IMPACT OF DIAPHRAGMS ON SEISMIC RESPONSE OF STRAIGHT SLAB-ON-GIRDER STEEL BRIDGES

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ABSTRACT: Many steel bridges have suffered diaphragm (cross frame) damage during recent earthquakes. Diaphragms provide an important load path for the seismically induced loads acting on slab-on-girder steel bridges, but their impact on seismic response is still unclear in many ways. The relative role played by intermediate and end diaphragms in providing lateral load resistance, along with the consequences of diaphragm damage on bridge seismic response, has not been studied. This paper quantitatively investigates the impact of diaphragms on the seismic response of straight slab-on-girder steel bridges. Typical 20 to 60 m span slab-on-girder bridges with and without diaphragms are considered and studied through elastic and inelastic static pushover analyses. Two hand-calculation analytical models are proposed to evaluate their period, elastic response, and pseudospectral acceleration at first yielding. It is shown that a small end-diaphragm stiffness is sufficient to make the entire superstructure behave as a unit in the elastic range. However, a dramatic shift in seismic behavior occurs once an end diaphragm fractures, with a sizable period elongation, considerably larger lateral displacements, and higher propensity to damage owing to P-Δ effects. It is also found that the presence of intermediate diaphragms does not significantly influence the seismic performance of these types of bridges, in either the elastic or the inelastic range.

INTRODUCTION

Several steel bridges have collapsed or suffered significant damage during recent earthquakes such as the 1989 Loma Prieta (Earthquake 1990), 1994 Northridge (Astaneh-Asl et al. 1994; Earthquake 1994; Mitchell et al. 1995), and 1995 Kobe earthquakes (Bruneau et al. 1996). Although the potential seismic vulnerability of bridges designed and constructed at a time when seismic-resistant provisions were nonexistent or ineffective has long been recognized [e.g., Tseng and Penzien (1973) and Kawashima (1990)], these more recent failures have triggered considerable seismic evaluation and retrofit activities throughout North America and generated renewed research interest on that subject. However, most of the current knowledge in earthquake-resistant bridge design is based on past studies of concrete bridges, and may require adjustments to effectively capture the seismic behavior germane to steel bridges. One such aspect of this behavior is reviewed here.

Currently, in the literature on the seismic evaluation or design of bridges [e.g., Applied (1981), Buckle et al. (1986), Ontario (1991), and Standard (1994)], when the lateral period of a slab-on-girder bridge is determined, the superstructure (deck and girders) is modeled as an equivalent beam supported on columns and/or foundation springs. The effective transverse stiffness of this equivalent beam is calculated considering that the deck and girders act as a single cross section. While this approach is acceptable for concrete bridges and box-girder superstructures, it may not be for some types of existing slab-on-girder steel bridges. Typically, in such bridges, the concrete deck slab is supported on I-shape beams interconnected by a few discrete diaphragms, and the mechanism by which the seismically induced inertia forces at the concrete slab level will be transmitted to the girders bearings can be quite different from that assumed by the equivalent beam analogy. The magnitude of this difference is tied to the effectiveness of the diaphragms, and can be quite large in bridges having flexible diaphragms. It is the objective of this paper to quantitatively investigate the lateral response of straight slab-on-girder steel bridges subjected to seismic lateral loads for various diaphragm conditions.

Here, a particular emphasis is placed on obtaining a proper representation of the superstructure's lateral stiffness, as this has a direct impact on bridge period and, consequently, on the intensity of earthquake excitation felt by the superstructure, bearings, and substructure. To achieve this, the behavior of bridges with and without any effective diaphragms is studied. Although the latter case may appear to be a theoretical situation, it is a model worth considering for the bridges having severely rusted diaphragms or with only nominal diaphragms (e.g., single channels bolted along their web as shown in Fig. 1) frequently encountered in eastern North America. Moreover, bridges having diaphragms with nonductile connection details can potentially become bridges without diaphragms once brittle failures develop at those connections. Comprehensive analytical expressions that capture the behavior germane to slab-on-girder steel bridges as a function of end-diaphragm characteristics are presented, and elastic and inelastic analyses are conducted to validate the proposed models.

PRELIMINARY INFORMATION

Diaphragm Design Requirements

Many bridge design codes require that slab-on-girder bridges be provided with end-span diaphragms as well as intermediate diaphragms (cross frames) at a spacing of no more than 7.6 m (Standard 1994) or 8 m (Ontario 1991). While these codes state that end diaphragms shall be proportioned to transmit all lateral forces to the bearings (and are designed by some departments of transportation to also serve as transfer members for jacks used to lift the superstructure during future bearing replacements), nothing is said about the role of the intermediate diaphragms. Although the intermediate diaphragms facilitate the construction process and stabilize the top compression flange of girders until the composite concrete deck is in place, no design guidance or detailing requirement is provided by these codes. In fact, even though some engineers have alleged that intermediate diaphragms can help evenly distribute the gravity and live loads among girders during service, recent research indicates otherwise (Azizinamini et al. 1995). As a result, diaphragm design has varied consid-
to the main girders. The effectiveness of such a diaphragm to transfer lateral loads is debatable. The much lower lateral stiffness by this type of diaphragm translates into a longer bridge structural period with correspondingly lower lateral forces, but larger drifts.

