Estimation of tensile load-bearing capacity of ductile metallic materials weakened by a V-notch: The equivalent material concept

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Ductile commercial steel was equated from the viewpoint of strain energy density with a virtual brittle material of the same elastic modulus by using a novel concept, called the equivalent material concept (EMC). By determining the ultimate tensile strength of the virtual material and also, by assuming that the values of the plane–strain fracture toughness for real and virtual materials are equal, the tensile load-bearing capacity of several V-notched samples of steel reported in literature was theoretically estimated by using the mean stress (MS) criterion as a well-known brittle fracture theory. It was found that the theoretical results of the MS-EMC criterion for the imaginary brittle specimens are in a very good consistency with the experimental results reported for real steel samples.

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1. Introduction

Unlike cracks which are usually unfavorable to appear in engineering components and structures, notches, particularly V-shaped ones, are utilized because of special design requirements. A V-notch concentrates stresses around its tip and hence can become prone to initiate cracks. Such cracks can propagate in the notched component and finally lead to fracture. From the viewpoint of fracture mechanics, the mechanisms of crack initiation and propagation from a notch tip are fundamentally different for ductile and brittle materials. For brittle materials, the emanation of crack from the notch tip consumes a great portion of the total fracture energy and the crack propagation is very less contributed in energy consumption. This is because the crack growth is such a rapid and unstable phenomenon that the final fracture occurs suddenly. Conversely, for ductile materials exhibit large plastic deformations around notches, both crack initiation and propagation consume considerable amount of energy during ductile rupture.

Fracture in notched metallic materials under fatigue loading conditions has been widely studied by several investigators during which the number of cycles to failure could be predicted (see for example [1–4]). However, the number of researches in open literature dealing with fracture in ductile metallic materials containing notches under static and monotonic loading conditions is very limited. For instance, J-integral has been evaluated under elastic–plastic conditions by Berto et al. [5] as a governing parameter in fracture assessment of U and V-notched components made of ductile materials obeying a power-hardening law. Susmel and Taylor [6] have published a paper dealing with predicting the load-bearing capacity for a type of ductile steel containing notches of different features by using the theory of critical distances (TCD) under pure tensile loading conditions (i.e. pure mode I deformation). Their tested material has been a commercial cold-rolled carbon steel exhibiting very ductile behavior. The notched specimens tested by them showed large plastic deformations around the notches after fracture [6]. They predicted the maximum load that each notched specimen can sustain by performing linear elastic and elastic–plastic stress analyzes in conjunction with the use of the theory of critical distances (TCD) with a maximum discrepancy of about 15%. They have clearly stated in their paper [6] that the good efficiency of TCD in the presence of large plastic deformations around notches is surprising. Also, stated in [6] is that the justification of the satisfactory accuracy of TCD accompanied by linear elastic stress analysis is very complicated for ductile V-notched samples. Although the experimental results reported in [6] have been in a good agreement with the results of TCD, its application in engineering design together with linear elastic analysis cannot be prescribed from the viewpoint of fracture mechanics principles. A set of elastic–plastic analyzes have also been performed in [6] accompanied by TCD to predict the load-bearing capacity of the
notched specimens under pure mode I loading conditions which have finally led to good consistency with experimental results. Since elastic–plastic analyzes are rather time-consuming and relatively complicated with respect to elastic analyzes, the author preferred to suggest a simple failure criterion to be conveniently used in the failure analysis of ductile notched domains.

Two of the first attempts to use simple elastic analysis instead of complicated elastic–plastic analysis have been made by Glinka and Molski [7] and Glinka [8]. They made use of the strain energy density (SED) approach for determining the elastic–plastic stress distribution around several notched components. In their works, first, the elastic stress concentration factor has been utilized to formulate the elastic stresses at the notch tip and then, the SED at the notch tip has been equated for elastic and elastic–plastic components in order to determine the stress distribution in the notched component made of ductile material. The equivalent strain energy density approach has been reformulated and applied to sharp V-shaped notches under localized and generalized plasticity by Lazzarin and Zambardi [9] for predicting the failure of components containing V-shaped notches. They used the strain energy over a finite volume around the notch tip as a governing failure parameter to predict the onset of failure in several notched specimens [9].

