TIME-BASED NOISE REMOVAL FROM MAGNETIC RESONANCE SOUNDED SIGNALS

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Received July 2010; revised January 2011

ABSTRACT. The Magnetic Resonance Sounding (MRS) is a new method for the ground-water exploration. In this method, an electromagnetic pulse with specific frequency equal to the Larmor frequency (the resonance frequency of the water molecules in the geomagnetic field) is sent to the underground from the surface. When the pulse is disconnected, after a few milliseconds, an electromagnetic field is returned from the water molecules and induces a voltage into the receiving antenna on the surface. As the inductive voltage generated by the protons is very small, the MRS method is very sensitive to the electromagnetic interferences and noise, which is the most important limitation for the practical application. Both the depth of investigation and precision of the MRS method depend on signal to noise ratio (SNR). In this paper, in order to improve the performance in the noisy environments, a time-based method is hereby proposed, so that the characteristics of the merely taken noise used to estimate the features of the interfered noise with the signal. By applying an optimization process, we can remove the noise from the signal to a high extent. Also, a frequency method is investigated for comparison. It will be observed that the proposed method is actually advantageous to the mentioned frequency method and the performance indexes, especially SNR, will increase significantly.

Keywords: Magnetic resonance sounding, Noise, Standard deviation, Larmor frequency, Parameter estimation

1. Introduction. A non-invasive method to explore the underground water resources, which has recently been provided, is the Proton Magnetic Resonance (PMR). This method is recently known as the Magnetic Resonance Sounding (MRS). A major limitation of MRS is the sensitivity to the natural and man-made electromagnetic (EM) noise. Magnetic storms, telluric currents, thunderstorms and so on can create natural EM noise. It always occurs in the MRS measurements and usually has the random Gaussian distribution. Electrical generators, radio transmitters and the like generate man-made EM noise. An alternative magnetic field which is produced by the precession of the proton magnetic moments in groundwater always varies between $10^{-12}$ and $4 \times 10^{-9}$ T. Therefore, the voltage created by the MRS signal varies between 10 nV and 4000 nV when using a wire loop of 100m diameter as a receiving antenna. In this method, contrary to many geophysical techniques, the signal cannot be amplified by increasing the transmitter power. Thus,
noting the same as well as the MRS signals are originally at nano-volt range, the noise will be very critical for them. In other words, even very low noises can intensely affect the signal. So, it is extremely important to carry out the de-noising on the MRS signals [4,5,12]. A low-pass filter with the Hanning edge smoothing and another filter proposed by Legchenko and Valla (2002) applied to suppress the random Gaussian distributed noise. Also, a high stacking rate of the single records leads to a reduction of the random and harmonic noise, but this process is bound to spend time (Plata and Rubio, 2007). The Subtraction technique to eliminate the harmonic noise has been developed by Butler and Russell (1993). Legchenko and Valla (2003) discussed the process of eliminating the power-line harmonics by applying the notch filters. Also, the useful methods to reduce the noise have been proposed in [13-16].

In this paper, a time-based method to remove the noise from the MRS signals is proposed. As the MRS method is relatively new, we believe the same has not been widely researched on. The works carried out by the team of the authors of this paper are actually a new method to have been proposed in the time domain. NUMIS equipment developed by the IRIS Instruments is applied to collect the concerning data. The parameters of the environment noise can be measured because this system records the noise automatically prior to the MRS measurement. The sampling frequency and the time length of the recorded noise time series and the recorded signals are the same [2], so elimination of the noise from the signal is possible due to the short interval between the noise and signal record as well as similarity between the recorded noise and the associated noise with the signal. The proposed method to remove the noise from the simulated signals as well as the actual MRS signals will be implemented and then the results and its performance will be evaluated and expressed accordingly. Finally, a comparison between the proposed method and the de-noising with the Hanning filter will be shown.


