Comprehensive data for rapid calculation of notch stress intensity factors in U-notched Brazilian disc specimen under tensile-shear loading

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\textbf{A B S T R A C T}

Stress distribution was analyzed around the tip of U-notches in a disc-type test sample, called U-notched Brazilian disc (UNBD), under combined tensile-shear loading. The notch stress intensity factors (NSIFs) which are vital parameters in brittle fracture investigation of U-notched engineering components were computed for UNBD specimen utilizing the finite element (FE) method for different notch geometries and wide range of mode mixities from pure mode I to pure mode II. To simplify the results to be used in engineering design, the NSIFs were converted to the dimensionless parameters called the notch shape factors (NSFs). These parameters are useful to compute more rapidly and conveniently the NSIFs in UNBD specimen for different notch tip radii. As a main result, it is shown that the NSFs presented in this work combined with the appropriate fracture criteria can be used to predict the load-cell capacity of the test machine required for fracture test of UNBD specimens made of various brittle materials.

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1. Introduction

Two major mechanisms of failure in engineering components are brittle fracture and ductile tearing. Ductile tearing often occurs in ductile materials and usually proceeded with moderate or large-scale yielding. On the contrary, brittle fracture is an instantaneous type of failure which occurs in brittle and quasi-brittle materials without any considerable plastic deformation. Due to its catastrophic consequences, brittle fracture in engineering components has been a topic of great interest to the researchers since many years ago.

In the presence of stress concentrators like defects, surface scratches, cracks and notches, brittle and quasi-brittle materials such as ceramics, rocks, brittle polymers, concrete, graphite and soda-lime glass etc. are so vulnerable to failure. Thus, many researchers have frequently studied the brittle fracture phenomenon in engineering components and structures containing cracks and notches experimentally and theoretically.

Various types of notches such as U and V-shaped ones are usually used in engineering elements because of special design requirements. For example, one can remember the use of V and U-shaped threads in screws [1], welded joints in structures [2–5] and V and U-notches in shafts [6,7] etc. Notches act as stress raisers and reduce considerably the load-bearing capacity of the notched component due to stress concentration at the notch tip vicinity. If the notched component is made of a brittle material and subjected to mechanical loading, fracture may occur abruptly from the notch. Thus, it is necessary to prevent brittle fracture in engineering applications by using reliable fracture criteria and/or by performing fracture tests.

Similar to cracks and V-notches, U-shaped notches can be generally subjected to three various in-plane loadings often called pure mode I, pure mode II and mixed mode I/II. Under pure mode I loading conditions, notch faces open with respect to the notch bisector line without any sliding. Any pure in-plane sliding of the notch faces with respect to the notch bisector line is called pure mode II deformation and combined opening and in-plane sliding of the notch faces is well-known as mixed mode I/II loading. Many evidences can be found in engineering applications in which U-notches are subjected to in-plane loading conditions. For instance, U-notches experience mode I loading conditions in screws [1] and mode I and mixed mode I/II conditions in axles [6].

Several failure theories can be found in literature for predicting brittle fracture in notched elements that almost all of them have been developed based on the linear elastic fracture mechanics (LEFM) such as that presented by Sih and Ho [8] based on the critical energy density theory and those suggested on the basis of the local strain energy density concept (see for example [9–14]). Like

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cracks, the stress intensity factors play a vital role in governing the brittle fracture phenomenon in notched components. For example, the notch stress intensity factors (NSIFs) have been used in Refs. [15–21] and [22,23] for estimating the onset of brittle fracture in V-notched and U-notched components, respectively.

