Failure assessment of notched polycrystalline graphite under tensile-shear loading

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The fracture load and the fracture initiation angle were experimentally measured for a V-notched specimen made of polycrystalline graphite under combined tensile-shear loading. The experimental results were obtained for several specimens with different notch angles and various notch tip radii. The experimental observations showed that for a constant notch tip radius, the fracture load in pure tensile loading conditions decreases as the notch angle increases. Moreover, for a constant notch angle, as the notch tip radius increases the fracture load in graphite specimens enhances in the entire domain between pure tensile and pure shear loading conditions. A recently developed failure criterion was then used to estimate the experimental values of the notch fracture resistance and the fracture initiation angle for the tested graphite specimens. The experimental results could be estimated very well by using the results of the proposed criterion.

1. Introduction

Graphite is widely utilized in many engineering applications such as power plants and steel-making industries as a refractory material. There are many mechanical components made of graphite that work in particular situations where the components are simultaneously subjected to both thermal and mechanical loads. Although graphite is well-known for keeping its mechanical properties at high temperatures, it is vulnerable to mechanical loads [1]. Mechanical elements may contain notches of different shapes, particularly V-shaped notches that generate a high stress concentration. Notched graphite components are prone to sudden fracture due to the brittleness of the material.

The research studies dealing with fracture in graphite materials have been mainly concentrated on thermal loading conditions. For example, the resistance of graphite materials against thermal shocks has been investigated in the past by several researchers [2–4]. Sato et al. [5] evaluated the thermal shock resistance and fracture toughness for several types of graphite by means of the arc-discharge method. Moreover, the irradiation effects on the thermal shock resistance and fracture toughness of HTGR-graphite were previously investigated by Sato et al. [6] both experimentally and theoretically.

Different mechanical properties of polycrystalline graphite materials have also been studied by several researchers, e.g. [1,7]. Roe and Torrance [8] studied and tested the surface failure and wear of graphite seals against steel sliders. Several failure criteria have already been suggested for predicting the load bearing capacity of graphite with no pre-existing cracks mainly based on the maximum principal stress (MPS) theory [9].

Fracture toughness has been experimentally determined for basic graphite by Lomakin et al. [10] and for carbon–carbon composites by Mirhabibi and Rand [11]. Awaji and Sato [7] conducted a wide range of fracture tests on the polycrystalline graphite samples containing a sharp crack under different loading conditions in order to evaluate the fracture toughness of the specimens experimentally.

There are several well-known failure criteria to study mixed mode I/II (i.e. combined tensile-shear) fracture in brittle materials like graphite. The maximum tangential stress (MTS) criterion [12], the minimum strain energy density (SED) criterion [13] and the maximum energy release rate or Griffith criterion [14] are three sample criteria which have been frequently used by the investigators. Recently, Ayatollahi and Aliha [15] employed the generalized MTS (GMCTS) criterion to predict tensile-shear fracture in two grades of commercial polycrystalline graphite. The same criterion was used by Ayatollahi and Aliha to provide very good estimates for the experimental results obtained from other brittle materials such as rock [16]. Ayatollahi and Torabi [17,18] have recently extended the application of the MTS criterion to U-notched domains in order to estimate the notch fracture toughness and the
fracture initiation angle in U-notched PMMA and soda-lime glass specimens when loaded under in-plane tension-shear (i.e., mixed mode I/II) and also pure shear (i.e., pure mode II). Additionally, Aytollahi and Torabi [19] made use of the mean stress (MS) criterion to estimate the notch fracture toughness of brittle components containing a round-tip V-notch. More recently, they made use of the MS criterion to analyze the experimentally obtained fracture toughness for three different V-notched polycrystalline graphite specimens in pure mode I loading conditions [20]. However, there are numerous practical applications in which a V-notched graphite component is subjected to a combination of tensile and shear (or mixed mode I/II) loading. A review of literature shows that the mixed mode I/II fracture behavior of polycrystalline graphite components containing a V-notch has not previously been investigated either experimentally or theoretically.