In this research, the mean stress (MS) criterion, proposed and formulated by Ayatollahi and Torabi [10] for predicting the onset of mode I brittle fracture in V-notched components, is employed in conjunction with the novel equivalent material concept (EMC) (presented in forthcoming sections) to estimate the experimental results reported in [6] for the tensile load-bearing capacity of several V-notched specimens made of ductile steel. It was found that the theoretical predictions of the MS-EMC criterion for the V-notched samples imaginarily fabricated from the virtual brittle material (i.e. the equivalent material) agreed well with the experimental results reported for real steel samples.

2. The equivalent material concept and the mean stress fracture criterion

2.1. The equivalent material concept

In this section, a novel concept, called the equivalent material concept (EMC), is introduced and utilized with the aim to equate a ductile material with a virtual brittle material from the viewpoint of strain energy density. By using EMC, one can imaginarily consider in fracture investigations a virtual brittle material exhibiting linear elastic behavior instead of the ductile material with elastic–plastic behavior. Finally, brittle fracture criteria may be simply utilized to study the fracture phenomenon in ductile materials.

The approach of EMC suggested and utilized herein in order to predict the load-bearing capacity of V-notched components made of ductile materials is relatively similar to that suggested by Glinka [8] but with fundamentally different target. According to the EMC, the strain energy density (i.e. the area under the stress–strain curve in uni-axial tension) for the existing ductile material is assumed to be equal to that for a virtual brittle material having the same modulus of elasticity. The strain energy density (SED) is, in fact, the strain energy absorbed by a unit volume of material. For a ductile material with considerable plastic deformations and with exhibiting power-law strain-hardening relationship in the plastic zone, one can write

\[
\sigma_p = K e_n^p
\]  

In Eq. (1), \(\sigma_p\) and \(\varepsilon_p\) are the plastic stress and the plastic strain, respectively. The parameters \(K\) and \(n\) are also the strain-hardening coefficient and exponent, respectively which depend upon the material properties. Fig. 1 displays schematically a tensile stress–strain curve for a typical ductile material.

In Fig. 1, \(E\), \(\sigma_Y\), \(\varepsilon_Y\) and \(\varepsilon_f\) denote the elastic modulus, the yield strength, the ultimate tensile strength and the strain to rupture, respectively. The total SED can be written in a general form of elastic–plasticity as

\[
\text{(SED)}_{\text{tot}} = \text{(SED)}_e + \text{(SED)}_p = \frac{1}{2} \sigma_Y \varepsilon_Y + \int_{\varepsilon_Y}^{\varepsilon_p} \sigma_p d\varepsilon_p
\]  

Substituting \(\varepsilon_Y = \sigma_Y / E\) and Eq. (1) into Eq. (2) gives

\[
\text{(SED)}_{\text{tot}} = \frac{\sigma_Y^2}{2E} + \int_{\varepsilon_Y}^{\varepsilon_p} K e_n^p d\varepsilon_p
\]  

Thus

\[
\text{(SED)}_{\text{tot}} = \frac{\sigma_Y^2}{2E} + \frac{K}{n+1} (\varepsilon_p^{n+1} - \varepsilon_Y^{n+1})
\]  

If \(\varepsilon_Y\) is considered to be equal to 0.002 (corresponding to 0.2% offset yield strength), then

\[
\text{(SED)}_{\text{tot}} = \frac{\sigma_Y^2}{2E} + \frac{K}{n+1} (\varepsilon_p^{n+1} - (0.002)^{n+1})
\]  

In order to calculate the total SED corresponding to the onset of crack initiation (i.e. the area under \(\sigma-\varepsilon\) curve from beginning of loading to \(\sigma_u\)), one can replace \(\varepsilon_p\) in Eq. (5) with \(\varepsilon_u\) (i.e. the strain at maximum load)

\[
\text{(SED)}_{\text{tot}} = \frac{\sigma_Y^2}{2E} + \frac{K}{n+1} (\varepsilon_u^{n+1} - (0.002)^{n+1})
\]  

From Eq. (1), \(\varepsilon_u\) can be obtained as

\[
\varepsilon_u = \left(\frac{\sigma_u}{K}\right)^{1/n}
\]
Substituting Eq. (7) into Eq. (6) gives

\[
(\text{SED})_{\text{lat}} = \frac{\sigma_f^2}{2E} + \frac{K}{n+1} \left( \frac{\sigma_f}{K} \right)^{(n+1)/n} - (0.002)^{n+1}
\]

The equivalent material considered in EMC is a virtual brittle material with the same values of the elastic modulus \(E\) and the plane–strain fracture toughness \(K_c\), but unknown value of ultimate tensile strength. Fig. 2 shows schematically a sample uni-axial stress–strain curve for the virtual brittle material.