Experimental investigations dealing with determining the fracture toughness (i.e. the critical values of the NSIFs) of notched elements have been performed by several researchers using a limited number of test samples. For example, one can find in literature the single-edge notch tension (SENT) [24], double-edge notch tension (DENT) [25], three-point bend (TPB) [26] and four-point bend [27] specimens. Recently, Ayatollahi and Torabi [23] suggested and used a new disc-type test specimen containing a bean-shape central slit and tested under compressive loading, called U-notched Brazilian disc (UNBD) specimen, for measuring experimentally the notch fracture toughness of plexi-glass (PMMA) and soda-lime glass under pure mode II loading conditions. The UNBD specimen is, in fact, a modified version of the centrally cracked Brazilian disc (CCBD) specimen which has been frequently utilized in mixed mode I/II fracture tests of brittle components containing sharp cracks (see for example [28–30]).

Due to the importance of the NSIFs in the brittle fracture assessment of notched components and also, the importance of rapid and convenient calculation of the NSIFs for widely used test specimens, Torabi and Taherkhani [31] have calculated more recently the NSIFs for the V-notched Brazilian disc specimen (V-BD) for various notch angles and different notch tip radii. Similar to their work, a total number of 250 finite element (FE) analyses were carried out in the present work to calculate the notch stress intensity factors for UNBD specimen under different mixed mode loading conditions from pure mode I to pure mode II for different notch tip radii. For more simplicity of using the obtained results in practical applications, the NSIFs are converted to the dimensionless parameters, called the notch shape factors (NSF). The NSFs which depend on the specimen diameter, specimen thickness, notch length and notch tip radius are presented in several suitable graphs. Once the NSFs and the geometric parameters are known for a UNBD specimen, one can directly determine the NSIFs for any combination of modes I and II loadings without requiring FE analysis. In the next section, closed-form expressions are presented in a general form for the linear elastic stress distribution around the tip of a U-notch. These expressions are then utilized to compute the NSIFs and the NSF in the forthcoming sections.

Nomenclature

- $K_{II}^{V,\rho}$: notch stress intensity factor (NSIF)-mode I
- $K_{II}^{U,\rho}$: notch stress intensity factor (NSIF)-mode II
- $K_{IC}$: mode I notch fracture toughness
- $K_{I}^{U,\rho}$: mode II notch fracture toughness
- $RNL$: relative notch length
- $RNR$: relative notch tip radius
- UNBD: U-notched Brazilian disc
- $Y_{I}^{U,\rho}$: notch shape factor (NSIF)-mode I
- $Y_{II}^{U,\rho}$: notch shape factor-mode II
- $2\alpha$: V-notch angle
- $\beta$: loading angle for UNBD specimen
- $\beta_{II}$: loading angle corresponding to pure mode II loading
- $\lambda_{I}$: eigenvalues
- $\mu_{I}$: eigenvalues (real parameters)
- $\sigma_{rr}$: radial stress
- $\sigma_{\theta\theta}$: in-plane shear stress
- $\sigma_{\rho\rho}$: tangential stress

2. Stress field around a U-notch

Filippi et al. [32] developed an expression for mixed mode I/II stress distribution around a round-tip V-shaped notch shown in Fig. 1. The stress distribution is an approximate formula because it satisfies the boundary conditions only in a finite number of points on the notch edge and not on the entire edge. They obtained the stress distribution using a conformal mapping in an auxiliary system of curvilinear coordinates “U and V” that are related to the Cartesian coordinates “X and Y” as $(X + iY) = (U + iV)^{q}$ [32]. The power “$q$” is a real positive coefficient ranging from 1 (for a flat edge) to 2 (for a crack).

The mixed mode I/II stresses can be written as

$$
\sigma = \frac{K_{I}^{V,\rho}}{\sqrt{2\pi r}} \left[ \begin{array}{c} f_{00}(\theta) \\ f_{0\theta}(\theta) \\ f_{\theta\theta}(\theta) \end{array} \right] + \frac{K_{II}^{V,\rho}}{\sqrt{2\pi r}} \left[ \begin{array}{c} f_{\theta\theta}(\theta) \\ f_{\theta\theta}(\theta) \\ f_{\theta\theta}(\theta) \end{array} \right] \left( \begin{array}{c} f_{00}(\theta) \\ f_{0\theta}(\theta) \\ f_{\theta\theta}(\theta) \end{array} \right)
$$

where $K_{I}^{V,\rho}$ and $K_{II}^{V,\rho}$ are the mode I and mode II notch stress intensity factors, respectively. The parameter $r_{0}$ is the distance between the origin of the polar coordinate system and the notch tip. The functions $f_{00}(\theta)$ and $f_{\theta\theta}(\theta)$ have been reported in Ref. [23] and the eigenvalues $\lambda_{I}$ and $\mu_{I}$ which depend upon the notch angle have been reported in Ref. [32].