In the first part of this research, the fracture resistance of a polycrystalline graphite containing a V-shaped notch was determined experimentally for various notch angles and different notch tip radii by conducting a series of experiments on a test specimen, called V-notched Brazilian disc (V-BD), under tensile-shear loading conditions. In the second part, a failure criterion called the V-notched MTS (V-MTS) was used to predict the notch fracture toughness and the fracture initiation angle in the graphite V-BD specimens. It is shown that the mixed mode fracture toughness and the fracture initiation angle of graphite samples can be estimated well by using the results of the V-MTS criterion.

2. Experiments

The notched components can be subjected to three different types of in-plane loading, often called pure mode I, pure mode II and mixed mode I/II loading. Under pure mode I loading, any two respective points along the notch faces open relative to the notch bisector line without any sliding. In such a loading condition, the notch is subjected to pure tensile deformation. In pure mode II, the two respective points along the notch faces slide relative to the notch bisector line without any opening and hence the notch is deformed under pure shear. Any combination of mode I and mode II deformation is called mixed mode I/II loading.

Several specimens have been earlier suggested in literature for the experimental investigation of brittle fracture in graphite components containing sharp cracks [21–23]. One of the specimens commonly used in the past for mixed mode I/II fracture tests in cracked elements is the centrally cracked Brazilian disc (CCBD) specimen (see for example [7]). A modified version of CCBD specimen called V-BD, was used in the present study to perform mixed mode I/II fracture experiments for V-notches. Fig. 1 shows the V-BD specimen schematically.

In Fig. 1, $\beta$ is the angle between the loading direction and the notch bisector line and the parameters $2\alpha$, $D$, $d/2$ and $P$ are the notch angle, the disc diameter, the notch depth and the applied compressive load, respectively. When the direction of the applied load $P$ is along the notch bisector line (i.e., $\beta = 0$), the upper and the lower corners of the rhombic hole are subjected to pure mode I deformation. When the angle $\beta$ increases gradually from zero, the loading condition varies from pure mode I towards pure mode II. For a specific angle, called $\beta_{II}$, pure mode II deformation is achieved. The mode II loading angle $\beta_{II}$ is always less than $90^\circ$ and depends on the notch length and its opening angle and also on the notch tip radius. The angles $\beta_{II}$ can be determined by using the finite element (FE) method as described in Section 4. The material used for fabricating V-BD specimens was a commercial grade of medium-grained equi-axed polycrystalline graphite with the properties presented in Table 1.

To measure the plane-strain fracture toughness ($K_{IC}$) and the tensile strength ($\sigma_t$) of brittle materials like graphite, a common method is to conduct compressive tests on the CCBD and the classical Brazilian disc (BD) specimens, as described in [7]. Five BD specimens of diameter 50 mm and thickness 8 mm and five CCBD specimens with the same dimensions but containing a central crack of length 25 mm were tested to obtain average values of $\sigma_t$ and $K_{IC}$ given in Table 1. The percentage discrepancies between the maximum and the minimum values of $\sigma_t$ and $K_{IC}$ for the tested graphite were about 7% and 6%, respectively. It was also necessary to examine whether the tested graphite material is isotropic at least in at least one direction.

### Table 1

Properties for the tested graphite.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bulk density (kg/m³)</td>
<td>1710</td>
</tr>
<tr>
<td>Mean tensile strength ($\sigma_t$) (MPa)</td>
<td>27.5</td>
</tr>
<tr>
<td>Young’s modulus ($E$) (GPa)</td>
<td>8.05</td>
</tr>
<tr>
<td>Plane-strain fracture toughness ($K_{IC}$) (MPa m0.5)</td>
<td>1.0</td>
</tr>
<tr>
<td>Mean grain size ($\mu$m)</td>
<td>320</td>
</tr>
</tbody>
</table>
the plane where the specimens were cut. Therefore, four BD specimens were fabricated and tested in two different loading directions perpendicular to each other (i.e., two tests for each direction). The results showed that the mean values of the tensile strength for the tested polycrystalline graphite in two different loading directions were almost identical with a maximum discrepancy of about 5%, demonstrating a nearly isotropic material behavior. However, we conducted all our V-notch fracture experiments from specimens cut in identical directions from the initial graphite block. By this arrangement, possible effects from this 5% difference would also be minimized.

For all the V-BD graphite specimens, the disc diameter ($D$), the notch depth ($d/2$) and the thickness were 60 mm, 15 mm and 8 mm, respectively.