In Fig. 2, the parameter \(\varepsilon_f\) and \(\sigma_f\) are the strain at crack initiation (i.e. the final fracture due to the brittleness) and the ultimate tensile strength, respectively. The SED for this material at the onset of crack initiation is

\[
(\text{SED})_{\text{EMC}} = \frac{\sigma_f^2}{2E}
\]

According to the requirements of EMC, SED values for both the real ductile and the virtual brittle materials must be equal. Therefore, Eqs. (8) and (9) are identical. Thus

\[
\frac{\sigma_f^2}{2E} = \frac{\sigma_f^2}{2E} + \frac{K}{n+1} \left( \frac{\sigma_f}{K} \right)^{(n+1)/n} - (0.002)^{n+1}
\]

Finally, \(\sigma_f^2\) can be obtained as

\[
\sigma_f^2 = \frac{2EK}{n+1} \left( \frac{\sigma_f}{K} \right)^{(n+1)/n} - (0.002)^{n+1}
\]

### 2.2. The mean stress (MS) fracture criterion

Different failure concepts have been previously suggested in the literature for predicting the onset of brittle fracture in sharp and blunt V-notched components such as the strain energy density [9,11–16], the generalized notch stress intensity factor [17] and the fictitious notch radius [18,19]. Meanwhile, several fracture criteria have been developed in the past based on determining a critical value for the notch stress intensity factor so-called the notch fracture toughness or the apparent toughness. In this area, one can refer to [11] and [20–25] for pure mode I and [26–31] for mixed mode I/II and pure mode II loading conditions. Under pure mode I loading, a simple and convenient failure criterion, called the mean stress (MS) criterion, has been published by Ayatollahi and Torabi [10] to predict the onset of fracture in brittle components containing V-shaped notches. Fig. 3 displays schematically a rounded-tip V-notch with its polar coordinate system.

The mode I notch fracture toughness of rounded-tip V-shaped notches has been given in [10] as a closed-form expression

\[
K_{cV}^{\ast} = \sqrt{\frac{2\pi}{r_0}} \sigma_{\infty},
\]

\[
q = \frac{2\pi - 2\alpha}{\pi}
\]

\[
d_c = \frac{2 \left( K_{cV}^{\ast} / \sigma_{\infty} \right)^2}{\pi}, \quad d_c^2 + d_c r_0
\]

In Eq. (12), \(\sigma_{\infty}\) is the critical value of the tangential stress which could be assumed to be equal to the ultimate tensile strength for brittle materials [10]. Eq. (12) has been developed by using the in-plane elastic stress distribution around a general rounded-tip V-shaped notch presented by Filippi et al. [32], Lazzarin and Tovo [33] and Azzori et al. [34]. Except \((\sigma_{\infty}), d_c, d_c^2\) and other parameters in Eq. (12) appear in the stress field expressions [32]. The parameters \(r_0, q, d_c\) and \(d_c^2\) are [10,32]

\[
r_0 = \frac{q - 1}{q}, \quad q = \frac{2\pi - 2\alpha}{\pi}
\]

\[
d_c = \frac{2 \left( K_{cV}^{\ast} / \sigma_{\infty} \right)^2}{\pi}, \quad d_c^2 = d_c + r_0
\]

In Eq. (13), the parameters \(2\alpha, \rho, r_0\) and \(K_{cV}^{\ast}\) are the notch angle, the notch tip radius, the distance between the notch tip and the origin of the polar coordinate system (see Fig. 3) and the plane–strain fracture toughness, respectively. The parameters \(\lambda_1\) and \(\mu_1\) are the mode I eigenvalues depend on the notch angle and can be found in [32–35]. \(n_{\infty}(0)\) is a function of the notch angle reported in [32] and also in the appendix of [10].

As previously mentioned, the only unknown parameter in simulating the ductile material with the virtual brittle material is the ultimate tensile strength of the virtual material (i.e. \(\sigma_f^2\) in Eq. (11)). By substituting \(\sigma_f^2\) from Eq. (11) into Eq. (12) instead of \((\sigma_{\infty}),\), one can theoretically determine the mode I notch fracture toughness for the virtual brittle material.

