuit models of a contactless system, the effects of different parameters including the compensating capacitors are analyzed for a specified frequency range and coupling coefficient. An algorithm is proposed to determine the optimal capacitors and resonance frequency. It is verified by experimental results.

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1. Introduction

In recent years, with the growth of portable electronic equipment such as notebooks and cell phones, new demands for contactless electrical power transfer (CEPT) are developed and the research on this kind of power transfer is gained more attentions. The CEPT may be achieved by different methods including microwave radiation [1], capacitor power transfer [2] and inductive power transfer (IPT) [3]. Inspired by the concept of mutual induction and resonance phenomenon, the IPT systems are attractive as a re-emerging technology. The IPT systems have many low and high power applications. They can be utilized to supply high power moving objects such as railway transportation systems [4]. Also, the systems can be applied to electric vehicles [5]. They can reduce the battery volume in hybrid electric vehicles [6] as well as hazards associated with plug-in vehicles [7]. The systems may have disadvantages such as effects of high electromagnetic fields on human health as a set of standards have been developed by the International Commission on Non Ionizing Radiation Protection (ICNIRP) [8].

An IPT system is composed of at least two coils, i.e. sender (primary) and the receiver (secondary) separated by an air gap. The most important aspect affecting a future wide spread application of such a system is the overall system efficiency. The IPT efficiency decreases rapidly by an increasing system air gap, which in turn increases the leakage inductances and magnetizing current. The efficiency also depends on the system operating frequency as the most influential factor to intensify the magnetic coupling between the primary and secondary sides. It is known that resonance frequencies on both sides contribute to a strong coupling, especially when the resonance frequencies are the same [9]. Compensating (resonant) circuits of different structures can be used in primary and secondary to achieve resonance conditions. Depending on the series (S) or parallel (P) connections of capacitors in the primary and secondary; four structures as SS, SP, PS and PP are built [10,11]. The combination of series and parallel connections are also used [12,13].

Different aspects of IPT systems are studied recently. A comparative study between resonance and non-resonance based inductive magnetic coupling methods is elaborated [14]. However, optimum resonance frequency for obtaining high efficiency is not considered. A contactless power delivery system is presented to transfer power to moving objects [15]. However, the optimal conditions are not investigated. An IPT system is proposed for a rotatable transformer in which the effects of load and air gap variations are studied for different compensation structures [10]. However, the effect of changing the operating frequency on the system efficiency is not studied. An IPT optimization algorithm is utilized to design an inductively coupled power transfer, with four compensation structures for a public transportation system [11]. But, the system efficiency is not analyzed for different operating conditions under varying coupling coefficient and frequency. A boundary frequency is found for optimal operation of contactless transformers under different loading conditions [16]. However, system parameters are not adjusted to obtain high efficiency for different cases. Keeping in mind this critical review, a thorough analysis of the factors influencing the IPT power transfer efficiency is essential for a practical system design.
In this paper the efficiency of an IPT system is obtained analytically in terms of coupling coefficient and resonance frequency; the former being a function of air gap, system structure and self and mutual inductances. A rather deep analysis is carried out to investigate the influences of the parameter variations on the system characteristics including efficiency and coupling coefficient. It is also shown that the efficiency strongly depends on the resonance frequency. An algorithm is developed to determine the optimal compensating capacitor and system resonance frequency. Finally, simulation results are verified by experimental results.

2. System model

An IPT system with a high frequency power source, two resonance circuits, a power conditioner and a load are shown in Fig. 1a. In Fig. 1b a system model with series resonant capacitors for the primary and secondary coils (SS), is shown. Also, Fig. 1c shows a model with a series resonant capacitor and a parallel capacitor (SP) for the primary and the secondary coils respectively. Fig. 1b (c) (b) shows a model with a series resonant capacitor and a parallel resonance circuit, a power conditioner and a load are shown in Fig. 1a. In Fig. 1b a system model with series resonant capacitors for the primary and secondary coils (SS), is shown. Also, Fig. 1c shows a model with a series resonant capacitor and a parallel capacitor (SP) for the primary and the secondary coils respectively. 