To study the effects of the notch angle and the notch tip radius on the fracture behavior of the graphite specimens, three values of notch angle $2\alpha = 30^\circ$, $60^\circ$, $90^\circ$ and three values of notch tip radius $\rho = 0$, $1$, $2$ mm were considered for fabricating the specimens. To prepare the graphite test specimens, first several plates of 8 mm thick were cut from a graphite block. Then, the specimens were precisely fabricated by using a 2-D CNC water-jet cutting machine. Before conducting the experiments, the cut surfaces of the graphite specimens were polished by using a fine abrasive paper. For sharp V-notches ($\rho \approx 0$), the notch tip was carefully sharpened by using a razor blade.

A total number of 108 mixed mode I/II fracture tests were performed for various notch geometry parameters and different loading angles $\beta$ from 0 (pure mode I) to $\beta_{II}$ (pure mode II). For $2\alpha = 30^\circ$, the experiments were performed according to the loading angles $\beta$ equal to $0^\circ$, $10^\circ$, $20^\circ$, $25^\circ$. Similarly, for $2\alpha = 60^\circ$ and $90^\circ$, the fracture tests were conducted for various angles $\beta$ of $0^\circ$, $15^\circ$, $25^\circ$, $30^\circ$ and $0^\circ$, $15^\circ$, $25^\circ$, $35^\circ$, respectively. For each geometry shape and loading angle, three fracture tests were performed by using a universal tension-compression test machine under displacement-control condition with a loading rate of 0.05 mm/min.

Fig. 2 shows two sample graphite specimens placed in the test machine for mode I and mixed mode I/II fracture tests, and Fig. 3 shows sample V-BD specimens broken after mixed mode I/II fracture tests.

The experimental observations showed that the load–displacement curves recorded during the fracture tests were all linear, and fracture occurred abruptly (see Fig. 4).

The mean values of fracture loads ($P_f$) recorded by the test machine are presented in Table 2, for each specimen. Table 2 shows that for a constant notch tip radius, the fracture load in pure mode I loading conditions (i.e. $\beta = 0$) decreases as the notch angle increases. Moreover, for a constant notch angle, as the notch tip radius increases, the fracture load enhances in the entire domain between pure mode I and pure mode II loading conditions. It is also seen in Table 2 that when both notch angle and notch tip radius are constant, the fracture load increases as the shear contribution of loading is increased (i.e. when $\beta$ increases).

In the next section, the elastic stress distribution around a V-notch is presented under general in-plane loading conditions. It will be shown that the presented stress field can be utilized to describe the procedure for determining the mode II loading angle $\beta_{II}$ and to interpret the results of the V-MTS failure criterion in estimating the notch fracture toughness and the fracture initiation angle for the tested V-BD specimens.

3. Elastic stress distribution around a V-notch

Filippi et al. [24] developed an expression for mixed mode I/II stress distribution around a V-shaped notch shown in Fig. 5. The
stress distribution is an approximate formulation because it satisfies the boundary conditions only in a finite number of points on the notch edge and not on the whole edge. Filippi et al. [24] obtained the stress distribution using a conformal mapping in an auxiliary system of curvilinear coordinates "U and V" that are related to the Cartesian coordinates "X and Y" as \((X + iY) = (U + iV)^\rho\). The power "\(q\)" is a real positive coefficient ranging from 1 (for a flat edge) to 2 (for a crack).

The mixed mode I/II stresses can be written as

\[
\begin{bmatrix}
\sigma_{II} \\
\sigma_{III}
\end{bmatrix} = \frac{K_{I}^{V,\rho}}{\sqrt{2\pi r^{1-\lambda_1}}} \begin{bmatrix}
f_{\theta}(\theta) \\
f_{r}(\theta)
\end{bmatrix}^{(I)} + \frac{K_{II}^{V,\rho}}{\sqrt{2\pi r^{1-\lambda_2}}} \begin{bmatrix}
f_{\theta}(\theta) \\
f_{r}(\theta)
\end{bmatrix}^{(II)}
\]

where \(K_{I}^{V,\rho}\) and \(K_{II}^{V,\rho}\) are the mode I and mode II notch stress intensity factors (NSIFs), respectively. The parameter \(r_0\) is the distance between the origin of the polar coordinate system and the notch tip.