According to the MS criterion, fracture occurs under mode I loading conditions when the value of the mode I notch stress intensity factor (NSIF) \(K_{nI}^{\ast}\) attains the notch fracture toughness \(K_{cV}^{\ast}\) [10]. The parameter \(K_{nI}^{\ast}\) has been presented in [10] as

\[
K_{nI}^{\ast} = \frac{2\pi \sigma_{\infty}(r_0, 0) r_0^{1-\alpha}}{\Gamma(1+\alpha)}
\]

In Eq. (14), \(\sigma_{\infty}(r_0, 0)\) is the tangential stress on the notch tip and \(\alpha\) is an auxiliary parameter depends on the notch angle and presented in [10]. After calculating the value of \(K_{nI}^{\ast}\) (i.e. the critical value of \(K_{nI}^{\ast}\) corresponds to the onset of crack initiation from the notch tip) for the virtual material from Eq. (12), it is
necessary to create a finite element model for the V-notched component with the aim to determine the applied load corresponding to the value of $K_{IC}^{V,R}$. To do this, first, a unit external load should be applied to the notched component and $K_{IC}^{V,R}$ should be computed. Then, the load is gradually increased until $K_{IC}^{V,R}$ attains $K_{IC}^{V,R}$. The load associated with $K_{IC}^{V,R}$ is, in fact, the load-bearing capacity of the V-notched component imaginarily made of the virtual brittle material.

In the next section, the theoretical results of the MS criterion in predicting the load-bearing capacity of several V-notched specimens reported in literature and imaginarily made of the virtual brittle material are compared with the experimental results exist for the same specimens made of a type of ductile metallic material.

3. Experimental verification of the MS-EMC failure criterion

To verify the validity of the results of the MS criterion in conjunction with the equivalent material concept (herein after referred to as MS-EMC criterion), some experimental results reported in [6] are utilized herein. The specimens tested have been several samples with different notch features, including V-shaped notches, made of a type of ductile commercial cold-rolled low-carbon steel, namely En3B, and tested under three-point bending (TPB) and uni-axial tension [6]. The mechanical properties of the material are presented in Table 1 [6].

The value of $K_c$ has been determined experimentally by means of testing the C(T) specimens of 85 mm thick in accordance with the ASTM E399 [36]. Fig. 4 represents the tested V-notched specimens, schematically [6].

The specimens of Fig. 4(a) and (b) have been tested under TPB loading and uni-axial tension, respectively. Also, Fig. 4(a) has been divided in two various specimens based on their notch length, namely V-short and V-long specimens [6]. In Fig. 4, the parameters $a$ and $d$ are the notch lengths and the parameters $l$, $P$, $S$, $W$, $2a$ and $ρ$ are the specimen length, the external load, the distance between two supports, the specimen width, the notch angle and the notch tip radius, respectively. The values of these parameters for the specimens (1)–(3) are presented in Table 2. Also, presented in Table 2 is the thickness (denoted by $t$) for the specimens [6].

The experimental values of the maximum loads that the specimens could sustain are presented in Table 3 [6]. In the sixth column of Table 3, the average values of the maximum loads recorded during experiments are presented [6]. The average values of the nominal elastic stresses on the net area according to the beam theory, i.e. $σ_{nom} = P_{av}/((W - 2a)^2t)$ for tensile specimens and $σ_{nom} = (P_{av}/2^2S^2)/(1/6^2t^2(W - d)^2)$ for three point bending specimens, are also presented in the last column of Table 3.

According to Table 3, for each specimen (1) and (2), two tests and for 3, four tests have been conducted [6]. The nominal elastic stresses on the net area seem to be surprisingly very high, particularly for three-point bending specimens. This phenomenon is described in detail in Section 4.

The theoretical results of the MS-EMC and the TCD criteria in predicting the load-bearing capacity of the tested V-notch specimens are presented in Table 4 together with the average values of the experimental results. Also, presented in Table 4 are the discrepancies between the theoretical and experimental results.

As seen in Table 4, the maximum discrepancy of the MS-EMC criterion is less than 12% (the mean discrepancy is about 5%)

Table 1
The mechanical properties of the En3B steel [6].

