Experimental and numerical investigation of the stability of overhanging riverbanks

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Although different types of riverbank mass failure have been studied by many researchers, many uncertainties remain in predicting cantilever failures, in part because of a lack of detailed observations of this type of bank collapse. In this study, a laboratory study of cantilever failure was carried out using two types of materials to form overhanging banks with three different densities. The laboratory results show that the occurrence of toppling failures is more probable than the simple shear-type mechanism that has been analyzed most frequently by prior researchers. We go on to model these toppling failures numerically. Specifically, a Mohr-Coulomb model, within the framework of SIGMA/W software (ver. 7.17), was used to simulate the stress–strain behavior of the experimental banks. The numerical results for loess materials are in good agreement with the laboratory observations, but the simulations do not replicate experimental failures observed in higher density, more cohesive soils.

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1. Introduction

Riverbank erosion is an important source of sediment production and remobilization in rivers, and as such, this phenomenon plays a key role in floodplain development and water resources management. Bank erosion also causes damage to agricultural lands and adjacent infrastructure and may be responsible for the delivery of large volumes of sediment, with associated sedimentation hazards in the downstream reaches of fluvial systems (Rinaldi and Casagli, 1999). Odgaard (1987) stated that the weight of silt and clay entrained into the water from cutbanks is estimated to be 30–40% of the suspended load for the East Nishnabotna and Des Moines Rivers in Iowa, USA. In the loess area of the Midwest United States, bank material contributes as much as 80% of the total sediment eroded from incised channels (Simon et al., 1996).

Accordingly, the assessment of riverbank stability is important in developing a full understanding of fluvial dynamics and in assessing the need for protection and stabilization of riverbanks in critical regions of erosion. A large number of riverbank stability analyses are already available (e.g., Thorne and Tovey, 1981; Van Ererdt, 1985; Osman and Thorne, 1988; Darby and Thorne, 1996; Rinaldi and Casagli, 1999; Simon et al., 2000; Amiri-Tokaldany, 2002; Micheli and Kirchner, 2002; Langendoen and Simon, 2008; Samadi et al., 2009; among others), but most of them are not able to analyze the overhanging bank profiles that are typically formed in composite or cohesive homogenous riverbanks (Samadi et al., 2011b). Composite riverbanks, typically consisting of a basal layer of relatively erodible material overlain by a less erodible layer (often of finer material), are widely observed on rivers flowing through alluvial deposits. Fluvial entrainment or seepage erosion of the erodible material from the lower bank usually occurs at a much higher rate than erosion of the less erodible material from the upper cohesive bank. In such circumstances this leads to the formation of a cantilevered (overhanging) bank profile, with upper bank retreat taking place predominantly by the failure of these cantilevers. However, relatively few studies have undertaken stability analyses of the overhanging blocks in composite riverbanks, though notable recent contributions have started to address the issue of undermining as a prerequisite for forming overhanging blocks (Fox et al., 2006, 2007a,b, 2010, among others). In addition, some workers have recently developed new bank stability relations for cantilevered riverbanks (Darby et al., 2007; Langendoen and Simon, 2008; Samadi et al., 2011b).

The seminal work on cantilevered riverbanks was by Thorne and Tovey (1981), who defined three possible failure mechanisms for such banks; i.e., shear, tension, and toppling (beam) failures. However, in the three decades since their paper, the critical assumptions...
employed in Thorne and Tovey's analysis and other similar analyses (e.g., Darby et al., 2007; Langendoen and Simon, 2008; Samadi et al., 2011a,b) have not yet been tested rigorously (Van Eerdt, 1985; C. Thorne, Univ. of Tehran, personal communication, 2007). Undoubtedly one of the reasons for this gap is related to the difficulties of undertaking field studies associated with cantilever failures (Samadi et al., 2011b). A particular problem concerns the hazardous nature of measuring the geometry of overhangs, while the situation is still more complicated if the failure takes place during the presence of water flow in the river. Thus, identifying the instantaneous mechanism of failure at the inception of failure is not easy.

By making some assumptions about the effective forces acting on the sliding surface, Van Eerdt (1985) introduced relationships to analyze the stability of toppling failures of overhanging salt marsh cliffs in Oosterschelde, The Netherlands, based on limit equilibrium and beam theorem. However, we can assume that the elastic–plastic behavior of cohesive materials forming overhanging blocks in riverbanks diverges from that of overhanging blocks in salt marsh sediments, meaning that the relationships introduced by Van Eerdt (1985) are unlikely to be transferable to riverbank contexts. Indeed, soil mechanics literature (see, for example, Muir Wood, 2004; Fang and Daniels, 2006) indicates that elastic models are unlikely to be satisfactory except under very restricted conditions. A specific difficulty concerns the assumption (Ajaz and Parry, 1975; Van Eerdt, 1985) of linear elastic stress–strain behavior during compression. A quick comparison of the stress–strain response implied by a linear elastic description of soil behavior with the actual stress–strain response of a typical soil shows that many features of soil response exist that the linear elastic model is unable to capture (Muir Wood, 2004). As such, choosing a soil constitutive relationship appropriate to the problem being considered is important (i.e., cantilever failure).

For these reasons, a study focused specifically on the overhanging failure phenomenon is necessary (see Fig. 1 for detail). Cantilever failure mechanisms have in fact recently been studied by constructing a physical model in the laboratory (Samadi, 2011; Samadi et al., 2011a); the clear advantage of this approach being the detailed observations that are possible. However, this form of physical modeling is limited to exploring a relatively narrow set of conditions because the lengthy times that are often required to construct the experiment. In contrast, numerical models may offer a more rapid setup that, in turn, implies that numerical models can be used to investigate a wider variety of different scenarios. In this paper we seek to employ the advantages of both physical and numerical modeling in a complementary approach designed to provide new insights into the process of cantilever failure. Specifically, herein we present numerical simulations of soil behavior (i.e., Mohr–Coulomb) using a finite element method for evaluating the overhanging failures observed in the laboratory experiments of Samadi et al. (2011a). The availability of high quality observations from the physical modeling provides the ability to evaluate rigorously how the choice of particular constitutive models affects the veracity of the numerical simulations. A particular advantage of the data presented in this paper is that we employ a new experimental procedure to investigate the behavior of fine-grained materials during the formation of overhanging bank profiles, in which digital photography and particle image velocimetry (PIV) techniques are used to quantify spatially explicit bank deformation fields, enabling a detailed comparison of experimentally derived and numerically simulated parameters. To our knowledge, ours is the first study to model overhanging failures using this methodological approach.