The functions \(f_{\theta}(\theta)\) and \(g_{\theta}(\theta)\) have been reported in Appendix A and the eigenvalues \(\lambda_1\) and \(\mu_1\), which depend upon the notch angle, have been reported in [24]. It can be shown that if the notch tip radius vanishes, Eq. (1) becomes the same as the stress field previously obtained by Williams [25] for sharp V-notches. According to a relation that exists between the Cartesian and the curvilinear coordinate systems, \(r_0\) can be written as [24]:

\[
r_0 = \frac{q - 1}{q} \rho, \quad q = \frac{2\pi - 2\alpha}{\pi} \tag{2}
\]

where "\(2\alpha\)" is the notch angle and \(\rho\) is the notch tip radius. The expressions for NSIFs are [26]:

\[
K_{I}^{V,\rho} = \sqrt{2\pi} \left( \frac{\sigma_{00}}{1 + \sigma_1 (r/r_0)^{1-\lambda_1}} \right)
\]

\[
K_{II}^{V,\rho} = \sqrt{2\pi} \left( \frac{\sigma_{00}}{1 + \sigma_2 (r/r_0)^{1-\lambda_2}} \right)
\]

where \(\sigma_{00}\) and \(\sigma_{0\rho}\) are the tangential and in-plane shear stresses, respectively. The auxiliary parameters \(\sigma_1\) and \(\sigma_2\) have been presented in Appendix A. If the values of the parameters \(\sigma_1\) and \(\sigma_2\) are known, the NSIFs can be obtained from Eqs. (3) and (4) as

\[
K_{I}^{V,\rho} = \sqrt{2\pi} \left( \frac{\sigma_{00}(r, 0)r^{1-\lambda_1}}{1 + \sigma_1} \right)
\]

\[
K_{II}^{V,\rho} = \lim_{r \to 0} \sqrt{2\pi} \left( \frac{\sigma_{00}(r, 0)r^{1-\lambda_2}}{1 - (r/r_0)^{2(1-\lambda_2)}} \right)
\]

NSIFs can be calculated by using the FE method as described in the next section. Note that the parameter \(r\) in Eq. (6) cannot be directly substituted by \(r_0\), because for \(r = r_0\), \(K_{II}^{V,\rho}\) becomes singular. Therefore, \(K_{II}^{V,\rho}\) is calculated from Eq. (6) at a point very close to the notch tip where \(r \to r_0\). When the notch tip radius \(\rho\) is zero (i.e., the case of a sharp notch), the stress values at the notch tip tend to infinity and hence the parameters \(K_{I}^{V,\rho}\) and \(K_{II}^{V,\rho}\) cannot be directly obtained by using Eqs. (5) and (6). In such conditions, the limits of the expressions given in Eqs. (5) and (6) should be calculated where \(r \to 0\).

### 4. FE analysis

In this section, the procedures needed for determining the NSIFs and the mode II loading angle \(\beta_\theta\) are elaborated. In order to compute NSIFs for the V-BD specimens used in the experiments, it is necessary to perform a FE analysis for each test specimen. Under an arbitrary load \(P\), the values of the tangential and shear stresses \(\sigma_{00}\) and \(\sigma_{0\rho}\) along the notch bisector are obtained from the FE analysis and the NSIFs are calculated by using Eqs. (5) and (6).

A plane-stress FE model with a total number of 52,800 elements was created for each specimen. Fine elements were utilized at the notch tip vicinity because of high stress gradient. In order to obtain the mode II loading angle \(\beta_\theta\), \(\theta\) was gradually increased from zero and the tangential stress at the notch tip \(\sigma_{0\rho}(r_0, 0)\) was obtained from the FE results. Under a fixed load \(P\), as \(\beta\) becomes larger the value of \(\sigma_{0\rho}(r_0, 0)\) decreases. According to Eq. (5), \(\beta_\theta\) is the angle for which \(\sigma_{0\rho}(r_0, 0)\) equals zero and hence \(K_{II}^{V,\rho}\) is equal to zero and the V-notch is subjected to pure mode II loading. The FE analyses showed that \(\beta_\theta\) increases for larger notch angles whereas for a constant notch angle, the angle \(\beta_\theta\) is not significantly affected by the notch tip radius. The FE results for the V-BD specimens of \(d/D = 0.5\) showed that \(\beta_\theta\) was approximately 25\(^\circ\), 30\(^\circ\) and 35\(^\circ\) for the notch angles 30\(^\circ\), 60\(^\circ\) and 90\(^\circ\), respectively.