According to a relation that exists between the Cartesian and the curvilinear coordinate systems, $r_{0}$ can be written as [23,31]:

$$
r_{0} = \frac{q - 1}{q} \rho, \quad q = \frac{2\pi - 2\alpha}{\pi}
$$

where “$2\alpha$” is the notch angle and $\rho$ is the notch tip radius. The expressions for NSIFs are [23,31]:

$$
K_{I}^{V,\rho} = \sqrt{2\pi} \frac{\sigma_{\rho\rho}(\theta)}{1 + \omega_{1}(r/r_{0})^{\mu_{1} - \lambda_{1}}}
$$

$$
K_{II}^{V,\rho} = \sqrt{2\pi} \frac{\sigma_{\rho\rho}(\theta)}{1 + \omega_{2}(r/r_{0})^{\mu_{2} - \lambda_{2}}}
$$

where $\sigma_{\rho\rho}$ and $\sigma_{\rho\rho}$ are the tangential and the in-plane shear stresses, respectively. The parameters $\omega_{1}$ and $\omega_{2}$ have been presented in
Ref. [23,31]. If the values of \(\omega_1\) and \(\omega_2\) are known, the NSIFs can be obtained from Eqs. (3) and (4) as

\[
k_I^{V,\rho} = \sqrt{2\pi} \frac{\sigma_0(r_0, 0) r_0^{1 - \lambda_1}}{1 + \omega_1}
\]

(5)

\[
k_{II}^{V,\rho} = \lim_{r \to r_0} \sqrt{2\pi} \frac{\sigma_0(r, 0) r_1^{1 - \lambda_2}}{1 - \rho/r_0^{1 - \lambda_2}}
\]

(6)

When the notch angle \(2\alpha\) is zero, then \(q = 2\) and the round-tip V-notch becomes a U-notch. Therefore, one can write

\[
r_0 = \frac{\rho}{2}
\]

(7)

By substituting Eq. (2) into Eq. (1), an expression for the elastic stress distribution in the K-dominant region around U-shaped notches can be derived. This expression coincides with that of Creager and Paris [33] which has been proposed to describe the stress field ahead of a rounded-tip crack.

Utilizing Eq. (7) and considering that \(\omega_1\) and \(\omega_2\) for U-notches are 1 and \(-1\), respectively [23], Eqs. (5) and (6) can be ultimately rewritten for U-notches as

\[
k_I^{U,\rho} = \sqrt{2\pi} \frac{\sigma_0(0, 0)}{2}
\]

(8)

\[
k_{II}^{U,\rho} = \lim_{r \to r_0} \sqrt{2\pi} \frac{(\sigma_{ref})(r, 0)}{1 - (\rho/r_0^{1 - \lambda})}
\]

(9)

Note that the parameter \(r\) in Eq. (9) cannot be directly substituted by \(\rho/2\) because the denominator becomes zero. Therefore, the limit of expression very close to the notch tip is utilized to compute \(k_{II}^{U,\rho}\).

The NSIFs can be calculated by using the FE method as elaborated in the next section. These parameters have the same unit of measure (MPa m\(^{1/2}\)) for mode I and mode II which is similar to the unit for the stress intensity factors for cracks.

It should be highlighted that the NSIFs are very important parameters in brittle fracture assessment of U-notched components because, wide range of the brittle fracture criteria in notched domains present their predicting expressions/envelopes (i.e. the fracture toughness) in terms of the NSIFs. Therefore, one should compute the NSIFs for given U-notched brittle component and compare them with the theoretically predicted values in order to specify whether or not the fracture occurs for the U-notched component.