<table>
<thead>
<tr>
<th>$ε_f$</th>
<th>$σ_f$</th>
<th>$n$</th>
<th>$K_f$ (MPa)</th>
<th>$K_c$ (MPa√m)</th>
<th>$E$ (GPa)</th>
<th>$σ_f$ (MPa)</th>
<th>$σ_c$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.56</td>
<td>851.8</td>
<td>0.06</td>
<td>882.7</td>
<td>97.4</td>
<td>197.4</td>
<td>606.2</td>
<td>638.5</td>
</tr>
</tbody>
</table>

Fig. 4. The tested V-notched specimens [6]. Specimens (a) and (b) correspond to TPB and tension tests, respectively.
demonstrating the effectiveness of the criterion. Moreover, a comparison of the last two columns of Table 4 shows clearly that the accuracy of the MS-EMC criterion is better than that of the TCD (the mean discrepancy is about 7.5%). It is necessary to remind that the theoretical values of the maximum loads for the MS-EMC criterion were computed by estimating \( K_{IC}^{VP} \) from Eq. (12) and performing linear elastic stress analysis as presented in the previous section.

4. Discussion

As represented in Fig. 4, all of the V-notched specimens have been subjected to pure mode I deformation (i.e. pure opening mode) regardless of the type of loading, e.g. TPB or uni-axial tension. A review of the expression for the mode I notch fracture toughness \( K_{IC}^{VP} \) in Eq. (12) obviously shows that the parameter \( K_{IC} \) is essential to calculate \( K_{IC}^{VP} \). This parameter is necessary in not only MS-EMC criterion but also in almost all of the failure criteria in the context of brittle fracture. If \( K_{IC} \) be not valid for a ductile material, the MS-EMC criterion will no longer be valid to be utilized. According to ASTM E399 [36], for metallic materials having yielding near the crack tip, the ratio of the maximum load to the load recorded at the end of the linear portion of the load-displacement curve during plane–strain fracture toughness test should not exceed 1.1. From the experimental point of view, this means that if the values of \( \sigma_Y \) and \( \sigma_s \) be considerably different (i.e. the strain-hardening exponent \( n \) becomes relatively great), the \( K_{IC} \) test would not probably be valid. In such cases, stable crack growth is usually seen during the test and ductile rupture occurs. For this type of metallic materials, one cannot use \( K_{IC} \) as a governing fracture parameter and should utilize instead the ductile fracture parameters such as J-integral, crack tip opening displacement (CTOD), crack tip opening angle (CTOA), R-curve etc. As a result, it seems that the MS-EMC criterion could be used to study the mode I fracture in V-notched metallic domains only when the yield and the ultimate strengths of material be adequately close together (like EN3B steel reported in [6]). Another important parameter to successfully achieve the \( K_{IC} \) parameter is the specimen thickness. According to [36], the thickness of \( (T) \) specimen should be greater than the reference thickness \( t_r = 2.5(K_{IC}/\sigma_Y)^2 \) in order to gain valid \( K_{IC} \) (\( t_r \) for EN3B steel is about 65 mm). Thus, for those ductile metallic materials having \( \sigma_Y \) and \( \sigma_s \) adequately close together, it is expected that selecting a specimen thickness greater than \( t_r \) will result in valid \( K_{IC} \) value. Consequently, the strain to rupture value (i.e. \( \varepsilon_f \)) for such ductile materials is not an important parameter to identify the type of fracture either being stable ductile or unstable brittle fracture. As a conclusion from the above statements, it can be said that the mean stress criterion in conjunction with the equivalent material concept (MS-EMC) can be utilized in both small-scale and large-scale yielding fracture conditions.

The TCD failure criterion presented in [6] for fracture analysis of V-notched components made of the ductile steel EN3B has been used based on the linear elastic stress analysis while the fracture phenomenon has been preceded by large amount of plastic deformations around the notch tip [6]. According to Susmel and Taylor [6], the successful predictions of linear elastic TCD are surprising and hence more studies are needed to justify the applicability of this criterion. In the author’s opinion, although TCD results have been satisfactory, the justification of the capability of using a failure criterion developed fundamentally based on the linear elastic fracture mechanics (LEFM) in situations having large plastic deformations (like for EN3B steel) is very difficult. This major shortcoming of TCD does not exist for MS-EMC criterion, because the ductile metallic material is initially simulated with a virtual brittle material having an ideal linear elastic behavior from beginning of loading up to final breakage. Beside linear elastic calculations, TCD has also been employed in [6] accompanied by elastic–plastic analyzes for predicting the load-bearing capacity of the V-notched specimens and its good results have been revealed. However, it should be noted that the elastic–plastic analysis based on the finite element modeling is very time-consuming and relatively complicated in comparison with the linear elastic analysis. Therefore, linear elastic calculations are generally preferred in real engineering applications. Consequently, it can be stated that from the viewpoint of engineering design, the MS-EMC fracture criterion is very easy and convenient to use, because its mathematical expression is closed-form and the corresponding analysis is completely linear elastic. The major advantage of the MS-EMC criterion is that it provides a short and justifiable path to estimate the tensile load-bearing capacity of V-notched components having large plastic deformations around the notch tip without requiring taking into account the fracture mechanism and performing plastic stress analysis. It is necessary to note that similar to TCD, the MS-EMC criterion can be utilized only to estimate the tensile load-bearing capacity of