In the next section, a recently developed mixed mode failure criterion is briefly described. This criterion will be used in Section 6 to estimate the experimental results for the graphite V-BD specimens.

### 5. Fracture criterion

The conventional MTS criterion is a well-known failure criterion frequently used for investigating mixed mode I/II brittle fracture for components containing a sharp crack [12]. According to the
MTS criterion, fracture occurs radially from the crack tip along the direction $\theta_0$ which is perpendicular to the direction of maximum tangential stress. The onset of fracture takes place when the tangential stress along $\theta_0$ and at the critical distance $r_c$ from the crack tip attains a critical value ($\sigma_{c0}$)\cite{12}.

Recently, Ayatollahi and Torabi\cite{27} have extended the MTS criterion to V-notches and proposed a failure criterion, called V-MTS, to estimate the fracture toughness and the fracture initiation angle for V-notched brittle components broken under combined tensile-shear loading. Using the V-MTS criterion, a set of fracture curves have been developed based on the notch stress intensity factors (NSIFs) with capability of predicting the fracture toughness and the fracture initiation angle for V-notches in the entire domain from pure mode I to pure mode II.

As shown in Fig. 4, the recorded load–displacement curves of the graphite V-BD specimens were linear from the beginning up to the fracture point. Therefore, the use of a brittle fracture theory (e.g. V-MTS criterion) based on the linear elastic notch fracture mechanics (LENFM) is adoptable. Using the V-MTS criterion, mixed mode I/II fracture in V-notched graphite specimens can be theoretically studied using a series of fracture curves that depend on the notch angle and the notch tip radius. The curves are similar to the fracture curve of the conventional MTS criterion which has been frequently used for analyzing the components having a sharp crack\cite{12}. To obtain these curves, a parameter, called the notch mode mixity parameter ($M_{0V}$) have been defined in\cite{27} as

$$M_{0V} = \frac{2}{\pi} \tan^{-1}\left(\frac{K_{Ic}^V - K_{Ic}^V}{r_c}\right)$$ \hspace{1cm} (7)

In Eq. (7), $r_{c,V}$ is the notch critical distance described in\cite{27}. $M_{0V}$ varies from zero (for pure mode II) to one (for pure mode I). By computing the fracture initiation angle $\theta_0$ in the entire domain between mode I and mode II by means of the V-MTS criterion, one can draw $\theta_0$ in terms of $M_{0V}$ to obtain the fracture initiation angle curves. The curves of fracture initiation angle can be obtained by using the procedure elaborated in\cite{27} for V-notches with different notch angles and various notch tip radii subjected to mixed mode I/II loading.

Fig. 6 presents a sample fracture initiation angle curve calculated for the V-notched graphite component with the notch angle $2\alpha = 60^\circ$ and the notch tip radius $\rho = 1$ mm. To use this curve in the engineering design, one should first compute NSIFs for the V-notched component under the given load and then simply calculate $M_{0V}^V$ from Eq. (7) and estimate the fracture initiation angle $\theta_0$.

Similar curves have been extracted in\cite{27} for estimating the onset of mixed mode brittle fracture using the V-MTS criterion. Fig. 7 shows a sample mixed mode I/II fracture curve calculated for the V-notched graphite component with the notch angle $2\alpha = 60^\circ$ and the notch tip radius $\rho = 1$ mm. Similar fracture curves can be derived for other notch angles and notch tip radii. In these curves, $K_{II}^V/\rho$ is the mode I notch fracture toughness which can be determined from a simple mode I fracture test on a relevant V-notch specimen.