2.1. Recording the noise prior to the data collection and the mean and variance. The standard NUMIS MRS equipment acquires data in the form of time series recorded before and after the pulse transmission (Figure 1) [18]. Each record prior to the pulse to be considered as the mere noise as the water molecules have not been stimulated, whereas the records after the pulse contain both the signal and noise [4]. This noise has actually different sources, a percentage is produced by the geophysical and environmental conditions of the area and another percentage is produced by the instrument. By investigating the noise it is concluded the recorded noise is statistically independent with the Gaussian distribution whose mean is approximately zero [8]. Therefore, noting the above mentioned features for the noise as well as the distinctive parameters (mean and variance) the same will hereby be proceeded to be removed. Initially we have to mention that the equations of the mean and variance are as follows.

\[
\bar{X} = \frac{1}{N} \sum_{i=1}^{N} x_i, \quad (1)
\]

where \(x_i\) is the amplitude of the \(i\)th sample, \(N\) is the number of the samples in the noise record and \(\bar{X}\) is the mean. The variance equation is as follows.

\[
\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2, \quad (2)
\]

where \(\sigma^2\) is the variance of the \(N\) noise samples, it shows the dispersion of the samples around the mean [9].
Noise removal from the MRS signal using the mean criterion. The returned MRS signal from the underground water molecules is an exponentially decaying sinusoidal signal. The frequency of this signal is 800 to 2800 Hz around the globe which alters according to the geomagnetic field [3]. This signal is as follows.

\[ e(t, q) = E_o(q) \exp(-t/T_2'(q)) \cos(\omega_t + \varphi_o(q)), \]  

(3)

where \( \omega_t = 2\pi f_l \) is the Larmor frequency (the resonance frequency of the water molecules in the geomagnetic field), \( \varphi_o \) is the phase shift of the MRS signal with respect to the energizing pulse, \( T_2'(q) \) is the spin-spin relaxation time (decay time), correlates with the mean size of pores of the water-saturated rocks and \( E_o(q) \) in fact the initial amplitude of the MRS signal which the same is related the water content, location and thickness of aquifers [1,2]. After the synchronous detection, the reference signal whose frequency is as high as the Larmor frequency is multiplied by the signal of Equation (3), consequently an addition and subtraction signal are created. The subtracted signal is however desired in the synchronous detection technique because its frequency being \( f_o = f_R - f_L \) is approached to zero as much as possible [1].

\[ e_{\text{detection}}(t) = E_o \exp(-t/T'_2) \exp(j(\omega_o t - \varphi_o)), \]  

(4)

where \( \omega_o = 2\pi f_o \). The equation shows the ideal form of the MRS signal. But generally this is not true and other factors are added to the signal as the noise. Thus, a description of the noisy MRS signal is as follows.

\[ e_{\text{noisy detection}}(t) = E_o \exp(-t/T'_2) \exp(j(\omega_o t - \varphi_o)) + \varepsilon_{\text{complex}}(t), \]  

(5)

where here \( \varepsilon(\varepsilon_{\text{complex}}(t)) \) is related to the statistical noise [1,4].

The main objective is to remove the statistical noise \( \varepsilon \) from Equation (5) and access to Equation (4). So, the noise parameters and the ideal signal features must be known correctly to remove the noise from the MRS signal [20,21]. The mean or the area under the curve is an exclusive feature of these two components. The mean of two signals or time series adding together, is equal to the summation of the mean of each one. This can actually be a major feature to be used for the noise and ideal signal identification. Generally, the following equation is used for the mean of the real signal of Equation (5) which includes the ideal signal and noise respectively:

\[ A = A_s + A_n, \]  

(6)

where \( A \) is the area under the curve of the real signal, \( A_s \) is the area of the ideal signal of Equation (5) and \( A_n \) is the noise area. The area of the random noise (or its mean) is assumed zero [9]. Therefore, the signal mean will not be changed after being noisy, then
by removing the noise, the mean must remain unchanged. Generally, Equation (5) can be expressed with the time series of each component:

\[ S(i) = S_s(i) + S_n(i), \]  

where \( S(i) \) is the \( i \)th sample of the real signal, \( S_s(i) \) is the \( i \)th sample of the ideal signal and \( S_n(i) \) is the \( i \)th sample of the noise time series. Thus,

\[ S_n(i) = S(i) - S_s(i). \]  

Then the following equation is used for calculation of the area under the noise curve:

\[
\sum_{i=1}^{N} S_n(i) = \sum_{i=1}^{N} (S(i) - S_s(i))
\]