3. The U-notched Brazilian disc specimen

Few specimens have been suggested in the past for experimental investigation of brittle fracture in engineering components containing sharp cracks [34]. A well-known specimen used most commonly in the past for performing mixed mode I/II fracture tests is the centrally cracked Brazilian disc specimen (see for example [28,29]). A modified version of the CCBD specimen, called UNBD, has been suggested by Ayatollahi and Torabi [23] which could be used to perform mixed mode (especially pure mode II) fracture tests for U-notches. Fig. 2 shows the UNBD specimen schematically.

Note that the UNBD specimen is, in fact, the CCBD specimen for which the central crack is replaced with a central bean-shape slit. In Fig. 2, \(\beta\) is the angle between the loading direction and the notch bisector line and the parameters \(D\) and \(P\) are the disc diameter and the applied load, respectively. When the direction of the applied load \(P\) is along the notch bisector line (i.e. \(\beta = 0\)), the round borders of the slit are subjected to pure mode I deformation. When the angle \(\beta\) increases gradually from zero, the loading condition varies from pure mode I towards pure mode II. For a specific angle, called \(\beta_{II}\), pure mode II deformation is achieved. The mode II loading angle \(\beta_{II}\) is always less than 90\(^\circ\) and depends upon the notch length and the notch tip radius. The angles \(\beta_{II}\) can be determined by using the finite element method as described in Section 4.

4. Finite element analysis

In this section, the FE procedures needed for determining the NSIFs and the mode II loading angle \(\beta_{II}\) are elaborated. To compute NSIFs for each UNBD specimen with arbitrary U-notch geometry, it is essential to create a FE model for the specimen and perform linear elastic stress analysis. Under an arbitrary load \(P\), the values of the tangential and shear stresses \((\sigma_{tt} and \sigma_{tt})\) along the notch bisector line are obtained from the FE analysis and the NSIFs can be calculated by using Eqs. (8) and (9).

A plane-stress FE model with a total number of about 60,000 Quad-8 elements was created for each UNBD specimen and mixed mode stress analyzes were entirely performed in four \(\beta\) angles of \(0^\circ, 10^\circ, 20^\circ\) and \(\beta_{II}\) (\(\beta\) angles were trivially selected between \(0^\circ\) and \(\beta_{II}\)) for computing the NSIFs. Fig. 3 shows a sample meshed UNBD specimen.

The commercial software ANSYS was utilized for conducting the stress analysis. Five values of the relative notch lengths (RNL) \(d/D\) (0.1, 0.2, 0.3, 0.4 and 0.5) and ten values of the relative notch tip radii (RNR) \(r/d\) (0.01 to 0.4) were considered for numerical analysis. Note that the parameter \(d\) is the overall length of the central slit (i.e. the distance between the two round notch tips). The RNL and RNR parameters are defined in the next section. The reference load \(P\), the specimen diameter and the specimen thickness were arbitrarily considered to be 1 kN, 80 mm (as presented in Ref. [23]) and 10 mm, respectively in the entire analyzes. Very fine grids were utilized at the notch tip vicinity because of high stress gradient. A total number of 250 analyzes were performed to compute the NSIFs for UNBD specimens.