In order to use these curves in practical cases and to predict if a V-notched graphite component fractures under a given load, first the mode I and mode II NSIFs related to the applied load should be calculated for the notched component. Then, these NSIFs are divided by $K_{II}^V/\rho$ to obtain $K_{II}^V/\rho_{II}$ and $K_{I}^V/\rho_{Ic}$. If the obtained point locates under the related fracture curve (e.g. the one presented in Fig. 7), the fracture will not occur. Otherwise, the fracture is expected to occur.

6. Results and discussion

In order to compare the experimental results with the theoretical fracture curves, it is essential to convert the fracture loads obtained for the tested graphite specimens (presented in Table 2) to the corresponding critical NSIFs. For this purpose, the FE method as described in Section 4 was employed.

Here, the results of the V-MTS criterion in predicting the mixed mode I/II fracture toughness and the fracture initiation angle of V-BD specimens are compared with the experimental results obtained for the polycrystalline graphite. Figs. 8–10 show the theoretical and experimental results related to the onset of fracture in the graphite specimens for the notch angles of 30°, 60° and 90° and three different notch tip radii $\rho = 0, 1, 2$ mm. The figures indicate that as the notch tip radius increases, the notch fracture toughness enhances because of lower stress concentrations near the notch tip. Note that the fracture loads presented in Table 2 are the mean values of the experimental results; however, all of the test results are shown in Figs. 8–10 in order to show the scatter in the experimental results. A good correlation is seen in Figs. 8–10 between the theoretical and experimental results obtained for the onset of mixed mode fracture in V-notched graphite specimens.

Both the theoretical and the experimental results confirm that for a constant notch angle, the fracture toughness enhances as the notch tip radius increases because of lower stress concentration around the notch tip. A review of Figs. 8–10 also indicates that for a constant notch tip radius, the numerical value of the notch frac-
Fig. 8. Experimental results obtained for the onset of fracture in the graphite V-BD specimens of $2\alpha = 30^\circ$ compared with the V-MTS curve. (a) $\rho = 0$ mm, (b) $\rho = 1$ mm and (c) $\rho = 2$ mm.

ture toughness dramatically increases as the notch angle becomes larger. From a mathematical point of view, this is because, by increasing the notch angle, the term $(1 - \lambda_2)$ for mode II decreases more rapidly compared to the term $(1 - \lambda_1)$ for mode I (see Eqs. (5) and (6)). Note that for a constant notch tip radius, a larger value of the notch fracture toughness does not necessarily correspond to a larger fracture load. Because, the fracture load of a V-notched specimen depends not only on the notch angle but also on the geometry and loading conditions of the notched specimen. For instance, the experimental results demonstrated that for the V-BD specimens with a constant notch tip radius loaded under pure mode I (i.e. $\beta = 0$), as the notch angle increases, the value of notch fracture toughness ($K_{IIc}$) increases while the fracture load decreases. Also note that the dimension of $K_{Ic}$ is $m^{1-\lambda_1}$ that trivially depends on the notch angle. In other words, the dimension of $K_{IIc}$ for two V-notches having the same notch tip radius but two different notch angles are different. It is noteworthy that for some of the specimens
proposed earlier such as the single-edge notched tension (SENT), and the rectangular plate subjected to three-point bend (TPB) or four-point bend (FPB) loading, both the fracture load and the mode I notch fracture toughness ($K_{V1c}$) become larger when the notch angle increases.

The V-MTS results presented in Figs. 8–10 for polycrystalline graphite illustrate that for a constant notch angle, the rate of enhancement for notch fracture toughness between $\rho = 0$ and $\rho = 1$ mm is much larger than that between $\rho = 1$ and $\rho = 2$ mm. Although the difference between the notch tip radii in these two cases is the same and equal to 1 mm, the first case (i.e. $\rho = 0$ and $\rho = 1$ mm) compares a nearly sharp notch and a round-tip notch. However, the second case (i.e. $\rho = 1$ and $\rho = 2$ mm) provides a comparison between two round-tip V-notches. This phenomenon may
be due to the transition from a sharp to a round-tip V-notch that results in disappearing the stress singularity and hence significant reductions in the stress concentration at the notch tip vicinity. Although not shown here, for V-notches having identical notch angles, when the notch tip radius is larger than $\rho = 5 \text{ mm}$, a unique fracture curve is obtained from the V-MTS criterion. Therefore, one can use the fracture curve corresponding to the notch tip radius $\rho = 5 \text{ mm}$ for the other V-notches having a notch tip radius larger than 5 mm.