**Description of Selected Bridges**

Simply supported single-span bridges, each supported by hinged bearings on one abutment and expansion bearings on the other, are considered for this study. Although multispan, simply supported steel bridges are known to be more vulnerable to earthquakes (Diceli and Bruneau 1995, 1996), the behavior of those more complex bridges can be modeled using the findings from the study reported here, by replacing each simply supported span with an equivalent beam whose mass and stiffness properties are selected to give the appropriate element stiffness using the equations developed hereafter.

The bridges designed by Dicleli and Bruneau (1995) are considered. To reflect the expected seismic performance of older existing highway steel bridges, these bridges were designed to be in compliance with the strength requirements of the 1961 edition of the American Association of State Highway Officials (AASHTO) code (Standard 1961). The characteristics of the bridges used in the current case study are listed in Table 1.

In all cases, 8-m-wide, two-lane straight bridges supported by four 300W grade steel girders spaced at 2 m center-to-center are considered. The bridge deck consists of a 200-mm-thick, 20 MPa concrete slab topped by 70 mm of asphalt. Finally, none of the bridges considered in this study have underside bracings; this was observed to be the case for those bridges in eastern North America having rather weak diaphragms.

**SLAB-ON-GIRDER STEEL BRIDGES WITHOUT DIAPHRAGM**

For bridges without diaphragm, preliminary analyses revealed that the concrete deck responded as a nearly rigid member in the transverse direction, irrespective of the pattern of the applied distributed lateral load. Hence, an equivalent static uniformly distributed load (UDL) applied at the deck level was deemed representative of the seismically induced inertia forces acting on this bridge structure. In the following, a generic load level corresponding to a pseudo-acceleration of 1g is considered. Obviously, all elastic analyses results presented in the following can be scaled as necessary.

The lateral response behavior of typical slab-on-girder steel bridges of various span lengths was first investigated using the program SAP90 (Wilson and Habibullah 1992). The resulting calculated first lateral period of vibration, resulting maximum drifts, and pseudospectral acceleration (PSa) required to produce first yielding are presented as a function of span length in Figs. 2(a–c), respectively (first yield being located in the vicinity of 0.05 g).

**TABLE 1. Geometric and Structural Characteristics of Steel Bridge Considered in Case Studies**

<table>
<thead>
<tr>
<th>Span (m)</th>
<th>Deck width (m)</th>
<th>Number of girders</th>
<th>Girder spacing (m)</th>
<th>Slab depth (mm)</th>
<th>Girder size and properties</th>
<th>Mass (10^3 kg)</th>
<th>I_w (10^4 m^4)</th>
<th>I_b (10^4 m^4)</th>
<th>I_d (m^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>200</td>
<td>WWF800 × 184</td>
<td>125</td>
<td>0.111</td>
<td>56.25</td>
<td>1.322</td>
</tr>
<tr>
<td>30</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>200</td>
<td>WWF1000 × 262</td>
<td>202</td>
<td>0.229</td>
<td>133.3</td>
<td>1.617</td>
</tr>
<tr>
<td>40</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>200</td>
<td>WWF1200 × 333</td>
<td>286</td>
<td>0.341</td>
<td>160</td>
<td>1.797</td>
</tr>
<tr>
<td>50</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>200</td>
<td>WWF1400 × 405</td>
<td>367</td>
<td>0.341</td>
<td>312.5</td>
<td>1.983</td>
</tr>
<tr>
<td>60</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>200</td>
<td>WWF1600 × 496</td>
<td>465</td>
<td>0.341</td>
<td>485.3</td>
<td>2.212</td>
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</tbody>
</table>

I_w, I_b, and I_d are the moments of inertia of girder web per unit length about bridge longitudinal axis, girder bottom flange about its strong axis, and superstructure about a vertical axis, respectively.

appropriate model should consider the displacement of the deck and the formulation of the proposed model described in the following section. Note that, throughout this study, it was found that stresses and strains in the concrete deck slab displace laterally nearly as a rigid body, while the flexible steel girders twist and deform laterally as necessary spanning between the slab and the supports. Closer examination of the steel beam reveals that they are most severely distorted near the supports; indeed, in each girder, the bearing supports are the only points that can counteract the lateral pull of the web to bring the lower flange under the slab. This visual observation of behavior has led to the formulation of the proposed model described in the following section. Note that, throughout this study, it was found preferable to model the entire bridge (i.e., all girders and full width of deck) to capture the correct seismic behavior. Furthermore, calculated stresses and strains in the concrete deck slab are small and not reported in this paper.

Proposed Model for Calculation of Period, Elastic Response, and PSAs to First Yielding

Clearly, based on the above description of behavior, an appropriate model should consider the displacement of the deck girder web, as described later). These terms vary nonlinearly as a function of span length in a complex manner because they depend on many other parameters that vary in the designed bridges of different lengths.

It is noteworthy that the resulting lateral periods and maximum lateral deflections are very large compared with values typically reported for slab-on-girder bridges in the literature, reflecting the extreme flexibility of the structural system in the absence of diaphragms. A typical deflected shape is presented in Figs. 3(a and b), where side views at support and along span show a big difference in relative displacements. Clearly, the concrete deck slab displaces laterally nearly as a rigid body, while the flexible steel girders twist and deform laterally as necessary spanning between the slab and the supports. Close examination of the steel beams reveals that they are most severely distorted near the supports; indeed, in each girder, the bearing supports are the only points that can counteract the lateral pull of the web to bring the lower flange under the slab. This visual observation of behavior has led to the formulation of the proposed model described in the following section. Note that, throughout this study, it was found preferable to model the entire bridge (i.e., all girders and full width of deck) to capture the correct seismic behavior. Furthermore, calculated stresses and strains in the concrete deck slab are small and not reported in this paper.

