Extensive data of notch shape factors for V-notched Brazilian disc specimen under mixed mode loading

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\textbf{A B S T R A C T}

The linear elastic stress field was numerically analyzed around the V-notch tip in a recently developed disc-type test specimen, called the V-notched Brazilian disc (V-BD), under mixed mode in-plane loading. The notch stress intensity factors (NSIFs) which are very important parameters in brittle fracture assessment of V-notched components were calculated for V-BD specimen by using the finite element (FE) method for different notch geometries and wide range of mode mixities from pure mode I to pure mode II loading conditions. In order to simplify the obtained results to be used in practical applications, the NSIFs were converted to the dimensionless parameters called the notch shape factors (NSFs). These parameters are useful to compute more rapidly and conveniently the NSIFs in V-BD specimen for various notch angles and different notch tip radii. It is shown that the notch shape factors presented in this work combined with the appropriate failure criteria can be utilized to estimate the load-cell capacity of the test apparatus required for fracture test of V-BD specimens made of different brittle materials.

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1. Introduction

Different failure modes can be recognized in brittle and quasi-brittle materials like ceramics, rocks, brittle polymers, concrete, graphite, soda-lime glass etc. among which the brittle fracture mode is very popular. In the presence of stress concentrators like cracks and notches, brittle materials are so vulnerable to failure. Therefore, many researchers have frequently investigated the brittle fracture in engineering structures containing stress concentrations both theoretically and experimentally.

Various types of notches, especially V-shaped ones, are usually utilized in engineering components and structures because of particular design requirements. V-notches act as stress raisers and reduce significantly the load-bearing capacity of the notched component due to stress concentration around the notch tip. If the notched component is made of a brittle material and subjected to mechanical loading, fracture may occur suddenly from the notch. Therefore, it is essential to prevent brittle fracture in engineering applications by using appropriate failure criteria and/or by conducting fracture tests.

Similar to cracks, V-shaped notches can be generally subjected to three different in-plane loadings usually called pure mode I, pure mode II and mixed mode I/II. Under pure mode I loading, notch faces open with respect to the notch bisector line without any sliding. Any pure in-plane sliding of the notch faces with respect to the notch bisector line is called pure mode II deformation and combined opening and in-plane sliding of the notch faces is well-known as mixed mode I/II deformation.

Several failure theories can be found in literature for predicting brittle fracture in notched elements that most of them have been developed based on the linear elastic fracture mechanics (LEFM) such as that presented by Sih and Ho [1] based on the critical energy density theory and those suggested on the basis of the local strain energy density concept (see for example [2–7]). Like cracks, the stress intensity factors play an important role in governing the brittle fracture phenomenon in notched components. For instance, the notch stress intensity factors (NSIFs) have been utilized in [8–10,15,16] to predict the onset of brittle fracture in V-notched and U-notched components, respectively.

Experimental investigations dealing with determining the fracture toughness (i.e. the critical values of the NSIFs) of notched elements have been performed by several researchers using a limited number of test samples. For example, one can find in literature the single-edge notch tension (SENT) [17], double-edge notch tension (DENT) [18], three-point bend (TPB) [19] and four-point bend [20] specimens. Recently, Ayatollahi and Torabi [12] suggested and used a new disc-type test specimen containing a rhombic central slit and tested under compressive loading, called the V-notched Brazilian disc (V-BD) specimen, for measuring the notch fracture toughness of plexi-glass (PMMA) [12], polycrystalline graphite [13]

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Nomenclature

- \( K_{I,\rho}^V \): notch stress intensity factor (NSIF)-mode I
- \( K_{II,\rho}^V \): notch stress intensity factor (NSIF)-mode II
- \( K_{Ic}^V \): mode I notch fracture toughness
- \( RNL \): relative notch length
- RNTR: relative notch tip radius
- V-BD: V-notched Brazilian disc
- \( V_1^\rho \): notch shape factor (NSF)-mode I
- \( V_II^\rho \): notch shape factor (NSF)-mode II
- \( 2\alpha \): notch angle
- \( \beta \): loading angle for V-BD specimen
- \( \rho_I \): loading angle corresponding to pure mode II loading
- \( \lambda_i \): eigenvalues
- \( \mu_i \): eigenvalues (real parameters)
- \( \sigma_{Itr} \): radial stress
- \( \sigma_{Ish} \): in-plane shear stress
- \( \sigma_{sh} \): tangential stress