<table>
<thead>
<tr>
<th>Specimen</th>
<th>( t ) (mm)</th>
<th>( a ) (mm)</th>
<th>( d ) (mm)</th>
<th>( t ) (mm)</th>
<th>( S ) (mm)</th>
<th>( W ) (mm)</th>
<th>( 2\alpha ) (deg.)</th>
<th>( \rho ) (mm)</th>
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<td>25</td>
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<td>25</td>
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<td>0</td>
<td>25</td>
<td>60</td>
<td>0.1</td>
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<table>
<thead>
<tr>
<th>Specimen</th>
<th>( P_{t} ) (kN)</th>
<th>( P_{c} ) (kN)</th>
<th>( P_{s} ) (kN)</th>
<th>( P_{t} ) (kN)</th>
<th>( P_{m} ) (kN)</th>
<th>( \sigma_{max,m} ) (MPa)</th>
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<td>(1)</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>119.4</td>
<td>1396</td>
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<tr>
<td>(2)</td>
<td>96.6</td>
<td>96.3</td>
<td>-</td>
<td>-</td>
<td>96.4</td>
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<tr>
<td>(3)</td>
<td>89.5</td>
<td>91.2</td>
<td>94</td>
<td>-</td>
<td>91.2</td>
<td>91.5</td>
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<table>
<thead>
<tr>
<th>Specimen</th>
<th>( P_{t} ) (kN) (MS-EMC criterion)</th>
<th>( P_{t} ) (kN) (experimental)</th>
<th>Discrepancy for MS-EMC criterion (%)</th>
<th>Discrepancy for TCD criterion (%)</th>
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<td>(1)</td>
<td>117.6</td>
<td>119.4</td>
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<td>(3)</td>
<td>92.6</td>
<td>91.5</td>
<td>1.2</td>
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V-shaped notches and not to identify the type of fracture and also not to study the propagation of cracks emanating from the notch tip.

As well-known in general solid mechanics, the area under the stress–strain curve of an engineering material is the strain energy per unit volume so-called the strain energy density (SED). This parameter is also called, material toughness. In a standard uni-axial tensile test for a ductile material, the test sample, usually dog-bone specimen; is subjected to uniform normal tensile stress. While the load increases gradually, the entire cross-section yields and the specimen experiences plastic deformations. The load enhances such that the necking phenomenon takes place. The final rupture ultimately occurs after a considerable and unstable deformation. The necking point is, in fact, the onset of crack initiation at the cross-section and this point on the engineering stress–strain curve associates with the ultimate tensile strength \( \sigma_u \) (i.e. the maximum load on the load-displacement curve). In pure mode I loading conditions applied to the components with stress concentrators such as V-shaped notches, the type of deformation around notches is purely tensile. However, the main difference between tensile deformations in notched and flawless components is that the severe deformation in notched component is local while in flawless one is global. In other words, plastic deformations in notched domains locate around the notch tip and crack initiates from the notch tip while in flawless domains, the entire cross-section often undergoes uniform plastic deformations. The SED for a ductile metallic material from beginning of loading \( \varepsilon \rightarrow 0 \) to \( \varepsilon = \varepsilon_t = \) is, in fact, the SED associated with the crack initiation. In the equivalent material concept (EMC), the SED values corresponding to crack initiation for the real ductile and the virtual brittle materials are assumed to be identical. Based on this fundamental assumption, it is expected that the amounts of energy absorbed by similar notched components of these two ductile and brittle materials up to crack initiation become equal since the deformations at the vicinity of the notches are purely tensile but only with local effects.