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FIG. 2. Comparison of Results for Different Bridges (Laterally Simply Supported) Obtained by SAP90 and Proposed Model: (a) Lateral Period; (b) Lateral Drift; (c) Required PSAs to Bring Bridge to First Yielding

<table>
<thead>
<tr>
<th>Bridge Span (m)</th>
<th>SAP90 Lateral Period</th>
<th>Proposed Lateral Period</th>
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<tr>
<td>20</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>40</td>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>60</td>
<td>0.8</td>
<td>0.9</td>
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<table>
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<tr>
<th>Bridge Span (m)</th>
<th>SAP90 Lateral Drift</th>
<th>Proposed Lateral Drift</th>
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</thead>
<tbody>
<tr>
<td>20</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>40</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>60</td>
<td>0.2</td>
<td>0.3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Bridge Span (m)</th>
<th>SAP90 PSA to First Yield</th>
<th>Proposed PSA to First Yield</th>
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</thead>
<tbody>
<tr>
<td>20</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>40</td>
<td>0.08</td>
<td>0.10</td>
</tr>
<tr>
<td>60</td>
<td>0.08</td>
<td>0.10</td>
</tr>
</tbody>
</table>

FIG. 3. SAP90 Deformed Shapes for Typical Bridges without Diaphragm: (a) Plan View; (b) Side Views and Schematic of Simplified Model without Diaphragm; (c) Plan View; (d) Side Views

Slab relative to the bottom flange of girders, the web of girders providing a stiffness link between these two components. As a result, stresses in the webs vary greatly along the span. In fact, in each girder, web stresses will be largest at both ends where the bottom flange is restrained by the bearings, and will be minimum at midspan. Figs. 3(c and d) show a schematic simplified model of this behavior for slab-on-girder steel bridges without diaphragms.

Analytical expressions to calculate the lateral period and deformations of a bridge can be developed using this model. For that purpose, since the relative displacement between the deck slab and the lower flanges of the girders plays a key role on behavior, it is convenient to define a relative displacement term, \( \Delta_{r}(x) \), as

\[
\Delta_{r}(x) = \Delta_{s} - \Delta_{b}(x) \tag{1}
\]

where \( \Delta_{s} \) and \( \Delta_{b}(x) \) are the displacements of the slab and bottom flanges of the girders, respectively. Since the deck slab nearly behaves as a rigid diaphragm in its plane, \( \Delta_{s} \) is assumed to be constant in this case. In light of the above observations, the behavior of a bridge without diaphragms was found to closely resemble that of a beam on elastic foundation, and that mathematical model was adopted here. In this analogy, the girder bottom flange and web, respectively play the roles of the beam on a flexible surface and the springs of uniform stiffness. Therefore, the differential equation describing the bottom flange relative lateral displacement, \( \Delta_{b}(x) \), is

\[
E I_b \frac{d^4 \Delta_{b}(x)}{dx^4} = -k_s \Delta_{b}(x) \tag{2}
\]

where \( E \) is modulus of elasticity; \( I_b \) is the bottom flange moment of inertia about its strong axis; and \( k_s \) is the web stiffness per unit length. If the bridge deck and bottom flanges can be assumed to remain always horizontal (i.e., without rotation about the bridge's longitudinal axis), the web stiffness can be modeled as

\[
k_s = \frac{1}{L_f^2} \int_{0}^{L_f} k(x) dx \tag{3}
\]

where \( L_f \) is the girder span length and \( k(x) \) is the web stiffness at any point along the girder. If the bridge is considered as a simple supported beam, the deflection of the beam at any point can be expressed as a function of the sum of the deflections at the support and midspan. In this case, the deflection at any point along the girder can be expressed as

\[
\Delta(x) = \Delta_{0} + \Delta_{m} \tag{4}
\]

where \( \Delta_{0} \) and \( \Delta_{m} \) are the deflections at the support and midspan, respectively. In this case, the deflection at any point along the girder can be expressed as

\[
\Delta(x) = \frac{1}{L_f} \int_{0}^{L_f} \Delta_{b}(x) dx \tag{5}
\]

where \( \Delta_{b}(x) \) is the bottom flange displacement along the girder span. If the bridge is considered as a continuous beam, the deflection of the beam at any point can be expressed as a function of the sum of the deflections at the support and midspan. In this case, the deflection at any point along the girder can be expressed as

\[
\Delta(x) = \frac{1}{L_f} \int_{0}^{L_f} \Delta_{b}(x) dx \tag{6}
\]

where \( \Delta_{b}(x) \) is the bottom flange displacement along the girder span. If the bridge is considered as a continuous beam, the deflection of the beam at any point can be expressed as a function of the sum of the deflections at the support and midspan. In this case, the deflection at any point along the girder can be expressed as

\[
\Delta(x) = \frac{1}{L_f} \int_{0}^{L_f} \Delta_{b}(x) dx \tag{7}
\]

where \( \Delta_{b}(x) \) is the bottom flange displacement along the girder span.
where $I_w$ is the web moment of inertia per unit length about the longitudinal axis of the bridge; and $h_w$ is the web height between top and bottom flanges. In reality, SAP90 analyses of the bridges under lateral seismic loading reveal that while the deck remains relatively horizontal, the bottom flanges rotate the above assumption makes the bridge model slightly too stiff. However, this is of little consequence in most cases, as will be demonstrated later. The classical solution of (2) is