2. Methods

In this section of the paper, we provide (i) an overview of the physical modeling and (ii) a description of the implementation of the numerical modeling.

2.1. Laboratory experiments

In recent years increasing interest has been focused on the potential of employing physical models to investigate riverbank erosion processes. For example, mass failures (including different mechanisms of cantilever failure) in a sandy gravel riverbank were investigated recently by Nardi et al. (2009, 2011). In addition, physical modeling of mass failure in riverbanks composed of sand and sandy silt material has been undertaken also by Taghavi et al. (2010). Various works have also been carried out to investigate dam-break flow and associated downstream sand bank failures (Spinewine et al., 2002; Soares-Frazao et al., 2007; Spinewine and Zech, 2007; Zech et al., 2008); and small scale experiments have been carried out on banks composed of fine-grained, sandy sediments, with a specific focus on the occurrence of seepage erosion and related mass failures (Howard and McLane, 1988; Fox et al., 2006; Chu-Agor et al., 2007; Fox et al., 2007a,b; Wilson et al., 2007; Chu-Agor et al., 2008; Lindow et al., 2009; Fox et al., 2010). In this study we use data from the experiments undertaken by Samadi (2011, also reported in Samadi et al., 2011a). Because the experimental design and aim of these experiments have been described previously in those publications, we herein present only a brief overview.

One of the traditional limitations of physical modeling is the effect of scale. Most physical models are constructed at much smaller scales.

![Fig. 1](image-url) (A) Fluvial erosion at the toe of a riverbank (bottom layer) results in the formation of an overhanging profile in cohesive banks of the Kordan River. (B) Destruction of the overhang from cantilever failure.
than the prototype, precisely because it is useful to obtain information about expected patterns of response more rapidly and with closer control over model details than would be possible with full-scale testing (Muir Wood, 2004). Consequently, ensuring dimensional consistency between the physical model and prototype is necessary. In the context of cohesive riverbank failures, scaling is not only concerned with maintaining geometrical scaling, but the physical properties (material scaling) of the soils must also be dimensionally consistent. Similar to Muir Wood (2004), Samadi (2011) and Samadi et al. (2011a) defined governing equations with regard to the distribution of effective forces on the slip surface of an overhanging bank as follows:

\[ FS_{cs} = f\left(\frac{C}{\gamma_s}, \frac{BW}{HB}\right) \]  

in which, \( FS_{cs} \) = the factor of safety with respect to overhanging bank failure (shear-type), \( C \) = the cohesion of the cantilever block, \( \gamma_s \) = specific weight of material, \( BW \) = the width of cantilever block, and \( HB \) = the height of the cantilever block. Based on Eq. (1), if an overhanging (prototype) is simulated in the laboratory, the values of the two dimensionless parameters must be similar in both the prototype and physical model.

Considering the forces applied to a bank subject to cantilever failure (Fig. 2) and because the cantilever height (\( HB \)) affects both the resisting and driving forces acting on unit width of an overhanging failure block in the same way (Samadi, 2011), this parameter can be ignored in the stability relation (Eq. (2)). Therefore, it is only necessary to evaluate the remaining three parameters (i.e., cohesion, unit weight, and block width) to model overhanging failures in the laboratory:

\[ FS_{cs} = FR_{cs}/W = C/(\gamma_s BW) \]  

We should mention that the main object of the current research is to discriminate the overhanging failure mechanism under a range of conditions.
different conditions. With this in mind, we used two types of susceptible soils and three different material densities to form overarching blocks. One soil type was gathered from cohesive banks of the Kordan River, Alborz Province, Iran; while the other was a noncohesive loess taken from tributaries of the Atrak River, Golestan Province, Iran. Because cantilever height has no effect in the stability analysis (Eq. (2)), we considered a fixed height in all tests; but the upper cantilever width, or related undermining depth, was increased in continuous steps. With regard to the field study results and elementary model tests reported in Samadi (2011), the height of undermining was assumed to be equal to the river flow stage. However, due to the lack of flowing water adjacent to the modeled bank, the undermining process was simulated by manually removing bank materials in five centimeter increments (see Samadi et al., 2011a for more detail).

2.1.1. Laboratory model construction

A physical model (tank) was made of steel with internal dimensions (length × height × width) of 200 × 100 × 100 cm (Fig. 3). To strengthen the model body during soil layer compaction, shields were welded onto the walls and floor of the tank; however, some parts of the tank were constructed with plexiglass walls (left part, i.e., walls AF, AB, EF in Fig. 3). To separate the soil block from the walls, two moving metal plates were located near the tank walls and were removed after block construction (walls BC and BE in Fig. 3). Furthermore, to remove friction between the soil block and the back wall of the tank (wall DE), three additional metal plates (3 mm thickness) were used near the tank wall before soil compaction (see Fig. 3 for detail). To facilitate the removal of these plates, both sides of the middle plate (plate number 3 in Fig. 3) were greased with oil. The left wall of the model that was in front of the soil block was designed as a movable plate (wall BE in Fig. 3). After construction of a soil block inside the tank, this plate was removed using a crane after opening the screws around them. The front wall of the model was built from plexiglass in order to see the position of the soil block during tests (i.e., wall BC in Fig. 3).

2.1.2. The experimental study method

To carry out the experimental tests, pieces of destroyed or eroded blocks of riverbanks were transferred to the laboratory and, after identifying the characteristics of the materials with various laboratory tests (see below), the soil materials were broken down using a hammer and a roller. These resulting materials were passed through a sieve with 5 mm openings, and the physical models were then constructed. Laboratory tests undertaken include grain size analysis, Atterberg (liquid and plastic) limit tests, bulk density, direct shear tests (DS), and a rapid triaxial test (UU). These tests were performed on two and four disturbed soil samples (cohesive material) taken from the Kordan River, near Abbas Abad and Najm Abad villages, respectively, along with two disturbed soil samples (noncohesive loess material) provided from tributaries of the Atrak River. However, direct shear (five samples) and triaxial tests (one sample) were also performed on the remolded materials forming the experimental blocks. The results obtained from the grain size and Atterberg limit tests show that bank materials at the Kordan study reach comprise silty clay (CL–ML) and lean clay (CL), whereas the bank materials at the Atrak study tributary are comprised of silt (ML). A summary of all test results is presented in Table 1.