where \( S(i) \) is a specified signal but \( S_s(i) \) and \( S_n(i) \) are actually random and indefinite signals, therefore, they will be estimated based on their distinctive features. If these two parts are correctly estimated, then the area of \( S_n \) is equal to zero. Equation (6) or Equation (9) demonstrates that if the mean of the added noise to the signal is zero then the integral or the area under the curve remains unchanged. Also \( S_s \) is an ideal exponentially decaying signal in the form of time series. The \( S_s \) signal will have to be estimated in a way that the mean of \( S_n \) be zero. In other words, an optimizing process is defined based on nullifying the mean of \( S_n \). The \( S_s \) signal will then be implemented according to the stimulated MRS signal and according to Equation (9) with initial amplitude \( E_o \) and decay time \( T_2^* \). Presently to simplify the mentioned process \( E_o \) is a constant and equal to \( S(1) \), the first recorded value of the real signal. The value may not be true and it may be destroyed by the noise, but it may be modified to the real value in the next stage. \( T_2^* \) is another value to be used for the primary signal, as for primary signal \( T_2^* \) is considered as equal to the signal recording time. Since an exponential signal is almost approached to the final value after about five times constant, this time is usually five times the true \( T_2^* \) value. According to the hypotheses, a primary signal equal to \( S_s \) is implemented, definitely its decay time is more than that of the real signal. The initial amplitude is constant too and it is equal to \( E_o \). The estimation and the optimization processes are such defined: \( T_2^* \) is estimated such that the mean of \( S_n \) is equal to zero, where the initial \( T_2^* \) value is more than the real one, the process may be stated more simply: The value of \( T_2^* \) is reduced until the mean of the noise will approach to zero. With the correct estimation of \( T_2^* \), \( S_s(i) \) is estimated so that it will approach to the ideal signal. We will therefore notice that the mean value of \( S_s \) be equal to the mean value of the real signal, namely \( S \). Since the mean of the noise is equal to zero, though the added value to the ideal signal will change many of its features but it dose not affect the mean value, consequently the area under the curve will almost be constant.

2.3. True initial amplitude estimation using the variance criterion. As mentioned in the previous section, an initial amplitude equal to \( S(1) \) is applied to estimate the \( S_s \) signal. Because the initial amplitude may have been changed due to the noise, that may result in error thus the initial amplitude must be improved based on a specified way. At the initial stage \( S_s \) was estimated so that its area be approached to the true one namely the area of the noiseless signal. According to the mentioned subjects, the area of the ideal signal after being noisy remains constant, thus it is a precondition for the estimation of the \( S_s \) signal correctly. According to Equation (10), much of \( T_2^* \) and \( E_o \) can satisfy the constant area for the exponential signal. Therefore, if the area under the curve is estimated correctly, in case there are wrong \( E_o \) or \( T_2^* \), the other component will be wrong too. Nevertheless, when the initial amplitude \( E_o \) is wrong, therefore, the estimated
value for \( T_2^* \) will be wrong too, though the area under the curve is correct. There is a
defined problem now while \( E_0 \) or \( T_2^* \) parameters are wrongly estimated then what will be
affected or what would the result be.

According to the investigation, it was realized that if \( E_0 \) or \( T_2^* \) be wrong, the mean of
the estimated noise will be zero despite the fact that its variance is incorrect. Since the
variance of the recorded noise is fully identified before the signal record, compared the
variance of the estimated and recorded noises enable us to realize the accuracy of \( E_0 \) as
well as \( T_2^* \). In case of any difference, the value of \( E_0 \) must be modified. So, an optimization
process is defined as: \( E_0 \) is estimated with two restriction which are the variance of the
noise and the area under the curve of the ideal signal. As already noted the area under
the curve of the ideal signal is estimated correctly. If \( E_0 \) is changed then \( T_2^* \) must be
changed too, but the area under the curve must remain constant (Equation (10)). Then
the change relation of the \( T_2^* \) and \( E_0 \) parameters are written based on Equation (4) [20]:

\[
E_0 = A/ (T_2^* (1 - \exp(-t_1/T_2^*))) ,
\]

(10)

where \( t_1 \) is at the time domain of the recording of the signal. \( E_0 \) is estimated using
Equation (10), then the removed noise parameters will be calculated according to the
estimation. Since the area under the \( S \) curve has not changed, then the mean of the
estimated noise will not change either but its variance will change. The calculated variance
is compared with the variance of the primary noise in order to understand the \( E_0 \) changes,
then the estimation and alteration of \( E_0 \) are continued until the variance of the estimated
noise will be approached to the variance of the recorded noise as much as possible. Then
\( E_0 \) is correctly estimated and when \( E_0 \) is correct we will be ensured that \( T_2^* \) is accurate
as well as the estimated ideal signal is correct and noiseless. Accordingly the removed
noise from the signal has a variance and mean approach to its corresponding recorded
noise values as much as possible, consequently the estimated signal will be about the
ideal signal with a high percentage of confidence.