$$\Delta(x) = C_1 \sin \beta x \sinh \beta x + C_2 \sin \beta x \cosh \beta x + C_3 \cos \beta x \sinh \beta x + C_4 \cos \beta x \cosh \beta x$$

where $C_1$ to $C_4$ are constant coefficients depending upon the boundary conditions; and $\beta$ is

$$\beta = \sqrt{\frac{k_w}{4EI}}$$

If both ends of each bottom flange are assumed to be simply supported in the transverse direction, then $\Delta(x)$ is

$$\Delta(x) = \frac{2R \beta}{k_w} \frac{\cosh \beta x \cos \beta(L - x) + \cosh \beta(L - x) \cos \beta x}{\sin \beta L + \sinh \beta L}$$

where $R$ is the support reaction resulting from bridge lateral loading tributary to one girder (based on the UDL applied at the deck level); and $L$ is the bridge span. To obtain the fundamental lateral structural period, the generalized mass and stiffness, $m^*$ and $K^*$, can be computed from the following:

$$m^* = \int_0^L \left( m_s + \frac{n_m m_t}{2} \right) \Delta^2 \, dx + \frac{n_m m_t}{2} \Delta^2(0)$$

$$K^* = n_s \left( \int_0^L \frac{12EI}{h_w^2} \Delta^2 \, dx + \int_0^L EI \Delta^2 \, dx \right)$$

where $n_s$ is the number of girders; and $m_s$ and $m_t$ are slab and girder's masses per unit length, respectively. Knowing that $\Delta_c = \Delta_s$ at the supports ($x = 0$), $R_c$ can be obtained as a function of $\Delta_c$ from (6). For simplicity, since the mass of the girders is much smaller than that of the slab, $m_s$ and $m_t$ can be lumped together, and

$$m^* = \int_0^L \frac{m}{L} \Delta^2 \, dx = m \Delta_c^2$$

where $m$ is the entire bridge mass per unit length. In all cases, the lateral period of the bridge is given by

$$T = 2\pi \sqrt{\frac{m^*}{K^*}}$$

The resulting periods and maximum drifts calculated according to this procedure are plotted in Figs. 2(a and b) and compare well with the more accurate SAP90 results. In percentages, the difference between the results obtained using the proposed model and SAP90 increases as span length increases. For example, the lateral period and drift obtained by the proposed model are, respectively 6% and 12% less than those given by SAP90 for the 20 m span bridge, whereas these differences increase to 9% and 17% for the 60 m span bridge. Note that periods are more closely approximated by this procedure than maximum displacements (beware that the vertical axis for the period graph does not start at zero).

Figs. 4 and 5 compare the lateral displacements of the bottom flange obtained using SAP90 and the proposed procedure, for 20 m and 60 m span bridges. In each case, three different web thicknesses were considered to illustrate sensitivity of response to this parameter. Clearly, the model works better for thinner webs and shorter spans. Although not shown on any figure, this is also true for the calculated fundamental period of vibrations, but with a lesser sensitivity. For example, the difference between the periods obtained by SAP90 and the proposed model for bridges having an 8 mm girder web thickness is only 4%, compared with 6% for the case with 11 mm girder web thickness.

The differences observed above are mainly a consequence of neglecting bottom flange torsional rotations in the proposed model. As shown in Fig. 6, where girder bottom flange twists are normalized per 100 mm drift, maximum normalized flange twist increases along with web thickness for a given bottom flange stiffness. Thus, for a thinner web, the normalized bottom flange twist is smaller, and the results of the proposed model and SAP90 [Figs. 4(a) and 5(a)] are in better agreement.

Maximum stresses in the girder webs will occur at the bridge end and can be calculated by assuming full fixity at top and end of the girder webs. This stress is given by

$$\sigma_{\text{max}} = \frac{M_{\text{max}}}{I_w} \frac{1}{2L} = \frac{6EI}{h_w^2} \frac{\Delta_c}{2L} = \frac{3EI}{h_w^2} \frac{\Delta_c}{h_w^2}$$

where $M_{\text{max}}$ is the maximum bending moment in the web, and $\Delta_c$ refers to maximum deck displacement. This equation is used to calculate the PSAs required to bring the bridge to first yielding. Results are presented and compared with SAP90.
results in Fig. 2(c). Neglecting bottom flange rotation is conservative in this case.

Interestingly, if the bottom flange of each girder is assumed laterally fixed at one end (which would be the case if bearings at that end were prevented from moving in the longitudinal direction of the bridge), then (6) describing the relative deflection, $\Delta_x(x)$, would become

$$\Delta_x(x) = \beta \frac{2R_2 \sin \beta L - R_1 \sinh \beta L}{k_w \cosh \beta L} \sin \beta x \sinh \beta x$$

$$+ \frac{R_1 \beta}{k_w} (\sin \beta x \cosh \beta x - \cos \beta x \sinh \beta x)$$

$$+ \beta \frac{2R_1 \cos \beta L + R_2 \cosh \beta L}{k_w \sinh \beta L}$$

where $R_1$ and $R_2$ are reactions at fixed and simple supports. For the new boundary conditions, Fig. 7 shows the lateral deflections of the bottom flange corresponding to the same three different web thicknesses considered previously for the same 20 m span bridge. Compared with the previous case, drifts are smaller and longer PSAs are needed to produce first yield. Hence, (6)–(11) can be used conservatively if expediency is desired.