The triaxial test is the most widely used test employed to determine the stress–strain behavior of structured soils, and validation of constitutive models is carried out based on the behavior observed during such tests (Liyanapathirana et al., 2005). Accordingly, the aim of the triaxial tests conducted in this study was to determine the soil behavior modules (i.e., elastic modulus and Poisson’s ratio) for subsequent use in the stress–strain numerical analysis. In this regard, the amounts of lateral pressure (σ3) in the triaxial cell were adjusted in accordance with the amount of preconsolidation pressure at 0.5, 0.8, and 1.3 (kg cm−2). To specify the soil behavior modules and shear resistance parameters, specific triaxial tests were done on the remolded samples with similar optimum moisture and density to the physical model tests (i.e., in an unsaturated condition). Before axial loading, the abovementioned lateral pressures were applied in the triaxial cell (around the soil sample), and water volume changes inside the triaxial cell were controlled from the burettes. After halting the volume change, the sample was strengthened (consolidated) and axial loading started. By applying an axial loading (i.e., fixed strain method) in the experiment, the pore pressure measured at the gauge attached to the cell bottom showed negligible changes that were mostly

<table>
<thead>
<tr>
<th>Sample</th>
<th>River</th>
<th>Location</th>
<th>USCS</th>
<th>Grain size</th>
<th>Atterberg limits</th>
<th>Bulk density</th>
<th>Cohesion</th>
<th>Internal friction angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>LL/PI (kN m−1)</td>
<td>DS/kPa</td>
<td></td>
<td>DS/LL (degrees)</td>
</tr>
<tr>
<td>1</td>
<td>Kordan</td>
<td>Abbas Abad</td>
<td>CL-ML</td>
<td>12.5 67.5 20</td>
<td>22.1 6.4 17.7</td>
<td>n/a n/a n/a</td>
<td>n/a n/a</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td>17.5 45 37.5</td>
<td>24.8 6.2 17.6</td>
<td>n/a n/a n/a</td>
<td>n/a n/a</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Kordan</td>
<td>Najm Abad</td>
<td>CL</td>
<td>16 46.5 37.5</td>
<td>25.6 11.1 17.8</td>
<td>22.7 19.6 27.8</td>
<td>21 n/a</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td>22.5 45 32.5</td>
<td>n/a n/a 17.5</td>
<td>n/a n/a n/a</td>
<td>n/a n/a</td>
<td></td>
</tr>
<tr>
<td>5*</td>
<td>Atrak</td>
<td>Dashli Boroun</td>
<td>ML</td>
<td>16 66 18</td>
<td>24 11.5 17.6</td>
<td>19 26 n/a n/a</td>
<td>n/a n/a</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td>14 35 51</td>
<td>n/a n/a 17.5</td>
<td>18.5 n/a 28.1</td>
<td>n/a n/a</td>
<td></td>
</tr>
<tr>
<td>7*</td>
<td></td>
<td></td>
<td></td>
<td>10 85 5</td>
<td>Non-plastic 15.3</td>
<td>9.6 n/a 31 n/a</td>
<td>n/a n/a</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td>22.5 77.5 0</td>
<td>Non-plastic 15.6</td>
<td>8.7 n/a 31.8</td>
<td>n/a n/a</td>
<td></td>
</tr>
</tbody>
</table>

Table 1

Summary of geotechnical tests on soil samples of Kordan River banks and Atrak tributaries riverbanks.

Table 2

The range of soil behavior modules used in the physical model tests.

<table>
<thead>
<tr>
<th>Test Soil class</th>
<th>Dry density (g cm−3)</th>
<th>Cohesion (kPa)</th>
<th>Internal friction angle (°)</th>
<th>Elastic modulus (MPa)</th>
<th>Poisson’s ratio (−)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 CL</td>
<td>1.5</td>
<td>6.0</td>
<td>15.0</td>
<td>1.2–2.4</td>
<td>0.32–0.4</td>
</tr>
<tr>
<td>2 CL</td>
<td>1.7</td>
<td>14.0</td>
<td>16.5</td>
<td>1.8–5.0</td>
<td>0.32–0.4</td>
</tr>
<tr>
<td>3 CL</td>
<td>1.8</td>
<td>17.0</td>
<td>17.0</td>
<td>2.4–8.0</td>
<td>0.30–0.36</td>
</tr>
<tr>
<td>4 ML</td>
<td>1.4</td>
<td>2.0</td>
<td>18.0</td>
<td>4.0–7.0</td>
<td>0.30–0.36</td>
</tr>
<tr>
<td>5 ML</td>
<td>1.5</td>
<td>2.5</td>
<td>21.5</td>
<td>6.0–8.0</td>
<td>0.30–0.35</td>
</tr>
<tr>
<td>6 ML</td>
<td>1.6</td>
<td>4.5</td>
<td>24.0</td>
<td>8.0–10.0</td>
<td>0.33–0.43</td>
</tr>
</tbody>
</table>

* Data obtained from special triaxial tests on the remolded unsaturated samples.
produced from pore air. Furthermore, little volume change in the soil sample was recorded in accordance to the volume change in the triaxial cell (volume change of cell’s water around the sample).

To construct the physical models, we used soil materials that were taken from the locations of samples 5 and 7 in the Kordan and Atrak Rivers, respectively. According to the specific weight of bank materials (Table 1) and the natural variability of these materials (see Samadi et al., 2009, 2011b), three densities of 1.5, 1.7, and 1.8 (g cm$^{-3}$) for the cohesive soil (CL) of the Kordan River (sample 5) and three densities of 1.4, 1.5, and 1.6 (g cm$^{-3}$) for the noncohesive soil (ML) of the Atrak tributaries (sample 7) were chosen for physical model tests. To determine the shear strength parameters of the soil materials (i.e., the cohesion and internal friction angle) used in the experiments, triaxial tests (following the above procedure) were carried out on these remolded samples (see Table 2 for geotechnical properties of each test). Unlike undisturbed sample, the arrangement, size, shape and frequency of the individual solid soil components within the remolded soil were rearranged and it does not provide the same strength and stability as in original soil (Rawai, 2009).

Most of the soil deposits showed a reduction in strength when the same strength and stability as in original soil (Rawai, 2009).

All model tests were conducted based on the optimum moisture of the soil as estimated by Proctor tests. After assessing the weight of dry soil and the required water for each layer, the soil was spread over several metal plates and the required water was added using a spray. After this stage, the prepared soil was moved into the model and compacted in different layers. In order to compact the soil layers, two different hand hammers with areas of 100 and 900 cm$^2$ and a vibration electrical hammer were used according to the required soil density. The height of the soil block was set to a constant value of 80 cm in all experiments, being constructed in eight layers of 10-cm thickness each.

