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Vulnerability in Graphs - a Comparative Survey

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ABSTRACT. The concept of tenacity of a graph G was introduced in References [5,6] as a useful measure of the "vulnerability" of G. In assessing the "vulnerability" of a graph one determines the extent to which the graph retains certain properties after the removal of vertices or edges. In this paper we will compare different measures of vulnerability with tenacity for several classes of graphs.

INTRODUCTION

If we think of the graph as modeling a network, the vulnerability measures the resistance of the network to disruption of operation after the failure of certain stations or communication links. In this survey we will restrict ourselves to the study of vertex versions of vulnerability.

Throughout this paper we will let n be the number of vertices of G, and we use \( \alpha(G) \) to denote the independence number of G. Let A be a subset of V(G). The neighborhood of A, N(A), consists of all vertices of G adjacent to at least one vertex of A. We define G-A to be the graph induced by the vertices of V-A. Also, for any graph G, \( \tau(G) \) is the number of vertices in a largest component of G and \( \omega(G) \) is the number of components of G. A cutset of a connected graph G is a collection of vertices whose removal results in a disconnected graph.

The connectivity of G, \( \kappa = \kappa(G) \) is the minimum order of a cutset of G.

The binding number of a graph G was introduced by Woodall in [17] and is defined as \( \text{bind}(G) = \min \left\{ \frac{|N(A)|}{|A|} \right\} \), where the minimum is taken over all \( A \subseteq V(G) \) with \( A \neq \phi \) and \( N(A) \neq V(G) \). The binding number has also been studied in [8,10,11,12,13,16] among others.

The concept of integrity of a graph G was introduced in [2] as a useful measure of the vulnerability of a graph G. The integrity of a graph G is

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