**Ultimate Nonlinear Behavior**

The program ADINA (ADINA R&D 1995) was used to investigate the nonlinear behavior of these steel bridges and the impact of $P-\Delta$ effects (second-order analysis) on this ultimate behavior. Results from push-over analyses shown in Fig. 8 indicate that, since lateral displacements are large in bridges without any diaphragms, $P-\Delta$ effects arising from the displaced weight of the deck are significant, leading to inelastic overturning and structural instability.

Inelastic analyses also revealed that, in the absence of end diaphragms, the presence of intermediate diaphragms does not greatly improve the seismic behavior of slab-on-girder bridges, since the largest girder web distortions occur near the girder.

**FIG. 5.** Comparison of Girder Bottom Flange Displacements Obtained by SAP90 and Proposed Model for 60 m Span Bridge (Simply Supported Laterally), Respectively, for Girder Web Thickness of (a) 8 mm; (b) 11 mm Corresponding to WWF800 x 184 In Original Design; (c) 20 mm

**FIG. 6.** Rotation of Girder Bottom Flange for 20 m Span Bridge (Simply Supported Laterally) for Normalized 100 mm of Maximum Transverse Displacement

**FIG. 7.** Comparison of Girder Bottom Flange Displacements Obtained by SAP90 and Proposed Model for 20 m Span Bridge (Laterally Fixed at One End), Respectively, for Girder Web Thickness of (a) 8 mm; (b) 11 mm Corresponding to WWF800 x 184 In Original Design; (c) 20 mm
senters equally spaced along the length of girders and added to enhance the shear resistance of these girders. (The trend in new designs, however, is to avoid transverse stiffeners as much as possible.) Bridges also have numerous intermediate transverse web stiffeners at the supports of steel bridge girders to prevent web crippling.

As bearing stiffeners can effectively be treated as end diaphragms in resisting the lateral loads, it is worthwhile to extend the above analytical study to investigate the impact of this small effective end-diaphragm action. Table 2 presents the resulting maximum displacements and lateral period of vibration in steel bridges of different spans subjected to a lateral uniformly distributed load equivalent to a pseudo acceleration of 1g at the deck level. As indicated in this table, even though relatively small bearing web stiffeners were considered, their presence has a large impact on the lateral period and displacement of bridges.

Second, when the bearing stiffeners at the supports are larger than the distributed intermediate web stiffeners (as they frequently are), they effectively contribute an additional stiffness at each support, essentially acting as end diaphragms, albeit weak ones. Finite element analyses were conducted to investigate the impact of this small effective end-diaphragm action. Table 2 presents the resulting maximum displacements and lateral period of vibration in steel bridges of different spans subjected to a lateral uniformly distributed load equivalent to a pseudo acceleration of 1g at the deck level. As indicated in this table, even though relatively small bearing web stiffeners were considered, their presence has a large impact on the lateral period and displacement of bridges.

As bearing stiffeners can effectively be treated as end diaphragms, it is worthwhile to extend the above analytical study to parametrically include the stiffness contribution of end diaphragms. This is done in the following.

**SLAB-ON-GIRDER STEEL BRIDGES WITH EFFECTIVE DIAPHRAGMS**

For bridges with diaphragms, preliminary analyses revealed that the concrete deck does not respond in a rigid-body motion during seismic excitations, as is the case for bridges without diaphragms. Rather, treating the structure as a generalized single degree of freedom (SDOF), it was found appropriate to express the transverse response displacement mode shape of the bridge, \( u(x) \), by

\[
   u(x) = \sin \frac{\pi x}{L} \tag{13}
\]

where \( L \) is the span length. The corresponding effective force, \( P_{\text{eff}} \), acting on this system is calculated as (Clough and Penzien 1993)

\[
   P_{\text{eff}} = \frac{L^2}{\pi^2} \text{PSa} = \frac{8m}{\pi^2} \text{PSa} \tag{14}
\]

where \( m \) is the mass of the bridge; \( \text{PSa} \) is the pseudo acceleration; and \( \zeta \), the earthquake excitation factor, is

**TABLE 2. Comparison of Elastic Lateral Deck Drift and Period of Steel Bridges, with and without Web Bearing Stiffeners, Subjected to 1g Pseudoacceleration**

<table>
<thead>
<tr>
<th>Bridge span (m)</th>
<th>Web stiffeners size</th>
<th>Without stiffeners</th>
<th>With stiffeners</th>
<th>Lateral Drift (mm)</th>
<th>Lateral Period (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2P1. 100 x 10</td>
<td>5.3</td>
<td>228</td>
<td>0.11</td>
<td>0.94</td>
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<td>30</td>
<td>2P1. 100 x 10</td>
<td>16</td>
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</tr>
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<td>40</td>
<td>2P1. 100 x 10</td>
<td>30</td>
<td>478</td>
<td>0.26</td>
<td>1.27</td>
</tr>
<tr>
<td>50</td>
<td>2P1. 120 x 12</td>
<td>37</td>
<td>785</td>
<td>0.26</td>
<td>1.66</td>
</tr>
<tr>
<td>60</td>
<td>2P1. 120 x 12</td>
<td>56</td>
<td>1,195</td>
<td>0.30</td>
<td>2.03</td>
</tr>
</tbody>
</table>

*Assuming simply supported laterally.