One of the remarkable advantages of the physical model tests undertaken in this research was the possibility of imaging a riverbank (overhang) cross section during the process of undermining and consequent formation of an overhanging block. To record the deformation of the soil block, four cameras were used including (i) a professional photography camera SONY Alpha 300 for continuous photographing in front of the soil block (at 10.2 megapixels resolution), (ii) a digital camcorder SONY XR500E recording at a rate of between 60 and 240 frames per second (fps) and installed in front of the soil block for continuous recording, (iii) a digital camcorder SONY SR200E installed at an incident angle of 45° to view block deformations during the experiment, and (iv) a Powershot CANON SX200IS (at 12 megapixels resolution) that was used to provide (still) photos of different parts of the soil block during the experiment. Note that the first camera was located exactly in front and in the middle of the soil block (in front of wall BC in Fig. 3).

2.2. Numerical simulation

The core purpose of undertaking the numerical simulations in this paper was to provide insight into the detailed stress–strain behavior of bank materials during the process of formation, and subsequent failure, of cantilevered overhangs. This stress–strain behavior depends on various factors, such as the type and the nature of soil, its loading history and path, as well as the loading speed. Numerous mathematical models have been developed to predict soil behavior (e.g., elastic, plastic, visco-elastic, elastic-plastic, hyperbolic, and work hardening and softening stress–strain behavior), and their limitations have been discussed by many investigators (e.g., Chen and Saleeb, 1982; Muir Wood, 2004; Fang and Daniels, 2006; Yu, 2006). Chen (1985) provided three basic criteria for model evaluation. The first criterion concerns the extent to which model behavior is consistent with the theoretical requirements of continuity, stability, and uniqueness (see Section 2.1 and Eqs. (1)–(2) of this paper for explanation of this criterion in this research). The second criterion involves evaluation of the models with respect to their fit against experimental data from a variety of available tests and in relation to the ease with which material parameters can be ascertained from standard test data (see the following details and Fig. 5 for explanation of this criterion in this research). The final criterion concerns the ease with which the models can be implemented in computer calculations (see next section for more details of this criterion). Wroth and Houlsby (1985) suggest that in order for a constitutive model to be useful in solving engineering problems, it should ideally be simple, but most importantly it must reflect the physical behavior of the soil.

In this research, SIGMA/W 2007 software, version 7.17 was used to model the stress–strain behavior of overhanging blocks. SIGMA/W is a two-dimensional finite element stress-deformation model. Stress is related to strain by the elastic modulus such that

$$\sigma = D\varepsilon$$

where $\sigma$ is the stress (in kPa), $\varepsilon$ is the dimensionless strain, and $D$ represents the material stiffness matrix (in kPa), which in turn depends on both the elastic modulus and Poisson’s ratio.

In contrast to the approach taken by Simon and Collison (2001), it is probable that in reality most cantilevered bank materials exhibit elastic-plastic rather than simple linear–elastic behavior. In a linear–elastic analysis, where stresses are computed without consideration of the soil strength, the soil is over stressed (i.e., the computed stress state is higher than the soil strength). However, Simon and Collison (2001) argued that given the small stresses associated with seepage forces, and relative to the elastic modulus in their scenarios, deformation is likely to remain in the elastic portion of the stress–strain curve. Duncan (1994) also states that simple elasticity models, such as the hyperbolic model, can be applied for stable structures where deformations are small, orientation of stresses are constant, and for fully drained or completely undrained conditions. In contrast, Swolfs and Brinkgreve (2008) argue that effective stresses at failure are quite well described using the Mohr–Coulomb failure criterion with effective stress parameters. Furthermore, the hyperbolic model is a hardening model that does not account for softening due to soil dilatancy and de-bonding effects. The major inconsistency of hyperbolic model for the present application is that, in contrast to the elasto-plastic type of model, a purely hypo-elastic model cannot consistently distinguish between loading and unloading. In addition, the model is not suitable for collapse load computations in the fully plastic range (Schanz et al., 1999).

In fact, the actual behavior of soils is more complicated and shows a great variety of behavior when subjected to different conditions. We must emphasize that no soil constitutive model can completely describe the complex behavior of real soils under all conditions. Soil is a material that behaves nonlinearly and often shows anisotropic and time-dependent behavior when subjected to stresses. It exhibits nonlinear behavior well below failure conditions with stress-dependent stiffness. Soil undergoes plastic deformation and is inconsistent in dilatancy (Ti et al., 2009). Thus, in all simulations undertaken for this study, an elastic–plastic stress–strain model (Muir Wood, 2004; Smith and Griffiths, 2004) was selected in which stresses are directly proportional to strains until the yield point is reached. Beyond the yield point, the stress–strain curve is perfectly horizontal (see Fig. 5). In SIGMA/W, soil plasticity is formulated using the theory of incremental plasticity (Hill, 1950). Once an elastic–plastic material begins to yield, an incremental strain ($\varepsilon_p$) can be divided into elastic and plastic components as follows:

$$\varepsilon = \varepsilon_e + \varepsilon_p$$
where \( \varepsilon_{de} \) and \( \varepsilon_{dp} \) are the elastic and the plastic strain increments, respectively. Only elastic strain increments will cause stress changes \( (d\sigma) \), as follows:

\[
d\sigma = C\varepsilon_{de}
\]

in which \( C \) is the elastic matrix.

Another function which describes the locus of the yield point is termed the yield function \( (F) \). In the elastic–plastic model employed herein \( (\text{Muir Wood, 2004; Smith and Griffiths, 2004}) \), the yield point depends only on the stress state. Consequently, the yield function can be written

\[
F = F\left(\sigma_x, \sigma_y, \sigma_z, \tau_{xy}\right)
\]

in which \( \sigma_x, \sigma_y, \) and \( \sigma_z \) are different components of normal stress, and \( \tau_{xy} \) is the shear stress. The following equation provides a common form of the Mohr–Coulomb criterion expressed in terms of the principal stresses used in SIGMA/W:

\[
F = \sigma_1 - \sigma_3 - (\sigma_1 + \sigma_3)\sin\phi - 2C\cos\phi
\]

where \( \sigma_1 \) and \( \sigma_3 \) are the major and the minor principal stresses, respectively.