The power converter (inverter) is modeled by a sinusoidal voltage source with an internal resistor (Rloss1). The inverter switching losses can be included into Rloss1. The power conditioner loss in the secondary is also modeled by a resistor (Rloss2). The windings resistors (Rcu) depend on the system operating frequency due to skin effect. Therefore, the following equations are valid:

\[ R_{1.2} = R_{cu1.2} + R_{loss1.2} \]
\[ R_{cu1.2} = R_{cu1.2} = F_R(X)R_{cu1.2} \]

where \( F_R(x) \) represents the ratio of ac to dc resistances. The core losses are ignored due to a rather large air gap and the use of ferrite material in the core.

The efficiency of the system in SS structure is obtained as [17]:

\[ \eta_s = \frac{1}{(R_2 + R_k) + \frac{R_k}{(R_2 + R_k)^2} + \left[(R_2 + R_k)^2 + (L_2\omega - \frac{1}{\omega C_2})^2\right]} \]

\[ \frac{1}{\sqrt{L_1C_1}} = \omega_1 = \omega_0 = \omega_2 = \sqrt{\frac{1}{L_2C_2}} \]

If (3) is satisfied, the efficiency of the system with SS structure is obtained as (4):

\[ \eta_s = \frac{R_k^2\omega_0^2L_1L_2}{(R_1 + R_2)(R_1R_2 + R_k^2\omega_0^2L_1L_2)} \]

where \( \omega_0 = 2\pi f_0 \) is the angular resonance frequency in rad/s and \( k = \sqrt{M/L} \) is the coupling coefficient.

The efficiency of the system with SP structure is given by:

\[ \eta_p = \left| \frac{R_k}{R_1 + R_2(1 + (R_kC_2\omega_0)^2) + \frac{R_k}{(R_1 + R_2 - R_kC_2\omega_0)^2} + \omega_0^2(L_2 + R_kC_2R_2)^2} \right| \]

\[ \frac{1}{\sqrt{L_1C_1}} = \omega_1 = \omega_0 = \omega_2 = \sqrt{\frac{1}{L_2C_2} - \frac{1}{R_k^2C_2^2}} \]

Fig. 1. System diagram (a) and circuit model of the contactless power transfer, (b) SS and (c) SP structures.
If (6) is satisfied, the efficiency of the SP structure in (5) is written as:

\[
\eta_p = \frac{R_1 \omega^2 L_1 L_2}{R_1 + R_2 (1 + (R_1 C_2 \omega)^2)} \left[ R_1 R_2 + \omega^2 L_1 L_2 + \frac{R_1 R_2}{1 + (R_1 C_2 \omega)^2} \right] \tag{7}
\]

Here, a maximum efficiency and an optimum resonance frequency are calculated for SS structure.

\[
\frac{\partial \eta_p}{\partial \omega} = 0
\]

\[-(R_1 + R_2)^2 + \left( L_2 \omega - \frac{1}{2 C_2 \omega} \right)^2 + \left( \frac{1}{C_2} - L_2 \omega^2 \right) \left( L_2 + \frac{1}{C_2 \omega^2} \right) = 0 \tag{8}
\]

\[
f_{op,SS} = \frac{1}{2 \pi \sqrt{L_2 C_2}} \left( R_2 + 2 \sqrt{R_1 C_2} \right)^2 \tag{9}
\]

Substituting (9) into (2), the maximum efficiency is achieved in terms of system parameters in which \((L_2 \omega - 1/C_2 \omega)^2\) can be neglected since it vanishes as the system frequency gets close to the resonance frequency. Finally, the maximum efficiency is obtained with a good accuracy as:

\[
\eta_{op,SS} \equiv \frac{R_1}{R_1 + R_2 + 2 \sqrt{R_1 R_2 C_2}} \tag{10}
\]

Also, the optimum resonance frequency is obtained for SP structure as:

\[
f_{op,SP} \equiv \frac{1}{2 \pi \sqrt{L_2 C_2}} \tag{11}
\]

Substituting (11) into (5), the system efficiency is calculated as:

\[
\eta_p = \frac{R_1}{R_1 + R_2 + \frac{R_1 R_2}{\omega^2 L_1} + \frac{R_1}{\omega^2 L_2} + \frac{R_1}{\omega^2 M_2} \left( \frac{L_2}{\omega} \right)^2} \tag{12}
\]

where \(R_1 R_2^2/\omega^2 M_2^2\) and \(R_1 R_2/\omega\) can be neglected due to the small values of resistors and a high value of angular frequency. The optimum system efficiency is then obtained in terms of system parameters as:

\[
\eta_{op,SP} \equiv \frac{R_1}{R_1 + R_2 + \frac{R_1 R_2}{\omega^2 L_1} + \frac{R_1}{\omega^2 L_2}} \tag{13}
\]

### 3. Efficiency analysis

The performance of IPT systems strongly depends on the system electromagnetic structure and its parameters such as coupling coefficient, operating frequency and load. To achieve a desired system performance, the parameters should be optimized. A procedure will be presented in this paper. However, the effect of the system parameter variations on the power transfer will be studied first.