3. The Comparison Criteria for the Efficiency Evaluation of the Proposed
Method. When the noise and the signal were simulated, a criterion must be defined for
the efficiency evaluation of the method to investigate the noise removal stages. Since the
noise parameters are statistical, then in order to evaluate the efficiency of the noise removal
we are not enabling to logically conclude on one single signal. The best method for the
same would then be to evaluate several signals simultaneously and the total efficiency of
the same method to be considered as the mean of the efficiency of each signal. A process
is developed to define a criterion such that: 10 different noises are implemented the ideal
signal, consequently 10 different noisy signals will be produced.

\[
X_n(i) = S(i) + N_n(i),
\]

(11)

where \( i \) is the index of the samples, \( n \) is the signal number. For instance, \( i = 1, \ldots, 100,
\( n = 1, \ldots, 10 \), \( N_n(i) \) is the \( i \)th sample of the \( n \)th noise component, \( S(i) \) identifies the
ideal signal and \( X_n(i) \) is the \( i \)th sample of the \( n \)th simulated noisy signal. 10 de-noised
signals will then be obtained when the mentioned algorithm is implemented:

\[
S'_n(i) = X_n(i) - N_n(i),
\]

(12)

where \( S'_n(i) \) is the \( i \)th sample of the de-noised \( n \)th signal. Then the standard deviation of
every signal point is calculated as follows:

\[
\sigma(i) = \sqrt{\frac{1}{10} \sum_{n=1}^{10} (S'_n(i) - S(i))^2}.
\]

(13)
in fact, $\sigma(i)$ is an array and every value of the array is correspondent with the dispersion of each point of the de-noised signal to its similar point of the ideal signal. To simplify and define a general error the dispersed mean is used along the signal. Then the estimated signal error namely the efficiency criterion of the method is used as follows:

$$\sigma_s = \frac{1}{N} \sum_{i=1}^{N} \sigma(i),$$

where $N$ is the whole of the signal samples which here is equal to 100. Also $T_2^*$ and $E_o$ are very important because their values are applied to specify some features of the aquifer [6,11]. Since the concerning features are directly related to $E_o$ and $T_2^*$ then their wrong estimation will result in wrong feature interpretation of the concerning aquifer. Consequently, these parameters must necessarily be correct and precise, so the mean of the estimation of both parameters is investigated for correct and precise estimation. Since the estimated signal parameters such as $T_2^*$ and $E_o$ are naturally statistical, it will not be concluded merely on one single de-noised signal. In order to achieve a better measure of the estimation efficiency of $T_2^*$ and $E_o$, the noise removal process is performed on 10 noisy signals, consequently a total mean is mentioned for the estimation of $T_2^*$ and $E_o$.

$$\mu_E = \frac{1}{M} \sum_{i=1}^{M} E_o(i),$$

where $M$ is the repetition number which here is equal to 10, $\mu_E$ is the mean of the initial amplitudes of all estimations and $E_o(i)$ is the initial amplitude of each signal. The standard deviation is another criterion which applied to measure the estimation of the initial amplitude such as follows:

$$\sigma_E = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (E_o(i) - \mu_E)^2},$$

where $\sigma_E$ is the standard deviation of the initial amplitudes, $M$ is the number of the initial amplitudes which here is equal to 10 and $\mu_E$ is the calculated mean by Equation (15). According to Equations (15) and (16), the mean and standard deviation for the decay times are defined as follows:

$$\mu_T = \frac{1}{M} \sum_{i=1}^{M} T_2^*(i),$$

and

$$\sigma_T = \sqrt{\frac{1}{M} \sum_{i=1}^{M} (T_2^*(i) - \mu_T)^2},$$

where $\mu_T$ is the mean of the decay times per $M$ repetition, $\sigma_T$ is the standard deviation and $T_2^*(i)$ is the estimated decay time for each signal separately. Since the above mentioned parameters (i.e., $\sigma_s$, $\mu_E$, $\sigma_E$, $\mu_T$, $\sigma_T$) are actually as the result of the analysis on many signals, they are into account as the reliable parameters to evaluate the applicability of the method.