---

**FIG. 6. Load-Displacement Curve for 40 m Span Bridge with and without Consideration of P-\( \Delta \) Effect**

**FIG. 7. Load-Displacement Curve for 40 m Span Bridge with and without End Diaphragms**

**FIG. 8. Load-Displacement Curve for 40 m Span Bridge with and without Consideration of P-\( \Delta \) Effect**

**FIG. 9. Impact of Intermediate Diaphragms on 40 m Span Bridge without End Diaphragm: (a) Lateral Load versus Midspan Drift; (b) Horizontal Shear Force in Braces of First and Second Intermediate Diaphragms from Bridge End**

**TABLE 2. Comparison of Elastic Lateral Deck Drift and Period of Steel Bridges, with and without Web Bearing Stiffeners, Subjected to 1g Pseudoacceleration**

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It is noteworthy that, for the simplified equivalent system considered above, the effective force must act on the system, instead of \( m^2 u_p \). For instance, in a 60 m span bridge with a total mass of 465,000 kg, a \( P_{\text{ext}} \) of 3,700 kN, distributed along the bridge as a sine curve, is obtained from a pseudo acceleration of 1g, while in the case without diaphragms a lateral loading of 4,560 kN uniformly distributed along the bridge was previously considered for the same pseudo acceleration.

In the following, this effective force corresponding to a generic pseudo acceleration of 1g is considered, and as before, results from elastic analyses can be linearly scaled as necessary.

**Proposed Model for Calculation of Period and Elastic Response**

In the presence of end diaphragms, the model previously derived must be modified to properly capture the lateral response of this type of steel bridge. A new model that may account for the stiffness of end diaphragms, in addition to all the behavior features described earlier, is schematically illustrated in Fig. 10.

The following mathematical approach was followed to develop an analytical expression capturing the lateral behavior of these steel bridges under transverse seismic excitation.

The differential equation of motion for the bridge deck with continuous mass, neglecting the effects of shear strain and rotary inertia, can be written as (Gorman 1975)

\[
\ddot{u}(x, t) + \frac{1}{\rho A} \frac{\rho A}{\sqrt{L}} \dot{u}(x, t) + \frac{E I}{L^4} \ddot{u}(x, t) = 0 \tag{16}
\]

where \( u(x, t) \) is a displacement function of the bridge deck in terms of longitudinal distance from the support, \( x \), and time, \( t \); \( I_d \) is the superstructure moment of inertia (slab and girders acting as a unit) about a vertical axis perpendicular to the deck; \( \rho \) is mass density; and \( A \) is the cross-sectional area of the entire superstructure. Hence, \( \rho A \) corresponds to the superstructure mass per unit length. Using a displacement function of \( u(x, t) = Z(t) Y(x) \) for bridge deck, the differential equation of motion becomes

\[
\ddot{Y}(x) - \frac{E I}{L^4} \ddot{Y}(x) = 0 \tag{17}
\]

This equation may be rearranged so that the left-hand side of the equality is a function of time only, while the other side is a function of distance, \( x \):

\[
\frac{1}{Z(t)} \frac{d^2 Z(t)}{d t^2} = \frac{E I}{\rho A \sqrt{L}} \frac{d^2 Y(x)}{d x^2} \tag{18}
\]

By making both sides equal to a constant, say \( \omega^2 \), and then introducing a dimensionless parameter, \( \alpha \), defined as

\[
\alpha^2 = \frac{\rho A g^2 L^4}{E I} \tag{19}
\]

the following expressions for the general solution can be developed:

\[
Y(x) = C_1 \sin \alpha x + C_2 \cos \alpha x + C_3 \sinh \alpha x + C_4 \cosh \alpha x \tag{20}
\]

\[
Z(t) = \cos(\omega t - \phi) \tag{21}
\]

where \( C_1 \) to \( C_4 \) are the coefficients depending on boundary conditions; and \( \phi \) is a phase angle of the periodic motion. For a steel bridge having end diaphragms and assumed simply supported at both ends in the transverse direction, \( \alpha \) can be obtained through trial and error by solving the following equality:

\[
\left( K_s^* \right)^2 - \frac{\alpha^2}{\omega^2} \left( 1 + C_4 \right) \left( \sin \alpha - \alpha \sinh \alpha \right) + \frac{\alpha^2}{2C_4} \left( 1 - \alpha \cos \alpha \right) = 0 \tag{22}
\]

where \( K_s \) is the ratio of right to left end-diaphragm stiffnesses \( K_{e1}/K_{e2} \), usually 1.0; and \( K_s^* \), the dimensionless stiffness, is

\[
K_s^* = \frac{K_s L^2}{E I} \tag{23}
\]

in which \( K_s \), the stiffness of lateral bracing systems at one end, depends on the geometry of the bridge and properties of the bearing stiffeners and diaphragm braces:

\[
K_s = \sum_{i=1}^n \frac{12EI}{h^4_i} + \sum_{i=1}^m 2EA_i \cos^2 \theta \tag{24}
\]

where \( I \) is the moment of inertia of the bearing web stiffener about the longitudinal axis of the bridge; \( h \) is the number of girders; and \( A_i, h_i, \theta \) are the cross-sectional area, length, and slope angle of braces. It is noteworthy that, in the absence of diaphragm braces, \( K_s \) would simply be the lateral stiffness of transverse bearing web stiffeners, i.e., \( \Sigma 12EI/h_i^4 \). For example, with two-sided P1. 100 \( \times \) 10 only, in a 20 m span bridge of four girders, \( I \) and \( K_s \) would be 7.8 \times 10^{-4} m\(^4\) and 170 kN/mm, respectively. With X-shape braces of 2L\(100 \times 100 \times 10 \) between every two girders at bridge ends, \( K_s \) would be 1,930 kN/mm (neglecting the contribution of the bearing stiffeners), and \( K_s^* \) would be 55.33, giving \( \alpha \) of 2.71 [see (22)].