Coupling coefficient, \(k\), represents the intensity of electromagnetic coupling. It depends on the parameters of primary and secondary windings, iron core and air gap. These parameters are usually fixed, but the air gap, which is in reciprocal to \(k\) and \(\eta\), can be varied. Fig. 2a shows a 3-D view of the efficiency versus coupling coefficient and frequency for the system with the specifications presented in Table 2 at the Appendix A. It is seen that a high efficiency corresponds to a high coupling coefficient most of the time. Also, the figure shows that for every coupling coefficient the efficiency reaches a maximum value as the frequency changes. Fig. 2b clarifies this fact by showing a vertical locus at a frequency of about 30 kHz for obtaining maximum efficiency as the value of \(k\) changes. It means that the frequency at which the efficiency becomes a maximum is independent of coupling factor. This frequency is the resonance frequency of the system. Therefore, the operating frequency must be selected near the fixed resonance frequency of the system regardless of the coupling coefficient.

The total impedance in the primary side changes with the variation of the load connected to the secondary side. In practice such a variation is common, i.e. the variation in the operating conditions (speed, acceleration, etc.) of a Maglev system. The impedance variation occurred by different means including changes in the load resistance as it happens here. The variation of the impedance, in turn, changes the resonance frequency of the primary side and the output power. The relationship between the efficiency and the frequency with variation of load resistance is illustrated in Fig. 3a. It is seen that a maximum efficiency is available for each load which occurs at a certain resonance frequency. The resonance frequency is rather independent of the load impedance at higher values of the load impedance. However, it increases as the impedance decreases. Fig. 3b clarifies this fact by showing a locus of the maximum efficiency in a 2D graph. The figure is especially useful in showing a locus of maximum efficiency as the load changes. It is interesting to see that the efficiency reaches to more than 80% at high load values. At a nominal load, the efficiency is at a moderate value of about 60%. It is also important to see that the maximum obtainable efficiency greatly depends on the load, where it

![Fig. 2. The efficiency versus coupling coefficient and frequency. (a) A 3-D view (b) A 2D view.](image-url)
decreases with load impedance. Therefore, it is wise to investigate the variation of maximum efficiency versus output power (the power of the secondary side), since there is not much gain with a high transfer efficiency at a low output power. Fig. 4 illustrates such a variation at a constant frequency of 30 kHz. It is seen that the efficiency reduces as the output power increases up to a maximum output power. As a conclusion, a contactless IPT system must be designed such that the efficiency is high over a range of practical output power.

4. System design

In this section, a practical design of contactless power transfer systems with a high efficiency is presented. The main objective is to find the compensating capacitors and the supply frequency. Compensating capacitors of the primary and the secondary sides rarely change once the system is designed. However, the primary and secondary inductances as well as the mutual inductance may change due to the air gap variations and load operating conditions. As a result, the efficiency may significantly change. In practice, an efficiency reduction can be prevented by tracing the system behavior and adjusting and applying the resonance frequency. In this section the effects of variations of different parameters on the efficiency are investigated and desirable resonance frequencies together with the corresponding resonant capacitors are determined for the two compensating capacitor structures.

An algorithm is developed as presented in Fig. 5 to achieve an optimal resonance frequency for a high efficiency. At first, the
Table 1
System parameters in different cases.

<table>
<thead>
<tr>
<th>Cases</th>
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<th>$C$</th>
<th>$f$</th>
<th>$L + C$</th>
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<td>Changes</td>
<td>Changes</td>
</tr>
<tr>
<td>Case II</td>
<td>Changes</td>
<td>Changes</td>
<td>Fixed</td>
<td>Fixed</td>
</tr>
<tr>
<td>Case III</td>
<td>Changes</td>
<td>Changes</td>
<td>Changes</td>
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</tr>
</tbody>
</table>

Fig. 6. Efficiency locus of the resonance frequencies in SP structure, case I.

initial values of the system are taken. Then, a set of constraints are considered including the limits of inductances, resonant capacitors and frequency. Next, using (2) and (5), the system efficiency is obtained for different structures and cases. The system frequency is concurrently changes with the change in capacitors and inductances as presented in Table 1. The resonant frequency and the related efficiency are recorded at each stage of the algorithm. Finally, the efficiency values and resonance frequencies are compared and the proper frequency with its corresponding efficiency is determined.