The theory of incremental plasticity dictates that the yield function \( F<0 \) and, when the stress state is on the yield surface, \( dF = 0 \). This latter condition is termed the neutral loading condition and can be written in the following matrix form (the bracket sets \( < > \), and \( \{ \} \), are used to denote a row vector and a column vector, respectively):

\[
dF = \left< \frac{\partial F}{\partial \sigma} \right> (d\sigma) = 0
\]

The plastic strain is also postulated to be

\[
d\varepsilon_p = \lambda \left< \frac{\partial G}{\partial \sigma} \right>
\]

where \( G \) is a plastic potential function, and \( \lambda \) is a plastic scaling factor.

The plastic potential function, \( G \), used in SIGMA/W \( (\text{Chen and Zhang, 1990; Geo-Slope International, 2008}) \) has the same form as the yield function, \( F \) \( (\text{i.e., } G=F) \), except that the internal friction angle, \( \phi \), is replaced by the dilation angle, \( \psi \).

In the numerical modeling undertaken for this research, soil behavior parameters were determined using the results of triaxial tests. Having determined these parameters, the mesh pattern was designed, and the material properties and boundary conditions were assigned appropriately. The soil behavior parameters \( (\text{i.e., the elastic modulus and Poisson’s ratio}) \) were estimated by numerical simulation of special triaxial tests \( (\text{see previous explanation of these special triaxial tests}) \), including pore pressure and volume change control. The numerical simulation is applied to simulate loading to predict the complete stress–strain characteristics of the soil materials.

The finite element mesh used for the analysis of triaxial specimens is shown in Fig. 4A. We can see that symmetry is assumed about the vertical and horizontal centre lines; consequently, only one-quarter of the specimen is simulated. The dimensions of the simulated portion of the specimen are 0.025 m x 0.05 m, which is half of the width and height of a conventionally sized triaxial specimen. We can see that each specimen has been discretized into 50 eight-noded axisymmetric quadrilateral elements with two nodal degrees of freedom \( (\text{i.e., two displacement components in the } x\text{-direction and the } y\text{-direction}) \). The node spacing in both radial and vertical directions is 0.005 m.

The first phase of the simulation seeks to replicate the consolidation phase of the triaxial test. Note that consolidation is isotropic with the confining pressure equal to preconsolidation pressures for each material, as determined previously from a consolidation test. The isotropic stress state is simulated by applying a normal stress on the top and right hand sides of the sample. The consolidation stage is set as the parent, that is the initial condition for the subsequent simulations involving shearing (axial loading). The shearing phase of the analysis is simulated as a strain rate controlled test. The definition of the strain rate involves defining the number of time steps and the displacement that occurs over each step. Although the time steps are being defined in this method, it is perhaps more appropriate to think of the time steps in the SIGMA/W strain rate modeling as load steps in the axial loading of the triaxial test. Absolute time has no meaning in the context of these analyses. Based on the
axisymmetric condition, the number of load steps defined in the shear stage simulations was 37 and the incremental \( y \)-displacement (i.e., the boundary condition) at the top of the specimen was defined as \(-0.0002\) m (per load step), where the negative sign indicates downward displacement. Consequently, 37 load steps, multiplied by a \( y \)-displacement of \(-0.0002\) m per load step, gives a total vertical displacement of 0.0074 m. Total vertical \( y \)-displacements of 0.0074 m produce axial strains of about 0.15 (15%), which is similar to the value (15%) for the triaxial test.

By drawing the deviatoric stress against the axial strain values of different tests, an attempt has been made to specify the best model of the soil behavior in each case using the elastic modulus \( (E) \) and Poisson’s ratio \( (\nu) \) parameters. The material cohesion and internal friction angle values were determined from each triaxial test, and behavior modules are applied to the model by trial-and-error fitting of experimental and numerical curves. The results for each physical model test (with a particular density) contain three curves that are produced for different lateral pressures.

In Fig. 5 the stress–strain curves obtained using SIGMA/W are conformed to the laboratory curves for different densities of the CL (tests 1 to 3) and ML soils (tests 4 to 6), respectively. The results show that the Mohr-Coulomb model, among others investigated in this study (i.e., linear, hyperbolic, and cam-clay models) has a better agreement for the ML versus CL soil type. Our investigation shows that in the

![Fig. 5. Calibration curves of numerical stress–strain analysis with laboratory results for CL soil types: (A) 1.5 g cm\(^{-3}\), (B) 1.7 g cm\(^{-3}\), and (C) 1.8 g cm\(^{-3}\). Calibration curves for ML soil types are also shown in subplots (D) 1.4 g cm\(^{-3}\), (E) 1.5 g cm\(^{-3}\), and (F) 1.6 g cm\(^{-3}\).]
Fig. 6. Images of overhanging failures as observed in the physical models for CL (tests 1 to 3) and ML soil types (tests 4 to 6). (A) Test 1 with a dry density of 1.5 g cm\(^{-3}\): (i) before undermining, (ii) tensile failure in lower layer, and (iii) toppling failure of overhanging block. (B) Test 2 with a dry density of 1.7 g cm\(^{-3}\): (i) before undermining, (ii) tensile failure in lower layer, and (iii) toppling failure of overhanging block. (C) Test 3 with a dry density of 1.8 g cm\(^{-3}\): (i) before undermining, (ii) tensile failure in lower layer, and (iii) toppling failure of overhanging block. (D) Test 4 with a dry density of 1.4 g cm\(^{-3}\): (i) before undermining, (ii) toppling failure of overhanging block. Note: subsequent destruction of block caused by lack of cohesion. (E) Test 5 with a dry density of 1.5 g cm\(^{-3}\): (i) before undermining, (ii) toppling failure of overhanging block. (F) Test 6 with a dry density of 1.6 g cm\(^{-3}\): (i) before undermining, (ii) tensile failure in lower layer, and (iii) toppling failure of overhanging block.
3. Results and discussion

3.1. Experimental results

Results of the images and films prepared in this research indicate the occurrence of tensile and toppling failure mechanisms in overhanging blocks composed of both cohesive fine materials and noncohesive loess materials. Shear-type failure mechanisms were not observed in our tests. Fig. 6 shows soil blocks before testing, along with the mechanism of failure (tensile and toppling) occurring in the overhanging blocks. In Fig. 7, the detailed geometry of the soil blocks observed during the cantilever failures are shown. During these experiments, initial evaluations indicated that, as expected, the undermining depth (i.e., block width) had the greatest effect on the incidence of overhanging instability. Therefore, the effect of this parameter on instability was investigated in further detail. The undermining process was simulated by manually removing bank materials in 5-cm increments. After each increment was removed, overhang was observed for about an hour. If the overhanging block did not fail, this procedure was repeated until failure of the overhanging block occurred.