and soda-lime glass [14] under various combinations of mode I and mode II loadings. The V-BD specimen is, in fact, a modified version of the centrally cracked Brazilian disc (CCBD) specimen which has been frequently utilized in mixed mode I/II fracture tests of brittle components containing sharp cracks [see for example [21–23]].

In this work, 276 finite element (FE) analyses were performed to calculate the notch stress intensity factors (NSIFs) for V-BD specimen under different mixed mode loading conditions from pure mode I to pure mode II for different notch angles and various notch tip radii. For more simplicity of using the obtained results in practical applications, the NSIFs are converted to the dimensionless parameters, called the notch shape factors (NSF). The NSF which depend on the specimen diameter, specimen thickness, notch length, notch angle and the notch tip radius are presented in several suitable graphs. Once the NSF and the geometric parameters are known for a V-BD specimen, one can directly determine the NSIFs for any combination of modes I and II loadings without requiring FE analysis. In the next section, closed-form expressions are presented in a general form for the linear elastic stress distribution around the tip of a rounded-tip V-notch. These expressions are then utilized to define and calculate the NSIFs and the NSF in the forthcoming sections.

2. Linear elastic stress distribution around a V-notch tip

Filippi et al. [24] developed an expression for mixed mode I/II stress distribution around a V-shaped notch shown in Fig. 1. The stress distribution is an approximate formula because it satisfies the boundary conditions only in a finite number of points on the notch edge and not on the entire edge. They obtained the stress distribution using a conformal mapping in an auxiliary system of curvilinear coordinates “\( U \) and \( V \)” that are related to the Cartesian coordinates “\( X \) and \( Y \)” as \( (X + iY) = (U + iV)^{q} [24] \). The power “\( q \)” is a real positive coefficient ranging from 1 (for a flat edge) to 2 (for a crack).

The mixed mode I/II stresses can be written as

\[
\begin{bmatrix}
\sigma_{I1} \\
\sigma_{I2}
\end{bmatrix} = \frac{K_{I,\rho}^V}{\sqrt{2\pi r}} \left[ \begin{bmatrix}
f_1(\theta) \\
f_2(\theta)
\end{bmatrix} + \left( \frac{r}{r_0} \right) \mu_1 \lambda_1 \begin{bmatrix}
\psi_{I1}(\theta) \\
\psi_{I2}(\theta)
\end{bmatrix} \right] + \frac{K_{II,\rho}^V}{\sqrt{2\pi r}} \left[ \begin{bmatrix}
f_1(\theta) \\
f_2(\theta)
\end{bmatrix} + \left( \frac{r}{r_0} \right) \mu_2 \lambda_2 \begin{bmatrix}
\psi_{II1}(\theta) \\
\psi_{II2}(\theta)
\end{bmatrix} \right]
\]

where \( K_{I,\rho}^V \) and \( K_{II,\rho}^V \) is the mode I and mode II notch stress intensity factors (NSIFs), respectively. The parameter \( r_0 \) is the distance between the origin of the polar coordinate system and the notch tip. The functions \( f_1(\theta) \) and \( f_2(\theta) \) have been reported in the Appendix and the eigenvalues \( \lambda_i \) and \( \mu_i \) which depend upon the notch angle have been reported in [24]. It can be shown that if the notch tip radius vanishes, Eq. (1) becomes equal to the stress field previously obtained by Williams [25] for sharp V-notches. According to a relation that exists between the Cartesian and the curvilinear coordinate systems, \( r_0 \) can be written as [24]:

\[ r_0 = \frac{q - 1}{q \rho} \quad q = \frac{2\pi - 2\alpha}{\pi} \]

where “\( 2\alpha \)” is the notch angle and \( \rho \) is the notch tip radius. The expressions for NSIFs are [26]:

\[
\begin{align*}
K_{I,\rho}^V &= \sqrt{2\pi} \frac{\sigma_{\theta\theta}(r_0, 0) r_0^{1-\lambda_1}}{1 + \omega_1 (r/r_0)^{1-\lambda_1}} \\
K_{II,\rho}^V &= \sqrt{2\pi} \frac{\sigma_{\theta\theta}(r_0, 0) r_0^{1-\lambda_2}}{1 + \omega_2 (r/r_0)^{1-\lambda_2}}
\end{align*}
\]

where \( \sigma_{\theta\theta} \) and \( \sigma_{\theta\theta} \) are the tangential and the in-plane shear stresses, respectively. The parameters \( \omega_1 \) and \( \omega_2 \) have been presented in Appendix A. If the values of \( \omega_1 \) and \( \omega_2 \) are known, the NSIFs can be obtained from Eqs. (3) and (4) as

\[
\begin{align*}
K_{I,\rho}^V &= \sqrt{2\pi} \frac{\sigma_{\theta\theta}(r_0, 0) r_0^{1-\lambda_1}}{1 + \omega_1} \\
K_{II,\rho}^V &= \lim_{r \to r_0} \sqrt{2\pi} \frac{\sigma_{\theta\theta}(r, 0) r^{1-\lambda_2}}{1 - (r/r_0)^{2-\lambda_2}}
\end{align*}
\]

The NSIFs can be calculated by using the FE method as elaborated in the next section. These parameters have different units of measure (MPa\( m^{1-\lambda_1} \) and MPa\( m^{1-\lambda_2} \)) for \( K_{I,\rho}^V \) and \( K_{II,\rho}^V \), respectively) and hence, cannot directly be compared. Note that the parameter \( r \) in Eq. (6) cannot be directly substituted by \( r_0 \), because for \( r = r_0 \), \( K_{II,\rho}^V \) becomes singular. Therefore, \( K_{II,\rho}^V \) is calculated from Eq. (6) at a point very close to the notch tip where \( r \to r_0 \). When the notch tip radius \( \rho \) is zero (i.e. in the case of a sharp notch), the stress values at the notch tip tend to infinity and hence the parameters \( K_{I,\rho}^V \) and \( K_{II,\rho}^V \) cannot be directly obtained by using Eqs. (5) and (6).

In such conditions, the limits of the expressions given in Eqs. (5) and (6) must be calculated where \( r \to 0 \). It should be highlighted that the NSIFs are very important parameters in brittle fracture assessment of V-notched components because, wide range of the

![Fig. 1. Round-tip V-notch and its polar coordinate system.](image-url)
brittle fracture criteria in notched domains present their predicting expressions/envelopes (i.e. the fracture toughness) in terms of the NSIFs. Therefore, one should compute the NSIFs for given V-notched brittle component and compare them with the theoretically predicted values in order to specify whether or not the fracture occurs for the V-notched component.

3. The V-notched Brazilian disc (V-BD) specimen

Few specimens have been previously suggested in the literature for the experimental investigation of brittle fracture in engineering components containing sharp cracks [27]. One of the most common specimens used in the past for conducting mixed mode I/II fracture tests is the centrally cracked Brazilian disc (CCBD) specimen (see for example [21,22]). A modified version of the CCBD specimen, called V-BD, could be used to perform mixed mode fracture tests for V-notches (see for example [11–14]). Fig. 2 shows the V-BD specimen schematically.

Note that the V-BD specimen is, in fact, the CCBD specimen for which the central crack is replaced with a central rhombic slit. In Fig. 2, $\beta$ is the angle between the loading direction and the notch bisector line and the parameters $2\alpha$, $D$, $d/2$ and $P$ are the notch angle, the disc diameter, the notch length and the applied load, respectively. When the direction of the applied load $P$ is along the notch bisector line (i.e. $\beta = 0^\circ$), the upper and the lower corners of the rhombic slit are subjected to pure mode I deformation. When the angle $\beta$ enhances gradually from zero, the loading condition varies from pure mode I towards pure mode II. For a specific angle, called $\beta_{II}$, pure mode II deformation is achieved. The mode II loading angle $\beta_{II}$ is always less than $90^\circ$ and depends upon the notch length and its opening angle and also slightly on the notch tip radius. The angles $\beta_{II}$ can be determined by using the finite element (FE) method as described in Section 4.