4.1. Case I: adjustable supply frequency systems

In this case, the resonant frequency, as the system operating frequency, is found for the systems with the capability of adjusting their supply frequency. This is the most practical case as variable frequency inverters are widely available. Any constraint regarding the supply frequency must be taken into account. The values of fixed resonant capacitors in both sides of the system are found by searching a wide capacitor range. Fig. 6 shows the efficiency of SP structure versus the same frequency of both sides for three capacitance values in the range. It is seen that the efficiency reaches a maximum value at a specific resonance frequency for each capacitor value. It is possible to repeat the case for many other capacitor values. A locus of the maximum efficiencies is also depicted by a thick line. When this locus is considered, the frequency axis must be regarded as the axis of resonance frequencies; since each point on the locus corresponds to a resonance frequency. The efficiency increases rapidly as the resonance frequency increases and then gradually saturates towards a maximum efficiency value. Since high efficiency is usually desirable at a low frequency with a low capacitor value, it is concluded that a practical design can be chosen at the knee of the locus, providing a high efficiency at a relatively low resonance frequency with a reasonable capacitor value. Following this design guideline, Fig. 6 shows an example of an optimum design with a capacitor value of less than 1 p.u, at a frequency of about 10 kHz, operating at an efficiency of above 80%.

The case is repeated with the capacitors in both sides are inserted in series (SS structure). Similar plots to those of Fig. 6 are shown in Fig. 7a. It is interesting to notice that the locus here is not monotonic with respect to the capacitance values. A zoom of Fig. 7a is shown in Fig. 7b to better grasp the fact. It is seen that the locus does not pass into the low frequency range, causing a maximum resonance frequency which can be regarded as an optimum value. This value resembles the frequency value corresponding to the knee of the locus in the SP structure (Fig. 6).

Now, with the determined compensating capacitors, the power transfer efficiency is studied under the variations of system inductances. In practice, the variations may occur as a result of changing operating conditions including air gap fluctuations. A multiplying factor is chosen to provide a common pattern of variation for all system inductances. Fig. 8a and b shows the efficiency for SP and SS structures respectively. A locus of maximum efficiencies is also depicted in each figure showing the maximum obtainable efficiency for each pair of $C$ and $L$. It is seen that the locus for SP structure declines with an increasing resonance frequency; while it is almost a flat line for the SS structure. Because of the effect of parallel capacitors at high resonance frequencies in the SP structure, a short circuit occurs on the load. Therefore, much less power is delivered to the load. Nonetheless, the efficiency is high at lower values of resonant frequencies in both structures. This is very fortunate since supplying the power to the IPT systems is not an easy task when the required frequency increases.

Fig. 7. (a) Efficiency locus of the resonance frequencies in SS structure, case I. (b) Zoom of (a).
4.2. Case II: adjustable resonant capacitors

In this case the IPT systems supplied by fixed frequency power supplies are investigated. Therefore, the resonance capacitors are adjusted such that the resonance frequency of the system, resulting in a maximum efficiency, is the same as this fixed frequency. In practice, this needs a variable capacitor bank. Referring to (6), a fixed resonance frequency means a fixed product of capacitor and inductance values at each side of the system. The capacitors are changed in the same way as in the search of case I. However, the inductances are found such that LC remains constant. The primary capacitor is in series while the secondary capacitor is in parallel. Fig. 9 shows the efficiency versus the same frequency of both sides for three capacitor values. It is seen that the efficiency reaches a maximum at a specific resonance frequency for all capacitor values. It is important to notice that the efficiency reduces when the capacitor value increases. A locus of maximum efficiency is also depicted by a thick line. It is a vertical line at the supply frequency, i.e. the common resonance frequency of all capacitor values.

Case II is repeated for SS structure. The corresponding plots are shown in Fig. 10a. It must be noticed that the locus here is not a vertical line any more, implying that the maximum efficiency occurs at different resonance frequencies. A zoom of Fig. 10a is depicted in Fig. 10b for the sake of clarity.