3.2. Numerical results

In this section numerically simulated deformation patterns and stress fields are compared to those obtained from the laboratory tests conducted for the six different overhangs. The mechanism of failure observed in four of these tests involved an initial tensile failure followed by whole block toppling. All such failures were associated with high shear strength materials. Beam failures were observed in the other two tests within overhanging banks composed of noncohesive, low density, loess material.

In Fig. 8 the numerical modeling results of soil behavior in the physical model tests, i.e., contour lines of x-y shear stress and patterns of soil deformation, are illustrated. According to the distribution of shear stress contour lines in Fig. 8, it is evident that the probable failure planes tend to cross in the vicinity of the zero shear stress line, though we must note that because of laboratory conditions and uncertainties small changes in the development of the failure plane at the top level of the overhang was possible. For instance, we could expect the failure to develop along one of the tension cracks at the top of the overhanging blocks.

3.2.1. Model validation

To assess the degree to which the numerical simulations undertaken in this study are reliable, we have validated the numerical model predictions by comparing them with the physical modeling test results. Specifically, we compared predictions of failure geometry and displacement with observations acquired using a particle image velocimetry (PIV) image processing technique. In this PIV...
method, patches of soil texture in the physical models were tracked through an image sequence. The PIV is a velocity-measuring technique that was originally developed in the field of experimental fluid mechanics (Adrian, 1991). The technique was originally implemented using photography of a seeded flow such that the resulting photographs contain image pairs of each seed particle. Within PIV analysis, the photograph is then divided into a grid of test patches with the displacement vector of each patch during the interval between image acquisition being identified by locating the peak of the autocorrelation function of each patch (White et al., 2003).

In the context of the experiments undertaken in this study, the soil deformation can be considered a low velocity flow process. For

Fig. 8. Numerical modeling results of soil block shear stresses at the threshold of cantilever failures for CL (tests 1 to 3) and ML soils (tests 4 to 6): (A) test 1, (B) test 2, (C) test 3, (D) test 4, (E) test 5, and (F) test 6. Note that observed failure planes are shown with dashed lines.
Table 3

<table>
<thead>
<tr>
<th>Test</th>
<th>Soil class</th>
<th>Dry soil density (g cm⁻³)</th>
<th>Cantilever mean displacement</th>
<th>Numerical model (mm)</th>
<th>Observed with PIV (mm)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>CL</td>
<td>1.5</td>
<td>9.0</td>
<td>8.5</td>
<td>+5.9</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.7</td>
<td>6.6</td>
<td>11.1</td>
<td>10.5</td>
<td>−4.5</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>ML</td>
<td>1.8</td>
<td>10.9</td>
<td>19.2</td>
<td>−43.0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.4</td>
<td>4.1</td>
<td>3.7</td>
<td>7.8</td>
<td>+13.1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.5</td>
<td>8.6</td>
<td>7.8</td>
<td>8.5</td>
<td>10.1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1.6</td>
<td>8.4</td>
<td>8.5</td>
<td>8.2</td>
<td>−0.2</td>
<td></td>
</tr>
</tbody>
</table>

*In the numerical model, the mean values of displacement have been calculated by using the displacement values estimated in all nodes within the cantilever block. But in the image processing method (PIV), the mean values of displacement have been calculated by using the displacement values measured in all test patches within the cantilever block.

instance, natural sand has its own texture in the form of different-coloured grains, and the light and shadow formed between adjacent grains when illuminating a plane of granular material. In the special case of nontextured fine material, similar to this research, texture can be added to an exposed plane of clay by the addition of coloured ‘flock’ material or fine sand (White et al., 2001). We took advantage of this by using the GeoPIV software, developed by White and Take (2002), to analyze the tests presented herein. The GeoPIV software is a Matlab module that implements the principles of PIV in a style suited to the analysis of the geotechnical tests undertaken herein. The development and performance of the software are described in detail by White (2002), with a more concise overview available in White et al. (2001, 2003); and we do not repeat these details here.

The simulated and observed deformation fields obtained from the numerical model and experimental observations are compared in Fig. 9. It is evident that the overall displacement patterns simulated in the numerical model are broadly similar to those estimated from PIV; however, the main simulated displacement values are less than the observations in tests 2 and 3. Furthermore, the simulated displacement contours are developed mainly from the right boundary of the soil block (i.e., behind the bank) to the extreme left boundary (i.e., representing the river side) of the model (except for ML soil types, tests 4 to 6, caused by weaknesses between soil particles and the low strength of loess material in which the overhanging block is deformed along a near vertical plane). Owing to errors near wall boundaries (attributable to factors such as crack development, falling surface soil particles, model edges in front of the soil block, etc.), the observed displacements (contour lines) in Fig. 9 are visualized in arbitrary domains that are smaller than the complete soil blocks in the main experiments. This is not a major limitation but simply reflects the point that, although the PIV analysis can discriminate tension and desiccation cracks developed in the surface of the soil block, the numerical finite element models are unable to model such features. The results also show that an observed sliding surface develops vertically in the soil block (in the vicinity of the zero shear stress surface, see also Fig. 8). This failure plane intersects the toe of the bank (i.e., undermining area) on a diagonal plane that is located at the section of minimum displacement (i.e., near the lower boundary). This area is under the maximum weight moment and, compared to other parts of the soil block, it changes faster from the elastic to the plastic phase.

The most important part of a soil block for comparing the simulated deformations is the overhanging part where the weight moment and internal resistance of the soil particles (in the tensile and compressive phases) are in competition at the prefailure stage. This area of the tested soil blocks was compared quantitatively (numerically and experimentally) in Table 3. In this table, the mean values of the displacement observed in all test patches within each cantilever block is compared with the mean value calculated at all numerical nodes located in the corresponding area of the soil block. It is evident that there is a relatively good agreement between the observed and numerical deformation results of tests 1, 4, 5, and 6. However, the amounts of displacement calculated by the numerical model are much less than the observed values in tests 2 and 3 (i.e., −40.5% and −43.0%, respectively). Fig. 9F also shows that the best estimation of cantilever displacement is obtained in test 6 (i.e., with a difference of only about −0.2% in Table 3). Seemingly, that the Mohr–Coulomb behavior model is not accurate enough for the CL soil types.