Note that if rocking of the girders on the end bearing occurs, the term \( \Sigma 12EI/h_i^4 \) should be replaced with \( \Sigma 3EI/h_i^4 \); rocking at the top of a noncomposite girder would further reduce the contribution of transverse bearing web stiffeners to lateral load resistance.

On the other hand, if the girders’ bottom flange lateral support conditions at one end of the simply supported bridge are considered fixed, then assuming \( C_4(K_{e1}/K_{e2}) \) to be 1.0, (22) becomes

\[
\left( K_s^* \right)^2 - \frac{\alpha^2}{\omega^2} \left( \sin \alpha \cos \alpha \sin \alpha - \alpha \sin \alpha \cosh \alpha \right) - \frac{K_{e1}}{K_{e2}} (1 + 3 \cos \alpha) \cdot \cosh \alpha + \sin \alpha \cosh \alpha + \sin \alpha \cos \alpha = 0 \tag{25}
\]

For the same 20 m span bridge, \( \alpha \) would be 2.96, giving a lateral period of 0.044 s.

In all cases, the displacement at the bridge ends is given by

\[
\Delta_{\text{ext}} = \frac{m (P S a)}{K_{e1} + K_{e2}} \tag{26}
\]
Rayleigh's method can be used alternatively to the proposed approach to calculate the period of these bridges, considering the bridge as a beam with continuous mass and elastic springs at the supports. This approach is a specific case of the method of generalized coordinates in which the shape function is determined on the basis of the static displacement of structure. The maximum potential and kinetic energies, \( V_{\text{max}} \) and \( T_{\text{max}} \), can generally be integrated from the following equations:

\[
V_{\text{max}} = \frac{1}{2} \int_0^L m(x) g \ddot{u}(x) \, dx \tag{27a}
\]

\[
T_{\text{max}} = \frac{1}{2} \omega^2 \int_0^L m(x)(u(x))^2 \, dx \tag{27b}
\]

where \( u(x) \) is the lateral displacement function assumed for the bridge; \( L \) and \( m(x) \) are span length and distributed mass of the bridge; \( g \) is gravitational acceleration; and \( \omega \) is the transverse frequency in rad/s. By equating the two energies,

\[
\int_0^L m(x)\ddot{u}(x) \, dx = \frac{\omega^2}{g} \int_0^L m(x)(u(x))^2 \, dx
\]

For a 20 m span simply supported bridge having only two web stiffeners of 100 \( \times \) 10 size at each support, \( \omega \) is computed as 57 rad/s from the above formula, corresponding to a lateral period of 0.11 s. By adding 2L100 \( \times \) 100 \( \times \) 10 braces to the ends, the lateral period decreases to 0.052 s if simply supported laterally and to 0.043 s in the presence of a fixed support. These results compare well with those obtained by SAP90.

Numerical Examples

Using SAP90, the lateral period was computed for bridges with different spans, considering various X braces as diaphragms. Note that many of the bridges' lower modes of vibration correspond to displacement response in the vertical direction. The discussion here addresses the lateral period corresponding to lateral response.

Fig. 11 shows the resulting lateral period versus brace cross-sectional area for 20, 40, and 60 m span bridges. As a zero cross-sectional area is assumed for the braces, periods of 0.82 and 1.77 s are obtained, respectively for 20 and 60 m long bridges, i.e., the same values obtained before. By using 2L45 \( \times \) 45 \( \times \) 5 and 2L100 \( \times \) 100 \( \times \) 10 X braces, lateral periods of 0.089 and 0.052 s are obtained for the 20 m span bridge, and 0.24 and 0.22 s, respectively, for the 60 m span bridge. It is observed that a very small end diaphragm is sufficient to produce a large "shift" in the period of these bridges, but that further increases in diaphragm stiffness have only a marginal impact on lateral structural period.

Table 3 presents the lateral periods for all braced bridges considered in this study and compares them with the periods obtained by SAP90. The difference between the results is smaller for larger span bridges, because shear deformations of the concrete deck are ignored in the proposed analytical model. For the 60 m span bridge, where the shear to flexural deformation ratio is the lowest, the proposed hand-calculation analytical model offers the results closest to those obtained by SAP90.

Ultimate Nonlinear Behavior

Frequently, end diaphragms have sufficient strength to remain elastic during earthquakes, and brittle damage instead occurs in the diaphragm connections or elsewhere in the structure. However, to investigate whether intermediate diaphragms can effectively contribute to lateral load resistance when end diaphragms undergo inelastic response, inelastic push-over analyses were carried out using ADINA. Results are shown in Fig. 12 for 40 and 60 m span bridges having X-brace diaphragms between the girders, namely 2L65 \( \times \) 65 \( \times \) 6 for end diaphragms and 2L45 \( \times \) 45 \( \times \) 5 intermediate diaphragms. Three pairs of braces are present in each diaphragm. The contribution resisted by flexure of the stiffened girders [per (24)] is not shown. As shown in Fig. 12, intermediate diaphragms take only a small percentage of the total applied load, even after buckling and yielding of the end-diaphragm braces, and would remain elastic until very large ductilities developed in the end diaphragms and at least until deck drifts in excess of 5%. The contribution of intermediate diaphragms is even less significant for the shorter bridges. This is because intermediate diaphragms are located 8 m from each other, the first one being at a greater percentage of the total span from the end in shorter bridges, and thus less likely to contribute, being more remote from the zone of greater girder transverse deformation.