4. Finite element analysis

In this section, the FE procedures needed for determining the NSIFs and the mode II loading angle $\beta_{II}$ are elaborated. In order to compute NSIFs for the V-BD specimen with arbitrary V-notch geometry, it is necessary to create a FE model for the specimen and perform linear elastic stress analysis. Under an arbitrary load $P$, the values of the tangential and shear stresses ($\sigma_{th}$ and $\sigma_{sh}$) along the notch bisector line are obtained from the FE analysis and the NSIFs can be calculated by using Eqs. (5) and (6).

A plane-stress FE model with a total number of about 50,000 Quad-8 elements was created for each V-BD specimen and mixed mode stress analyses were entirely performed in four $\beta$ angles of $0^\circ$, $10^\circ$, $20^\circ$ and $\beta_{II}$ ($\beta$ angles were trivially selected between $0^\circ$ and $\beta_{II}$) for computing the NSIFs. Fig. 3 shows a sample meshed V-BD specimen.

The commercial software ABAQUS was utilized for conducting the stress analysis. Since the V-BD specimens reported in the literature have the notch angles of $30^\circ$, $60^\circ$ and $90^\circ$ [11–14] (these are the favorite notch angles for the researchers in mixed mode fracture experiments), the V-BD models were created and analyzed according to these notch angles. A wide range of the relative notch lengths (RNL) $d/D (0.1–0.5)$ and the relative notch tip radii (RNR) $r/D (0–0.075)$ were considered for analysis. The RNL and RNR parameters are defined in the next section. The reference load $P$, the specimen diameter and the specimen thickness were arbitrarily considered to be 10 kN, 80 mm (as presented in [12–14]) and 10 mm, respectively in the entire analyzes. Very fine meshes were utilized at the notch tip vicinity because of high stress gradient. A total number of 276 analyzes were performed to compute the NSIFs for V-BD specimens.

In order to obtain the mode II loading angle $\beta_{II}$, $\beta$ was gradually increased from zero and the tangential stress at the notch tip $(\sigma_{th} (r_0, \theta))$ was obtained from the FE results. Under a fixed load $P$, as $\beta$ becomes larger, the value of $\sigma_{th} (r_0, \theta)$ decreases. According to Eq. (5), $\beta_{II}$ is the angle for which $\sigma_{th} (r_0, \theta)$ and hence $K_{I/II}^p$ are equal to zero and the V-notch is subjected to pure mode II loading. The FE analysis of V-BD specimens demonstrated that $\beta_{II}$ enhances for larger notch angles and also larger RNR for each value of RNL in its entire domain of variation. These results are illustratively presented in the next section.

It is essential to highlight that the stability of the computed NSIFs is a vital parameter because; the investigators require stable and reliable NSIFs values for their designs and calculations. Thus, this important fact should be evaluated and proved for our calculations in V-BD specimen. For this purpose, first, the validity of the numerically determined stress distribution around the notch tip should be verified. Therefore, several mesh patterns with different numbers of elements were created for each V-BD specimen and FE stress analysis was performed accordingly. This procedure was continuously followed until the mesh-independency of the analysis was demonstrated. Then, the validity of the NSIFs computed...
in accordance with Eqs. (5) and (6) should be proved. As seen in Eq. (5), the value of the tangential stress exactly at the notch tip is required for determining $K_{II}^{\psi,\rho}$. Therefore, there is not any reason to make doubt about the validity and reliability of the $K_{II}^{\psi,\rho}$ values. The only doubtful parameter in our calculations is the role of distance $r$ ahead of the notch in computing $K_{II}^{\psi,\rho}$ (see Eq. (6)) since $K_{II}^{\psi,\rho}$ is obtained from a limit computation. To reach to a stable value for $K_{II}^{\psi,\rho}$, this parameter was computed for several nodes on the notch bisector line and the final value was selected for $K_{II}^{\psi,\rho}$ by conducting well-known extrapolation technique. Finally, Eq. (1) was plotted for each V-BD specimen based on the calculated NSIFs and the obtained analytical curve was compared with the curve of numerical stress distribution resulted from FE analysis in a unified graph. The mode II NSIF was slightly modified everywhere required till the curves match each other at the K-dominant region (i.e. the notch tip vicinity) as more as possible. The outputs of this procedure are presented in this paper as the final mode II NSIFs for V-BD specimens.