In Fig. 10, the distribution of the effective horizontal stress along the probable and actual failure planes is shown for all the tested overhanging blocks. In these tests, actual failures tended to be observed within the internal zones to the right of the probable failure planes discriminated by the numerical modeling. It follows that the integration of horizontal effective stresses (tensile and compressive) along these failure plane locations is greater than for the probable failure planes. The probable and actual failure planes were shown in Fig. 7. The probable failure planes (i, ii, iii) are vertical dashed lines that passed through points 6, 6, and 13 toward the top of the soil block for cohesive soils in tests 1 to 3, respectively (Fig. 7A). Similarly for tests 4 to 6 with noncohesive ML soil types, the vertical dashed lines that passed through points 6, 10, and 18 toward the top of the soil block are the probable failure planes (Fig. 7B).

Fig. 11 shows that the evolution of the horizontal effective stress distribution along the probable failure plane as the soil blocks are progressively undermined. We see that, during undermining, cantilevers tend to be inclined toward the tank (left) because of the moment weight of the overhanging soil block. This deviation tends to create tensile and compressive stresses on the upper and lower parts of the soil block, respectively. The important issue here is the development of this tensile zone toward the right part of the soil block in association with the increase of undermining width. Simulated horizontal stress distributions along the vertical line indicate that by increasing the undermining width the area of the tensile zone on the probable failure plane (i.e., a vertical surface developed from the end of undermining into the upper side of the soil block) will increase. This means that by increasing the undermining, the moment of the overhanging weight will increase, while the location of the rotation center is also moved upward.

3.2.2. Generalizing the numerical model
In this section we extend the numerical modeling domain to consider a wider range of materials, without constraining the model input (i.e., cohesion, internal friction angle, density, and soil behavior modulus) parameters to equate to the properties of the blocks employed in the laboratory tests. To achieve this we employ SIGMA/W in a series of sensitivity analyses to simulate the response to undermining beneath an overhanging block that consists of varying material properties. Note that the undermining steps and block geometries used in these simulations are, however, similar to those used in the laboratory tests. For this purpose, materials susceptible to cantilever failure were used for further analysis. The selected rivers were chosen on the basis of four main criteria: (i) the riverbank was composite; (ii) the riverbank was actively retreating; (iii) the geotechnical data were available; and (iv) the prevailing mechanism of bank retreat was cantilever failure. The first selected river was the Missouri River, in eastern Montana, where the riverbanks comprise relatively fine-grained soils in addition to banks of silty clay and clay loam, with some fine sand, that were suitable for cantilever failure (Simon et al., 2002). Secondly, Goodwin Creek in Mississippi is another example of a river that is experiencing excessive erosion and bank instability through the cantilever failure mechanism (e.g., Langendoen and Simon, 2008). Similarly, Dapporo et al. (2003) reported that cantilever failures prevailed in the upper reaches of the Arno River.
Fig. 9. Comparison between predicted (diagrams labeled i) and observed (diagrams labeled ii, as estimated from PIV) soil block deformations (in meters) at the threshold of cantilever failure for the CL (tests 1 to 3) and ML (tests 4 to 6) soils: (A) test 1, (B) test 2, (C) test 3, (D) test 4, (E) test 5, and (F) test 6. Note that the PIV results show the deformation domain for an arbitrary centered defined mesh that is smaller than the physically modeled soil block because of the introduction of errors near boundaries owing to factors such as crack development, falling surface soil particles, model edges in front of the soil block, etc. Observed failure planes are indicated with dashed lines.
Geotechnical data collected for the soil materials of these rivers are summarized in Table 4.

We previously reported that obtaining values for the elastic modulus and Poisson’s ratio of riverbank materials is highly problematic. Therefore, most researchers have used estimated values derived from the literature (e.g., Simon and Collison, 2001). In this research, we first identified the typical range of these soil behavior modules as reported in the soil mechanics literature (Bardet, 1997; Bowles,
We have then selected mean, minimum, and maximum values of the elastic modulus and Poisson’s ratio based on these data. Fig. 12 shows the effects of different combinations of soil behavior modules (i.e., mean, minimum, maximum) on cantilever displacement during the undermining process. It is evident that different materials have different responses to the stress–strain analysis (i.e., cantilever displacement). The materials used in test cases 1 and 5 were more resistant than the others, while cases 6 and 7 (which are comprised of silt and sandy-silt materials) were more unstable to the extent that cantilever failures occurred in the first and second steps of undermining, respectively (5 and 10 cm). The results also show that by changing the soil behavior modules from their mean to the minimum and maximum values, it was shown that the ranges of cantilever displacement change from 10–14 to 0.4–0.6 times the cantilever displacement for the mean value, respectively. It is, therefore, evident that the magnitude of simulated cantilever displacement is highly sensitive to soil behavior modules (i.e., elastic modulus and Poisson’s ratio). To explore this further, two additional sensitivity analyses were performed.

3.2.3. Sensitivity to elastic modulus and Poisson’s ratio

According to the data in Table 4, the modulus of elasticity of soils can vary from a minimum value of 2000 (kPa) to a maximum of 50,000 (kPa). Therefore, it is expected that this parameter exerts a considerable influence on the magnitude of cantilever displacement.
Boundary conditions were identical to those of the initial six simulations for the laboratory experiments, with Poisson’s ratio set to a minimum of 0.3 for the ML soil. The elastic modulus was varied between 2000 and 50,000 kPa, based on published values for materials likely to be encountered in an overhanging bank (Bardet, 1997). To investigate the effect of the soil modulus of elasticity ($E$) on cantilever displacement, calculations were performed for block models over a range of undermining widths, though the simulations were performed with the same basic block geometry as in the laboratory tests.

The resulting simulations (Fig. 13A) show an inverse nonlinear relationship between elastic modulus and cantilever displacement, with great sensitivity between 2000 and 10,000 kPa and somewhat less sensitivity above this value. The results show that assigning various values of elastic modulus in the above-mentioned range may result in as much as 90% variation in the corresponding values of cantilever displacement (Fig. 13A). In other words, changing the elastic modulus is not related to the undermining width and caused similar change to the cantilever displacement in each stage of undermining. Fig. 13B shows a positive nonlinear relationship between cantilever displacements and undermining values. Similarly, the elastic modulus has a similar effect on cantilever displacement during the undermining process.