Figs. 11–13 show the theoretical curves of the V-MTS criterion for the fracture initiation angle together with the experimentally measured fracture initiation angles for the graphite specimens described in Section 2. The vertical axis represents the fracture initiation angle and the horizontal axis represents the parameter $M^o_V$ that varies from 0 (for pure mode II) to 1 (for pure mode I). Again the V-MTS curves provide very good estimates for the fracture initiation angles obtained experimentally.
According to Figs. 11–13, as $M_f^2$ decreases from 1 (pure mode I) to 0 (pure mode II), the fracture initiation angle ($\theta_f$) is raised from 0° up to a mode II fracture initiation angle. Moreover, by increasing the notch tip radius, the fracture initiation angle is slightly reduced. Figs. 11–13 also indicate that for a constant notch tip radius, the notch angle has almost no effect on the fracture initiation angle when mode I is dominant (i.e. $M_f^2 > 0.6$). However, as the contribution of mode II enhances the effect of notch angle on the fracture initiation angle increases slightly such that its maximum influence takes place in pure mode II loading conditions. For example, the mode II fracture initiation angle decreases about 10° when the notch angle increases from 30° to 90°.

In order to investigate the effects of notch geometry and loading conditions on the load-bearing capacity of the V-BD graphite specimens and also to compare the physical behavior of V-notches with a planar sharp crack more clearly, a dimensionless parameter, called relative notch fracture load (RNFL), is defined herein. This parameter represents the ratio of experimentally measured fracture load for any V-BD specimen over the experimental mode I fracture load for the cracked BD specimen of the same flaw length which was obtained during the plane-strain fracture toughness test.

Fig. 14 displays the variation of the RNFL parameter versus the notch tip radius for different notch angles ($2\alpha$) and various loading angles ($\beta$).

It is observed from this figure that for all of the notch angles, RNFL increases as the notch tip radius increases, which is obviously due to lower stress concentrations around the larger radii. It is also seen in Fig. 14 that for a constant notch tip radius, the fracture load for V-BD specimen in mode I loading conditions (i.e. $\beta = 0$) decreases as the notch angle increases. Although the curves in Fig. 14 are plotted for the experimentally measured fracture loads, it was found that similar curves could be obtained by using simultaneously the FE analysis and the V-MTS criterion. Fig. 14 shows that in pure mode I loading conditions (i.e. $\beta = 0$), the RNFL values for all the V-BD graphite specimens having sharp notch tip ($\rho = 0$) are less than 1. This implies that the fracture load for a cracked BD specimen becomes larger than that for V-BD specimens having zero notch tip radius and non-zero notch angles. In order to evaluate this apparently surprising experimental finding, we attempted to study the stress distribution around the tip of a V-notch in the V-BD specimens numerically for arbitrarily chosen values of notch tip radius ($\rho = 3$ mm) and reference load of $P = 6$ kN. Fig. 15 shows the radial distribution of tangential stress obtained from finite element analysis for the V-BD specimens along the notch bisector line.

Fig. 15 illustrates that for a constant load $P$, as the notch angle increases, the tangential stress ahead of the notch tip enhances. Taking into account that all the specimens have the same material properties, it can be concluded from an experimental point of view that for larger notch angles, the tangential stress attains its critical value under a lower value of the applied load.

Fig. 14 represents that for V-BD graphite specimens, the fracture load increases as the loading mode tends from pure mode I to 0 (pure mode II), the fracture initiation angle decreases about 10° when the notch angle increases from 30° to 90°.

In order to investigate the effects of notch geometry and loading conditions on the load-bearing capacity of the V-BD graphite specimens and also to compare the physical behavior of V-notches with a planar sharp crack more clearly, a dimensionless parameter, called relative notch fracture load (RNFL), is defined herein. This parameter represents the ratio of experimentally measured fracture load for any V-BD specimen over the experimental mode I fracture load for the cracked BD specimen of the same flaw length which was obtained during the plane-strain fracture toughness test.