5. Results

In order to present the NSIFs for the V-BD specimen more compactly and to make use of them convenient in the engineering applications, two dimensionless parameters called the mode I and mode II notch shape factors (NSIFs) are defined herein as

$$\quad Y_{I}^{\psi,\rho} = Y_{I} \left( \beta, \frac{d}{D}, \frac{\rho}{D}, 2\alpha \right) = K_{I}^{\psi,\rho} d^{(\beta+1)} \frac{D t}{P}$$

(7)

$$\quad Y_{II}^{\psi,\rho} = Y_{II} \left( \beta, \frac{d}{D}, \frac{\rho}{D}, 2\alpha \right) = K_{II}^{\psi,\rho} d^{(\beta+1)} \frac{D t}{P}$$

(8)

The parameter $t$ in the above equations is the specimen thickness. As seen in Eqs. (7) and (8), the NSIFs $Y_{I}^{\psi,\rho}$ and $Y_{II}^{\psi,\rho}$ depend on the loading angle $\beta$, the relative notch length (RNL), the relative notch tip radius (RNR) and the notch angle $2\alpha$. The ratio $D t / P$ (mm$^2$/N) is, in fact, the reversed far-field reference stress applied to the V-BD specimen. Unlike $K$, $Y$ does not depend on the disc dimensions ($D$ and $t$) and also on the applied load $P$. Note that Eq. (1) has...
been presented for the K-dominant or singular stress field around the V-notch tip. Therefore, the mode II NSIF presented in Eq. (6) and hence, the mode II stress component becomes non-singular. For these angles, \( \lambda_2 \) becomes greater than 1 and hence, the mode II stress component becomes non-singular.

Figs. 4 and 5 show the variations of the mode I and mode II NSFs, respectively versus the loading angle \( \beta \) for \( 2\alpha = 30^\circ \) and different values of RNL and RNR. Notice that the values of the parameters \( K_{\|}^{V,\rho} \) and hence \( Y_{\|}^{V,\rho} \) are negative. So, the absolute values are presented entirely in the present manuscript for more convenience.

It is seen in these Figs. that as \( \beta \) enhances from zero (i.e. the loading angle corresponding to pure mode I) to \( \beta_{\|} \), \( Y_{\|}^{V,\rho} \) decreases from a finite positive value to zero and \( Y_{\|}^{V,\rho} \) enhances from zero to a finite value. For a constant value of RNL, as RNR increases, the

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**Fig. 5.** The variations of the mode II notch shape factor versus the loading angle \( \beta \) for \( 2\alpha = 30^\circ \) and different values of RNL and RNR.
mode I NSF increases and the mode II NSF decreases. Moreover, for a constant RNR, increasing the RNL leads to the enhancement of $Y_{V,II}$ and $Y_{II,II}$ because of higher stress concentration at the notch tip vicinity. Similar results are presented in Figs. 6 to 7 and 8 to 9 for $2\alpha = 60^\circ$ and $90^\circ$, respectively.