The procedure for determining the mode II loading angle \(\beta_{II}\) has been elaborated in Ref. [23], but for making more convenience to follow the calculations, it is briefly reviewed herein. In order to obtain the mode II loading angle \(\beta_{II}\), \(\beta\) was gradually increased from zero and the tangential stress at the notch tip \((\sigma_{tt}(\rho/2, 0))\) was obtained from the FE results. Under a fixed load \(P\), as \(\beta\) becomes larger, the value of \(\sigma_{tt}(\rho/2, 0)\) decreases. According to Eq. (8), \(\beta_{II}\) is the angle for which \((\sigma_{tt}(\rho/2, 0))\) and \(K_{II}^{U,\rho}\) are identical to zero and the U-notch is subjected to pure mode II loading. The FE analysis of UNBD specimens demonstrated that \(\beta_{II}\) enhances for larger RNRs for each value of RNL in its entire domain of variation. These results are presented by means of illustrations in the next section.
It is essential to highlight that the stability of the computed NSIFs is a vital parameter because the engineers require stable and reliable NSIFs values for their designs and calculations. Thus, this important fact should be evaluated and proved for our calculations in UNBD specimen. For this purpose, first, the validity of the numerically determined stress distribution around the notch tip should be verified. Therefore, several mesh patterns with different numbers of elements were created for each UNBD specimen and FE stress analysis was performed accordingly. This procedure was continuously followed until the mesh-independency of the analysis was demonstrated. Then, the validity of the NSIFs computed in accordance with Eqs. (8) and (9) should be proved. As seen in Eq. (8), the value of the tangential stress exactly at the notch tip is required for determining $K_{\text{II}}^{\text{UL}}$. Therefore, there is not any reason to make doubt about the validity of the $K_{\text{II}}^{\text{UL}}$ values. The only doubtful parameter in our calculations is the role of distance $r$ ahead of the notch in computing $K_{\text{II}}^{\text{UL}}$ (see Eq. (9)) since $K_{\text{II}}^{\text{UL}}$ is obtained from a limit computation. To reach to a stable value for $K_{\text{II}}^{\text{UL}}$, this parameter was calculated for several nodes on the notch bisector line and the final value was selected for $K_{\text{II}}^{\text{UL}}$ by conducting extrapolation technique. Finally, Eq. (1) was plotted for each UNBD specimen based on the calculated NSIFs and the obtained analytical curve was compared with the curve of numerical stress distribution resulted from FE analysis in a unified graph. The mode II NSIF was slightly modified everywhere required till the curves match each other at the K-dominant region (i.e. the notch tip vicinity) as more as possible. The outputs of this procedure are presented in this paper as the final mode II NSIFs for UNBD specimens.

5. Results

In order to present the NSIFs for the UNBD specimen compactly and to make use of them convenient in the engineering design, two dimensionless parameters called the mode I and mode II notch shape factors are defined herein as

$$Y_{\text{I}}^{\text{UL}} = \frac{\sqrt{\pi} Dt}{2d^2} \frac{R}{P}$$

$$Y_{\text{II}}^{\text{UL}} = \frac{\sqrt{\pi} Dt}{2d^2} \frac{R}{P}$$

The parameter $t$ in the above equations is the specimen thickness. As seen in Eqs. (10) and (11), the NSIFs $Y_{\text{I}}^{\text{UL}}$ and $Y_{\text{II}}^{\text{UL}}$ depend on the loading angle $\beta$, the relative notch length (RNL) and the relative notch tip radius (RNR). The ratio $Dt/P$ (mm$^2$/N) is, in fact, the reversed far-field reference stress applied to the UNBD specimen. Unlike $K$, $Y$ does not depend on the disc dimensions ($D$ and $t$) and also on the applied load $P$.

Fig. 4 represents the variations of the mode II loading angle $\beta_{\text{II}}$ versus RNR for different RNLs. It is clear from this figure that for the entire values of RNLs, as RNR increases from 0.01 to 0.4, $\beta_{\text{II}}$ enhances typically about 6–8°. Also shown in Fig. 4 is that for a constant value of RNR, $\beta_{\text{II}}$ decreases as RNL increases.

Figs. 5 and 6 show the variations of the mode I and mode II NSFs, respectively versus the loading angle $\beta$ for different values of RNL and RNR.

It is seen in these figures that as $\beta$ enhances from zero (i.e. the loading angle corresponding to pure mode I) to $\beta_{\text{II}}$, $Y_{\text{I}}^{\text{UL}}$ decreases from a finite positive value to zero and $Y_{\text{II}}^{\text{UL}}$ enhances from zero to a finite value. For a constant RNR, increasing the RNL results in the enhancement of $Y_{\text{I}}^{\text{UL}}$ and $Y_{\text{II}}^{\text{UL}}$ because of higher stress concentration at the notch tip vicinity.