Note that results previously obtained for bridges without end diaphragms and presented earlier should be used if end diaphragms are deemed incapable of developing the ductile behavior assumed here.

Further Observations on Seismic Behavior

Deformed shapes for a 40 m span bridge at one end and at a distance of 8 m from the end are shown in Fig. 13, and it is noteworthy that even though the lateral inertia force is applied at the deck level and that supports are provided only at the level of the lower bottom flanges at the bridge end, the bridge deck rotates conversely to the direction of the resulting couple, or counterclockwise at the middle. This is due to the axisymmetric cross section of the bridge, which has a shear center located above the concrete slab.

<table>
<thead>
<tr>
<th>Bridge span (m)</th>
<th>( pA ) (kpg/m)</th>
<th>( I_0 ) (m(^4))</th>
<th>X-brace size</th>
<th>( T_{\text{SAP90}} ) (s)</th>
<th>( T_{\text{Proposed}} ) (s)</th>
<th>Difference (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>6,300</td>
<td>1.322</td>
<td>2L100 ( \times ) 100 ( \times ) 10</td>
<td>0.0515</td>
<td>0.0477</td>
<td>8</td>
</tr>
<tr>
<td>30</td>
<td>6,733</td>
<td>1.617</td>
<td>2L100 ( \times ) 100 ( \times ) 10</td>
<td>0.0766</td>
<td>0.0711</td>
<td>7</td>
</tr>
<tr>
<td>40</td>
<td>7,150</td>
<td>1.797</td>
<td>2L100 ( \times ) 100 ( \times ) 10</td>
<td>0.12</td>
<td>0.112</td>
<td>6.6</td>
</tr>
<tr>
<td>50</td>
<td>7,340</td>
<td>1.983</td>
<td>2L100 ( \times ) 100 ( \times ) 10</td>
<td>0.17</td>
<td>0.16</td>
<td>6</td>
</tr>
<tr>
<td>60</td>
<td>7,750</td>
<td>2.212</td>
<td>2L100 ( \times ) 100 ( \times ) 10</td>
<td>0.223</td>
<td>0.213</td>
<td>4</td>
</tr>
</tbody>
</table>

*It is assumed there is one laterally fixed end; \( pA \) is the superstructure mass per unit length.*
Linear elastic and nonlinear inelastic analyses were conducted to investigate the impact of diaphragms on the seismic behavior of straight slab-on-girder steel bridges. The results of this limited analytical study demonstrate that a small end-diaphragm stiffness is sufficient to make the entire superstructure behave as a unit in the elastic range. However, the above results also illustrate that a dramatic shift in seismic behavior could occur once rupture of the end diaphragms occurred, with a sizable period elongation, considerably larger lateral displacements, and a higher propensity to damage in response to instability and $P-\Delta$ effects. It is also found that the presence of intermediate diaphragms does not significantly influence the seismic performance of these bridges, both in the elastic and inelastic range, regardless of whether end diaphragms are present or not.

Moreover, these analyses confirmed that effective end diaphragms constitute critical structural elements along the main seismic load path, and that they should be designed accordingly. Therefore, in new bridges, they should be designed to resist in an elastic manner the forces induced by the maximum credible earthquake. Alternatively, they could be designed and detailed as ductile members to preclude brittle member or connections failure. This is not warranted for intermediate diaphragms. Nonductile end-diaphragm members and connection details in existing steel bridges should be retrofitted, because of their impact on seismic response.

FIG. 12. (a) and (d) Lateral Loads Imposed to Braced 40 and 60 m Span Bridges, Respectively, versus End and Midspan Drifts; (b) and (c) Tension and Compression Axial Forces in One Pair of Diaphragm Braces for 40 m Span Bridge; (e) and (f) Same for 60 m Span Bridge

FIG. 13. End Views of Deformed Shapes for 40 m Span Bridge at (a) One End; (b) Distance of 8 m from Bridge End

CONCLUSIONS

Seismic and Nonlinear inelastic analyses were conducted to investigate the impact of diaphragms on the seismic behavior of straight slab-on-girder steel bridges. The results of this limited analytical study demonstrate that a small end-diaphragm stiffness is sufficient to make the entire superstructure behave as a unit in the elastic range. However, the above results also illustrate that a dramatic shift in seismic behavior could occur once rupture of the end diaphragms occurred, with a sizable period elongation, considerably larger lateral displacements, and a higher propensity to damage in response to instability and $P-\Delta$ effects. It is also found that the presence of intermediate diaphragms does not significantly influence the seismic performance of these bridges, both in the elastic and inelastic range, regardless of whether end diaphragms are present or not.

Moreover, these analyses confirmed that effective end diaphragms constitute critical structural elements along the main seismic load path, and that they should be designed accordingly. Therefore, in new bridges, they should be designed to resist in an elastic manner the forces induced by the maximum credible earthquake. Alternatively, they could be designed and detailed as ductile members to preclude brittle member or connections failure. This is not warranted for intermediate diaphragms. Nonductile end-diaphragm members and connection details in existing steel bridges should be retrofitted, because of their impact on seismic response.

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APPENDIX. REFERENCES


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