6. Discussion

The V-BD specimen has originally suggested by Ayatollahi and Torabi [12] and used frequently for measuring the fracture toughness of V-notched engineering components made of brittle materials under mixed I/II mode loading. As well-known, the standard fracture tests on notched specimens are always conducted with the aim to determine the notch fracture toughness. The main parameter that usually obtained from the simple fracture toughness tests on brittle materials is the fracture load while the results of the fracture criteria that estimate theoretically the fracture toughness of notched components are widely available in the form of the NSIFs (see for example [8–16,28]). Therefore, the researchers require essentially converting the experimentally obtained fracture loads to the corresponding values of the critical NSIFs in order to compare the experimental results with the theoretical ones. In order to calculate the NSIFs related to a measured fracture load of V-BD specimen more conveniently, one can simply follow a straight-forward procedure described herein. As previously mentioned, the finite element (FE) method can be used to analyze the elastic stress distribution around the V-notch tip for V-BD specimen and to calculate the notch stress intensity factors (NSIFs). To compute the NSIFs for a V-BD specimen under a given load and mixed mode I/II loading conditions by using the graphs of NSIFs (i.e. Figs. 4–9), one should first calculate the parameters RNL and RNR for the specimen and obtain the values of the NSIFs from Figs. 4–9. Then, by substituting NSIFs into Eqs. (7) and (8), the NSIFs can be directly achieved by knowing the given fracture load $P_{cr}$ and the specimen dimensions $D$ and $t$. 
Figs. 4–9 illustrate that for the notch angles of 30°, 60° and 90°:

a. For a constant RNL, as RNR increases, $Y_{V,\rho}$ increases and $Y_{II,\rho}$ decreases.

b. For a constant RNR, as RNL increases, both $Y_{I,\rho}$ and $Y_{II,\rho}$ increase. This is because the stress values in Eqs. (5) and (6) increase while the other parameters are constant.

c. For a constant RNL, as RNR increases from 0 to 0.075, the mode II loading angle $\beta_{II}$ increases approximately 5°.

d. For a constant RNR, as RNL increases from 0.1 to 0.5, the mode II loading angle $\beta_{II}$ decreases always less than 10°.

The same approach utilizing FE analysis has been frequently employed by Ayatollahi and Torabi [12–14] to calculate the critical NSIFs (called the notch fracture toughness) for numerous V-BD specimens made of PMMA [12], polycrystalline graphite [13] and soda-lime glass [14] tested under different mixed mode loading conditions from pure mode I to pure mode II. They utilized the calculated NSIFs in order to verify the theoretical results of a recently developed brittle fracture criterion called RV-MTS [12–14] and found very good agreements. More recently, Ayatollahi et al. [28] have computed the critical NSIFs for V-BD test specimens containing sharp V-notches (i.e. RNR = 0) made of PMMA in order to evaluate the results of the SV-MTS fracture criterion. In almost all of
the experiments performed by Ayatollahi and his co-workers on V-BD specimens, a very good correlation has been found between the theoretical values of the critical NSIFs and the experimental ones computed by using the same approach described in the present paper. Although not shown here, comparing the values of the NSIFs obtained comprehensively in the present work with those numerous values reported in [12–14] shows very good consistencies and demonstrates the validity of the present calculations.

Rather similar FE results of the shape factors have been reported in literature by Ayatollahi and Aliha [29] for the Brazilian disc specimen containing a central sharp crack (i.e. the CCBD specimen). They presented a wide range of data for the stress intensity factors (SIFs) and also the shape factors which are useful parameters in estimating the mixed mode I/II fracture toughness of brittle materials [29]. Another way of verification was followed by the authors to ensure the validity of the calculations presented in this work.

From a view point of geometry, a sharp crack can be considered as a rounded-tip V-notch with zero notch angle and also zero notch tip radius. Therefore, the V-BD specimen in such conditions becomes a CCBD specimen with the same flaw length. Consequently, we could compute the NSFs for the CCBD specimen by using our described procedure and compare the results with those reported in [29] and obtained from another computational procedure. For this purpose, first, the geometrical parameters corresponding to $2a = 0$ and $\rho = 0$ were entirely substituted into Eqs. (5) and (6) and then into Eqs. (7) and (8) to obtain the NSIFs and NSFs expressions for the CCBD specimen. Then, two FE models of the CCBD specimen with $\text{RNL} = 0.3$ and 0.5 were created and the stress analyzes were performed. Finally, comparing the obtained results of the shape factors (named as the geometry factor in [29]) with those reported in Fig. 10 of [29] (obtained by the combination of the displacement and the J-integral methods) revealed a very good consistency with a
discrepancy of less than 3% demonstrating again the validity of our calculations. It is noteworthy that the idea to suggest Eqs. (7) and (8) was taken from similar expressions presented in the past by Ayatollahi and Aliha [29] for sharp cracks in CCBD specimens. The main difference is the fact that despite the relative flaw (crack or notch) length, the number of the parameters affecting the NSFs for V-BD specimen (i.e. 3 parameters) is more than those affecting the stress intensity factors (SIFs) for CCBD specimen (i.e. 1 parameter).