As stated in [36], the plane–stress fracture toughness test may be valid for metallic materials if the specimen thickness is selected to become larger than \( t_c = 2.5(K_{IC}/\sigma_t)^2 \). The value of \( t_c \) for the ductile steel reported in [6] (i.e. the En3B grade) is calculated to be equal to about 65 mm. Therefore, the thickness selected in [6] has been equal to 85 mm for CT specimens. The thicknesses considered for the notched specimens in [6] (also used in the present work) have been equal to 6 and 25 mm. Since both values are lower than 65 mm, the plane–stress fracture conditions are not completely satisfied and the \( K_{IC} \) value should be modified to \( K_t \) values when applied to these notched specimens. The \( K_t \) values depend basically on the specimen thickness as stated in [37]. Because the \( K_t \) values for En3B steel have not been reported in [6] as a function of thickness, \( K_t \) has been utilized instead of \( K_{IC} \) in both TCD [6] and MS–EMC models. It is necessary to highlight that although the very good accuracy of the MS–EMC criterion in predicting the tensile load-bearing capacity of V-notched steel samples was revealed, the applicability of this model should be examined for similar ductile metallic materials and also non-metallic ductile materials having valid \( K_{IC} \) values.

It should be reminded that the equivalent material concept simulates the ductile material with a virtual brittle one. The brittle fracture criteria (e.g. the MS criterion) are applied to the virtual brittle material after this simulation. Since the mode I failure of V-notched specimens was considered in this research, the EMC was used together with the MS criterion. It is expected that the EMC can be used accompanied by different failure criteria in the context of brittle fracture to estimate the load-bearing capacity of V-notched components made of ductile materials and loaded under mode I, Mode II, mixed mode I/II and also mode III contributed conditions (mode III is commonly produced by a torque). The applicability of EMC in different loading conditions other than pure mode I is planned to check by the author in his further investigations.

Another important comment about the present study is that the constraint effects in elastic–plastic conditions such as the local effects of test supports and loadings in finite size thicknesses cannot be studied by this approach. The fundamental assumption in the present research is that the effects of constraints in fracture investigation of V-notched ductile components are local and small enough such that the main deformations exist around the V-notch tip (like that happens for brittle materials). If plastic deformations around constraints are relatively large and comparable with those around the notch tip, the EMC will no longer be effective. Moreover, if the constraints are close enough to the notch section so that the local deformations and therefore, the stress distributions around the constraints and the V-notch tip are mixed, the EMC will be ineffective.

The nominal elastic stresses on the net area presented in Table 3 seem to be very high, particularly for three-point bending specimens. Correspondingly, the level of stresses on the notch tip is expected to be much higher than the nominal stresses due to the effects of stress concentration. This fact is somewhat surprising because these stresses are considerably higher than the material ultimate tensile strength. The stress distributions reported in [6] (see Fig. 8 in [6]) showed that the stresses in both linear elastic and elastic–plastic conditions decrease dramatically as the distance from the notch tip becomes larger and the stresses at the critical distance attain close to the ultimate tensile strength of the material. In experiments, the level of elastic–plastic stresses around notches is considerably lower than the elastic one because of the small value of the material strain-hardening exponent. It is necessary to highlight that because of the existence of a highly stressed zone near the notch tip, this zone is damaged during loading and hence the stress value at the critical distance is considered for real load-bearing consideration (see the TCD failure concept). The larger distance from the notch tip in three-point bending specimens means trivially the shorter distance from the neutral axis where the level of nominal stresses is very low. Consequently, this phenomenon is not surprising because of the combined effects of the critical distance and the variations of stresses through the width of the three-point bending specimens. For tensile specimens, although the stress reduction through the width does not exist, the effects of the critical distance from the notch tip still exist and lead to significant reduction of stress.

5. Conclusions

The main conclusions of the present research can be summarized as

1. The mean stress (MS) criterion, proposed fundamentally for estimating the mode I fracture toughness of V-notched components made of brittle materials, was successfully utilized in conjunction with the novel equivalent material concept (EMC) in order to predict the tensile load-bearing capacity of several V-notched specimens made of a type of ductile steel.
2. The MS–EMC criterion is valid for those metallic materials having valid \( K_{IC} \) values or at least valid \( K_t \). The ductile rupture cannot be analyzed by this criterion.
3. The MS–EMC criterion may be valid for ductile materials with both small and large-scale plastic deformations. This fact was demonstrated in this work for V-notched specimens with considerable yielding area around notches.
4. The applicability of the MS–EMC criterion in practical engineering design is very convenient since it does not require time-consuming elastic–plastic analysis.
References