4.3. Case III: target efficiency

In this case the capacitors and the inductances change with the same variations. The efficiency for the two structures can be seen in Fig. 11a and b. The efficiency in both SP and SS structures remains constant at the target value.
5. Comparison and discussion

In case I, the two loci are compared in Fig. 12a, showing that a higher efficiency is obtained in a SS structure than in a SP structure at the cost of a higher resonance frequency where the dash lines are plotted by using (9) and (10) for SS structure and (11) and (13) for SP structure. In both structures, the effect of capacitor variations on the optimal resonance frequency and system efficiency are investigated for selecting appropriate capacitors. It is shown that selecting smaller capacitors leads to larger resonance frequencies and higher efficiencies. Also, it is shown that the results of the algorithm confirm the analytical results.

In case II, the two loci are compared in Fig. 13a, showing that a higher efficiency is obtained in the SS structure than in the SP structure. A reduction in inductance values increases the value of resonance capacitor. This condition does not change the optimal resonance frequency of the system in SP structure. However, the efficiency of the system significantly declines although, the resonance frequency increases and the efficiency decreases in SS structure. In case III, the resonance frequency is traced with inductance variations and similar changes are applied to the capacitors to keep the efficiency constant. However, the resonance frequency is changed. Fig. 13b shows that difference between efficiencies of the two structures in this case.

6. Experimental verification

A laboratory setup including a variable frequency converter and a contactless IPT system is built as shown in Fig. 14 to verify the simulation results. The converter consists of a rectifier, a DC link and an inverter feeding the primary coil of the system. The Converter switches are controlled by a phase shift method covering a specified frequency range of 100 Hz to 30 kHz. Due to hardware restrictions the experimental setup is built with a low power rating of 100 W. The system power losses are low due to the use of ferrite material in the core with $\mu_r = 5000$ and $B_{sat} = 0.4$ T. The solid wire with a rather large cross-section area is used to avoid skin effect. The contactless system is capable of working with variable turns and adjustable air gap. Electric power is delivered to a vari-
able load through a rectifier. Resonance circuits are designed in the primary and the secondary sides by using variable capacitors. The system variables such as power, current, voltage and frequency in the primary and secondary sides are measured by using a precise power analyzer and a digital oscilloscope.

Fig. 15a shows a 3-D view of the experimental results of the efficiency versus frequency and coupling coefficient. An experimental result of system efficiency is shown in Fig. 15b where the efficiency follows the same trend as that of Fig. 6. Fig. 16a shows the experimental results of the efficiency versus frequency in SS structure for three air gap values. The results are comparable with the simulation results presented in Fig. 10a and b. The dash line in Fig. 16a is a locus of maximum efficiency. It is similar to the locus shown by a bold line in simulation results of Fig. 10a. Fig. 16b shows the experimental results of locus of maximum transfer efficiency in SP structure. The figure verifies the simulation results of Fig. 11a. However, there are little discrepancies between the simulation and experimental results. This is mainly due to the mismatch between the actual capacitors used in the setup with those of the simulation. Also, the accuracy limits of the measuring equipment contribute to the mismatch.
7. Conclusion

In this paper, the efficiency of contactless electric power transfer systems with inductive method has been investigated by simulations and experiments. Analytical analysis shows that the conditions of resonance must be met to achieve high efficiency. Furthermore, an optimal selection of resonance frequency is needed to yield a maximum efficiency. An algorithm is devised to find the optimal resonance frequency. The optimal conditions are obtained by means of adding resonance capacitors to both primary and secondary sides of the system. Even though air gap influences the coupling coefficient, the resonance frequency is independent from the air gap.

In both series and parallel connections of capacitors an optimal resonance frequency, leading to a maximum efficiency, exits. In both structures it is possible to adjust system parameters to reach a rather low resonance frequency for obtaining an optimal or suboptimal efficiency. This is useful in high power applications where a high resonance frequency is difficult to be provided in practice.

It is shown that among different resonance frequencies produced by variations of primary and secondary inductances and capacitors, system efficiency is enhanced at some frequencies. Therefore, a trade-off between a low resonance frequency and a high system efficiency is required. A procedure is proposed to determine the compensated capacitor to have both desirable resonance frequency and efficiency. It can be shown that the above conclusions are true for PS and PP structures. More complex compensating structures such as LCL and LCC can also be analyzed by a similar approach.

Appendix A

(see Table 2)
References