Fig. 14 displays the variation of the RNFL parameter versus the notch tip radius for different notch angles ($2\alpha$) and various loading angles ($\beta$).
I towards pure mode II [i.e. as $\beta$ enhances]. The theoretical results obtained from the notch stress analysis together with the V-MTS failure criterion also confirmed this experimental finding.

It is noteworthy that the V-MTS fracture curves presented in this paper for graphite have been plotted in terms of NSIFs. Therefore, they can be employed for estimating the mixed mode I/II fracture resistance for any graphite V-notched component regardless of its overall geometry, notch length and type of loading. In other words, these curves depend only on the notch angle and notch tip radius. On the contrary, the RNFL curves (i.e. the curves of normalized fracture load) cannot be utilized for other types of V-notched test specimens or engineering components made of the same graphite, since the RNFL curves depend on the geometry of specimen and its loading conditions. However, as stated earlier, the RNFL curves were presented only for more convenient understanding of the fracture behavior in the V-BD specimens.

It should be finally noted that some types of graphite materials exhibit nonlinear stress–strain responses in a standard tension (or compression) test conducted on a plain test specimen. When using a plain specimen with no stress concentration, a considerable volume of material often undergoes nonlinear deformation before the final failure. This may give rise to a nonlinear stress–strain curve for the tested materials. However, in test specimens containing a stress concentrator (like a crack or notch); the material nonlinearity is often localized only around the crack or notch tip and the load–displacement curve is linear even for the same graphite material. In such conditions, a fracture criterion like the V-MTS criterion is anticipated to be still applicable as long as the notch specimen fails in a brittle manner with a nearly linear load–displacement curve.

7. Conclusions

Fracture in V-notched polycrystalline graphite specimens was examined both experimentally and theoretically under combined tensile-shear loading conditions. For the experiments, a test sample called the V-BD specimen was used which could provide the entire loading domain from pure mode I to pure mode II. A fracture criterion, called V-MTS, was employed to estimate the test results related to the mixed mode fracture resistance and the fracture initiation angle for V-notched specimens made of polycrystalline graphite. Very good agreement was found between the theoretical estimates and the experimental results for both the fracture resistance and the fracture initiation angle.

Appendix A.

(a) Functions used in the stress field for rounded V-shaped (modes I and II) [24]:

\[
\begin{align*}
&f_{I0} = \frac{1}{1 + \lambda_1 + \chi b_1 (1 - \lambda_1)} \\
&f_{I1} (1 + \lambda_1 \cos (1 - \lambda_1) \theta) + \chi b_1 (1 - \lambda_1) \sin (1 - \lambda_1) \theta) \\
&g_{I0} = \frac{q}{4(q - 1) [1 + \lambda_1 + \chi b_1 (1 - \lambda_1)]} \\
&g_{I1} (1 + \mu_1 \cos (1 - \mu_1) \theta) + \chi c_1 (1 - \mu_1) \sin (1 - \mu_1) \theta) \\
&f_{II0} = \frac{1}{1 - \lambda_2 + \chi b_2 (1 + \lambda_2)} \\
&f_{II1} (1 + \lambda_2 \sin (1 - \lambda_2) \theta) + \chi b_2 (1 + \lambda_2) \cos (1 - \lambda_2) \theta) \\
&g_{II0} = \frac{1}{4(\mu_2 - 1) [1 - \lambda_2 + \chi b_2 (1 + \lambda_2)]} \\
&g_{II1} (1 + \mu_2 \sin (1 - \mu_2) \theta) + \chi c_2 (1 - \mu_2) \cos (1 - \mu_2) \theta)
\end{align*}
\]

(b) The expressions for parameters $\omega_1$ and $\omega_2$ [24]:

\[
\begin{align*}
\omega_1 &= \frac{q}{4(q - 1)} \left[ \chi d_1 (1 + \mu_1) + \chi c_1 \right] \\
\omega_2 &= \frac{1}{4(\mu_2 - 1)} \left[ \chi d_2 (1 - \mu_2) - \chi c_2 \right] = -1
\end{align*}
\]

The values of the parameters $\lambda_1, \lambda_2, \mu_1, \mu_2, \chi b_1, \chi b_2, \chi c_1, \chi c_2, \chi d_1, \chi d_2$ are reported in [24] for various notch angles.

References