In contrast to the elastic modulus, the Poisson’s ratio of the soils lies within a shorter range of variation, usually between 0.1 and 0.5 for various soil types (Bardet, 1997). Results of calculations on the

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**Fig. 11.** Evolution of horizontal effective stresses along the probable failure plane as a function of increasing undermining width in each step for the CL (tests 1 to 3) and ML (tests 4 to 6) soil types: (A) test 1, (B) test 2, (C) test 3, (D) test 4, (E) test 5, and (F) test 6.
effects of Poisson’s ratio for various undermining widths in case 1 indicate that the range of cantilever displacements will be extended by increasing the value of Poisson’s ratio. In contrast to the unique form of the response curve for the elastic modulus, varying the Poisson’s ratio (Fig. 13C) causes different relationships for different undermining widths, from a positive quasilinear relationship (minimum undermining) to an inverse nonlinear relationship (maximum undermining). The results of computations for a 5-cm undermining increment (minimum value) show that, increasing the value of the Poisson’s ratio from 0.1 to 0.5 will lead to an increase in cantilever displacement of about 32%. For the maximum value of undermining width, the effect is about 24%. Notably, these values are the results of computations only for case 1 considered here. Obviously, for other cases with smaller stiffness, the effect will be less. In contrast to Fig. 13C, and similar to Fig. 13B, the resulting simulations for different Poisson’s ratio (Fig. 13D) show positive nonlinear relationships between cantilever displacement and undermining values. Fig. 13D illustrates that the effect of Poisson’s ratio on cantilever displacement changed from about 70% (for \(\nu = 0.1\)) to about 40% (for \(\nu = 0.5\)) at the maximum width of undermining in case 1.

4. Conclusion

This study has presented an integrated laboratory and numerical study of cantilever failures. In the physical model tests, two types of erodible soil with three different densities were investigated. Deformations of these experimental blocks were also simulated numerically, and the stress and deformation distributions obtained from the numerical model were compared with the laboratory observations. A summary of key results is provided as follows:

- At least for this laboratory study, the dominant failure mechanism is observed to be toppling. This is in contrast to the assumptions employed by previous researchers who have suggested that the shear-type mechanism of cantilever failure may be prevalent (e.g., Darby et al., 2007; Rinaldi et al., 2008).
- The undermining depth in riverbanks consisting of CL soil types, especially of low density, is greater than riverbanks consisting of loess materials (ML soil type).
- Numerical modeling of overhangs utilizing the Mohr–Coulomb behavior model gives a good agreement with laboratory observations, except for high density CL soil types for which minor differences were observed between model results and laboratory observations.
- Numerical modeling of triaxial tests shows that the Mohr–Coulomb model has a better agreement for ML than CL soil types.
- Numerical modeling results show that by increasing the undermining depth the location of maximum tensile stress on the bank surface moves backward (right) to the soil block and its location is not along the top of the vertical overhang. This is in accordance with laboratory results, in which all failures take place in greater upper width than undermining depth.
- Numerical simulations showed that the magnitude of cantilever displacement was highly sensitive to elastic modulus but relatively insensitive to Poisson’s ratio.

Acknowledgement

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References


Table 4

Geotechnical properties of appropriate streambanks for cantilever failure occurrence as reported in a range of previous studies.

<table>
<thead>
<tr>
<th>Number</th>
<th>River</th>
<th>Location</th>
<th>USCS</th>
<th>Soil type</th>
<th>Bulk unit weight (kN m(^{-3}))</th>
<th>Cohesion (kPa)</th>
<th>Internal friction angle (°)</th>
<th>Elastic modulus a (MPa)</th>
<th>Poisson’s ratio a</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Missouri</td>
<td>Nobly</td>
<td>ML</td>
<td>Silt</td>
<td>14.2</td>
<td>13.2</td>
<td>30.1</td>
<td>2–20</td>
<td>0.3–0.35</td>
<td>Simon et al. (2002)</td>
</tr>
<tr>
<td>2</td>
<td>Culberston</td>
<td>Vournas</td>
<td>CL-CH</td>
<td>Clay (low to high plasticity)</td>
<td>16.6</td>
<td>8.7</td>
<td>29.5</td>
<td>2–50</td>
<td>0.1–0.5 b</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Vournas</td>
<td>Fraizer pump</td>
<td>CL</td>
<td>Clay and clay</td>
<td>16.6</td>
<td>8.5</td>
<td>20.9</td>
<td>2–50</td>
<td>0.1–0.5 b</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Fraizer pump</td>
<td>Milk river</td>
<td>CL</td>
<td>Clay (low plasticity)</td>
<td>15.4</td>
<td>8.1</td>
<td>28.8</td>
<td>2–50</td>
<td>0.1–0.5 b</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Milk river</td>
<td>Missouri</td>
<td>CH</td>
<td>Clay (high plasticity)</td>
<td>16.3</td>
<td>27.7</td>
<td>9.9</td>
<td>2–50</td>
<td>0.1–0.5 b</td>
<td></td>
</tr>
</tbody>
</table>

a Because the lack of data for soil behavior modules in the previous studies (above references), these data are arbitrary from the available literature (e.g., Bardet, 1997; Bowles, 1997).
b For computational purposes, Poisson’s ratio can never be 0.5. Physically, this means that the volumetric strain tends toward zero as Poisson’s ratio, \(\nu\), approaches 0.5. Even values > 0.49 can cause numerical problems. Consequently, SIGMA/W limits the maximum value for Poisson’s ratio, to 0.49.
Fig. 12. Effects of soil behavior modules (i.e., elastic modulus and Poisson’s ratio) on cantilever displacement with step-by-step undermining process: (A) case 1, (B) case 2, (C) case 3, (D) case 4, (E) case 5, and (F) case 7. Note: Because of low strength of the material used in case 6, the model simulation result shows cantilever failure in the first step (i.e., 5 cm undermining).


Fig. 13. (A) Effect of elastic modulus on cantilever displacement of case 1, in different undermining widths (Poisson's ratio assumed to be constant, $\nu = 0.3$). (B) Effect of undermining development on cantilever displacement of case 1, in different elastic modules ($\nu$ assumed to be constant, $E = 2$ MPa). (C) Effect of Poisson's ratio on cantilever displacement of case 1, in different undermining widths (elastic modulus ratio assumed to be constant, $E = 2$ MPa). (D) Effect of undermining development on cantilever displacement of case 1 in different Poisson's ratio (elastic modulus assumed to be constant, $E = 2$ MPa).


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