One of the most important applications of the curves of the NSFs is to estimate more conveniently and quickly the fracture load for the V-BD specimens made of different brittle or quasi-brittle materials and hence to select an appropriate load cell for the test apparatus prior to the experiments. To do this, one should first obtain the NSFs for the V-BD specimen from the curves presented in Figs. 4–9. Correspondingly, the notch fracture toughness $K_{IC}$ or $K_{IVC}$ (i.e. the critical values of the NSIFs) should be predicted by using appropriate fracture criteria such as the cohesive zone model (CZM) [9] and the mean stress criterion (MSC) [10,11] in mode I loading conditions and the V-notched maximum tangential stress criterion (V-MTS) [12–14,28] in mixed mode and pure mode II loadings. Ultimately, substituting the predicted critical NSIFs and the obtained NSFs into Eqs. (7) and (8), also considering that the

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**Fig. 9.** The variations of the mode II notch shape factor versus the loading angle $\beta$ for $2\alpha = 90^\circ$ and different values of RNL and RNR.
7. Conclusions

The main conclusions obtained from the present work are as follows:

1. The NSF values can be conveniently used by researchers and engineers in the context of brittle fracture tests.

2. A comparison of the present results with the results reported in literature demonstrated the validity of the calculations conducted in this research.

3. The NSF combined with appropriate fracture criteria can be used rapidly and more conveniently to estimate the fracture load for the V-BD specimen made of different brittle materials and hence to select a proper load cell for the test apparatus prior to experiments.

4. The NSF curves provide a compact toolbox for the researchers and engineers to compute the NSF for the V-BD specimen loaded under mixed mode conditions without requiring creating the FE models and performing time-consuming analysis.

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Appendix A.

(a) Functions used in the stress field for rounded-tip V-shaped notches (modes I and II) [24]:

\[
\begin{align*}
\{f_0\} & = \frac{1}{[1 + \lambda_1 + X_{b_1}(1 - \lambda_1)]} \left[ (1 + \lambda_1)\cos(1 - \lambda_1)\theta + X_{b_1}(1 - \lambda_1) \right] \\
\{f_r\} & = \frac{1}{[1 + \lambda_2 + X_{b_2}(1 - \lambda_2)]} \left[ (1 + \lambda_2)\sin(1 - \lambda_2)\theta + X_{b_2}(1 + \lambda_2) \right] \\
\{g_0\} & = \frac{q}{4(q - 1)[1 + \lambda_1 + X_{b_1}(1 - \lambda_1)]} \left[ (1 + \lambda_1)\cos(1 - \lambda_1)\theta - \cos(1 + \lambda_1)\theta \right] \\
\{g_r\} & = \frac{q}{4(q - 1)[1 + \lambda_2 + X_{b_2}(1 + \lambda_2)]} \left[ (1 + \lambda_2)\cos(1 - \lambda_2)\theta - \sin(1 + \lambda_2)\theta \right]
\end{align*}
\]

(b) The expressions for parameters \(\omega_1\) and \(\omega_2\) [24]:

\[
\begin{align*}
\omega_1 & = \frac{q}{4(q - 1)} \left[ X_{d_1}(1 + \mu_1) + X_{c_1} \right] \\
\omega_2 & = \frac{1}{4(\mu_2 - 1)} \left[ X_{d_2}(1 - \mu_2) - X_{c_2} \right] = -1
\end{align*}
\]

The values of the parameters \(\lambda_1, \lambda_2, \mu_1, \mu_2, X_{b_1}, X_{b_2}, X_{c_1}, X_{c_2}, X_{d_1}, X_{d_2}\), are reported in [24] for various notch angles.
References