6. Discussion

Ayatollahi and Torabi [23] has originally suggested and utilized the UNBD test sample for measuring the load-bearing capacity
(i.e. the fracture toughness) of U-notched components made of two quasi-brittle and brittle materials (i.e. PMMA at room temperature and soda-lime glass) under pure mode II loading conditions. As well-known, the standard fracture tests on notched specimens are usually conducted with the aim to determine the notch fracture toughness. The main parameter that usually obtained from the simple fracture toughness tests on brittle materials is the fracture load while the results of the fracture criteria that estimate theoretically the fracture toughness of notched components are widely available in the form of the NSIFs (see for example [15–23]). Therefore, the researchers require necessarily converting the experimentally obtained fracture loads to the
corresponding values of the critical NSIFs in order to compare the experimental results with the theoretical ones (see [23]). In order to calculate the NSIFs related to a measured fracture load of UNBD specimen more conveniently, one can simply follow a straight-forward procedure described herein. As previously mentioned, the finite element method can be used to analyze the elastic stress distribution around the notch tip for UNBD specimen and to calculate the notch stress intensity factors. To compute the NSIFs for a UNBD specimen under a given load and mixed mode I/II loading conditions by using the graphs of NSFs (i.e. Figs. 5 and 6), one should first calculate the parameters RNL and RNR for the specimen and obtain the values of the NSFs from Figs. 5 and 6. Then, by substituting NSFs into Eqs. (10) and (11), the NSIFs can be directly achieved by knowing the given fracture load $P_c$ and the specimen dimensions $D$ and $t$. 

Fig. 6. The variations of the mode II notch shape factor versus the loading angle $\beta$ for different values of RNL and RNR.
Results below can be achieved from Figs. 4–6:

a. For a constant RNR, as RNL increases, both $\psi_{I}^{U}$ and $\psi_{II}^{U}$ enhance because of higher stress gradient.

b. For a constant RNL, as RNR increases from 0.01 to 0.4, the mode II loading angle $\beta_{II}$ increases typically 6–8°.

c. For a constant RNL, as RNR increases from 0.1 to 0.5, the mode II loading angle $\beta_{II}$ decreases typically 5–8°.

The same approach using FE analysis has been recently employed by Ayatollahi and Torabi [23] to calculate the critical NSFs (called the notch fracture toughness) for several UNBD specimens made of PMMA and soda-lime glass tested under pure mode II loading conditions. They utilized the computed NSFs in order to verify the theoretical results of a recently developed failure criterion called UMTS [23] in pure mode II loading conditions and found satisfactory agreements. Although not shown here, comparing the values of the mode II NSFs obtained comprehensively in the present work with those numerous values reported in Ref. [23] shows very good consistency and demonstrates the validity of the present calculations.

Rather similar FE results of the shape factors have been reported in literature by Ayatollahi and Aliha [35] for the Brazilian disc specimen containing a central sharp crack (i.e. the CCD specimen). They presented a wide range of data for the stress intensity factors (SIFs) and also the shape factors which are useful parameters in estimating the mixed mode II fracture toughness of brittle materials [35]. Another way of verification was followed by the authors to ensure the validity of the calculations presented in this work. From a view point of geometry, a sharp crack can be considered as a U-notch with zero notch tip radius. Therefore, the UNBD specimen in such conditions becomes a CCD specimen with the same flaw length. Consequently, we could compute the NSFs for the CCD specimen by using our described procedure and compare the results with those reported in Ref. [35] and obtained from another computational procedure. For this purpose, first, the geometrical parameters corresponding to $\rho = 0$ were entirely substituted into Eqs. (8) and (9) and then into Eqs. (10) and (11) to obtain the NSFs and NSFs expressions for the CCD specimen. Then, two FE models of the CCD specimen with RNL = 0.3 and 0.5 were created and the stress analyses were performed. Ultimately, comparing the obtained results of the shape factors (named as the geometry factor in Ref. [35]) with those reported in Fig. 10 of Ref. [35] (obtained by the combination of the displacement and the J-integral methods) revealed a very good consistency with a discrepancy of about 2% demonstrating again the validity of our calculations. It is worth mentioning that the idea to suggest Eqs. (8) and (9) was taken from similar expressions presented in the past by Ayatollahi and Aliha [35] for sharp cracks in CCD specimens. The main difference is the fact that despite the relative flaw (crack or notch) length, the number of the parameters affecting the NSFs for UNBD specimen (i.e. 2 parameters) is more than those affecting the stress intensity factors (SIFs) for CCD specimen (i.e. 1 parameter).

An interesting application of the curves of the NSFs is to estimate more conveniently and quickly the fracture load for the UNBD specimens made of different brittle or quasi-brittle materials and hence to select an appropriate load cell for the test apparatus prior to the experiments. To do this, one should first obtain the NSFs for the UNBD specimen from the curves presented in Figs. 5 and 6. Correspondingly, the notch fracture toughness $K_{Ic}$ or $(K_{Ic}^{U}$ and $K_{II}^{U}$) (i.e. the critical values of the NSFs) should be predicted by using appropriate fracture criteria such as the cohesive zone model (CZM) and the mean stress criterion (MSC) [16] in mode I loading conditions and the U-notched maximum tangential stress criterion (UMTS) [23] in mixed mode and pure mode II loadings. Finally, substituting the predicted critical NSFs and the obtained NSFs into Eqs. (10) and (11), also considering that the geometrical values (i.e. $D$ and $r$) in the equations are known, one can determine the fracture load $P_{cr}$ related to the critical conditions and select an appropriate load cell.

The U-notched Brazilian disc and the V-notched Brazilian disc specimens are completely similar in shape except in the type of their notch. Similar to other specimens utilized in brittle fracture testing, the Brazilian disc specimen, including CCBD, UNBD and VBD, has some advantages/disadvantages. Two major advantages are $i$. It does not require complicated and additional test fixtures and $ii$. It can be used to measure the experimental fracture toughness of brittle materials in the entire domain of mixed mode I/II from pure mode I to pure mode II. This can be simply done by rotating the specimen on the bottom fixture of the test machine. The major disadvantage of the BD specimen is that it is tested under compressive loading which may result in buckling before brittle fracture. Although selecting the specimen dimensions in accordance with the ones suggested in the literature guarantees the success of the test, buckling analysis is essential before brittle fracture test for those BD specimens having arbitrary dimensions to estimate the minimum buckling load. If the theoretically predicted fracture load (as previously mentioned, this can be performed by using a combination of the NSFs and the appropriate fracture criterion) is considerably lower than the computed buckling load, the test will be successful. Otherwise, the specimen dimensions must be changed to satisfy the above requirement.

7. Conclusions

The dimensionless parameters NSFs were calculated and illustratively represented for the UNBD specimen for different notch lengths and various notch tip radii in a range of mode mixity from pure mode I to pure mode II. These extensive results can be conveniently utilized by engineers in fracture tests of UNBD specimens in order to convert the experimentally obtained fracture loads to the corresponding notch fracture toughness values. Comparing the NSFs values computed in the present work with those reported in literature demonstrated the validity of the computations. As previously mentioned, the NSFs combined with appropriate failure criteria can be rapidly and more conveniently utilized for estimating the fracture load for the UNBD specimen made of any brittle material. Therefore, one can select an appropriate load cell for the test machine prior to the experiments. The main advantage of the present work is that the curves of NSFs provide a useful toolbox for the researchers and engineers to calculate the NSFs for the UNBD specimen under mixed mode in-plane loading conditions without requiring creating FE models and conducting time-consuming analysis.

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