4. The Implementation of the Method for the Simulated Data. In real data since the added noise to the ideal signal is not specified, the percentage of the noise reduction cannot be surveyed, in other words, the performance cannot be tested numerically, so the signals with the identified noise level are used to specify the performance of the
method and the percentage of the noise removal to calculate the remaining noise level. The simulated data must be very close to the real data as it is quite important. As it was mentioned in the previous sections the ideal MRS signal is an exponentially decaying sinusoidal signal where the synchronous detection technique results in the elimination of the Larmor frequency or its sinus wave, consequently it will be revealed as an exponentially decaying signal merely [1,4]. Thus the ideal signal is simulated mathematically as follows:

\[ S(t) = E_o \exp(-t/T_2^*) \]

where \( S(t) \) is the general form of the ideal signal according to time \( t \), \( E_o \) is the initial amplitude and \( T_2^* \) is the decay time. The added noise to the simulated ideal signal must be simulated, too. To simulate the noise the process seems to be a little more complicated because the noise is more versatile, even type of the noise may be different due to the area of the recording of the signal. Thus all the mentioned and effective factors must be studied one by one to be input into the simulated noise. In fact, some of them are regional factors and random, independent with the Gaussian distribution. This random noise is generally present in all signals. The first source of the noise is due to the each system, although it might be trivial and negligible. The secondary noise sources are physical and geophysical conditions of the region, which may be more intense than the first group. Generally, the mentioned factors are present in most of the MRS processes [2,3].

4.1. The signal simulation applying the real noises. Actually, the recorded noises received by the instrument are used to be fully ensured of the noise simulation instead of producing artificial noises. Several sites were selected in this survey and for each site at least 10 noises have been recorded which have different time series but with similar statistical parameters. Firstly, an exponentially decaying signal with initial \( T_2^* = 100 \, \text{ms} \) and \( E_o = 50 \, \text{nv} \) is produced and then the real noises are added to it. Since there are 10 different noises so we may produce 10 different simulated signals. Then the mentioned method is implemented for the signals to calculate the statistical parameters for the estimated signals. It is impossible to add or reduce noise level to change SNR because all the regions have their specific recorded noise and we do not know the specifications of each region. But to change SNR, the added ideal signal may be changed for every recorded constant noise to produce different SNR. Then the related values are calculated for producing 10 different MRS signals and different produced SNRs. The removal of the noise is implemented in two stages here, too: The mean criterion is just implemented at the first stage, at the second stage the variance criterion is implemented as well. The results of the noise removal and the concerning parameters are hereby displayed in Table 1, the results in the table are only for the mean criterion implementation.

In the first column of the Table 1 the amplitude is equal to 50 signifying that the first simulated decaying exponential signal has the initial value equal to 50 nv. Then a noise signal has been added to the same so that the appropriate signal will be produced. In fact, 10 different noises have been added to the same signal to produce 10 different noisy signals. These signals contain the initial value (amplitude) of \( E_o = 50 \, \text{nv} \) and the decay time \( T_2^* = 100 \, \text{ms} \) which have been changed their initial amplitude and decay time due to the noise. By implementing the suggested algorithm each 10 signal has been de-noised and the mean of the initial amplitudes as well as the decay time has been again recovered. These values have been specified in the table by \( \mu_T \) and \( \mu_E \) also the values of \( \sigma_s, \sigma_E, \sigma_T \) which have been gained according to the relations 14, 16 and 18 that describing the dispersion of the estimation, which have been mentioned in Tables 1-3. The closer the value of the \( \mu_T \) to 100 and the \( \mu_E \) to the first value in the related column, meaning close to the initial ideal value, it will therefore signify the more efficiency of the algorithm.
the next columns of the table, the initial value of the ideal signal has been increased as per one constant noise, this means the SNR has increased. As it is obvious from the data in the tables, the performance of the same method has improved by increasing the SNR and the estimated values of $T_2^*$ and $E_o$ will approach to the ideal values. The first line in Tables 1, 2 and 3 define the initial value of the ideal signal, the values are respectively equal to 50 nv, 100 nv, 150 nv and 200 nv. For all the signals, the decay time is identical and equal to $T_2^* = 100$ ms.

**Table 1.** $\mu_E, \sigma_E, \mu_T, \sigma_T, \sigma_S$, per different SNRs and the mean criterion application

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Amplitude (nv)</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_E$</td>
<td></td>
<td>40.23</td>
<td>89.35</td>
<td>138.4</td>
<td>187.4</td>
</tr>
<tr>
<td>$\sigma_E$</td>
<td></td>
<td>20.43</td>
<td>20.34</td>
<td>20.26</td>
<td>20.22</td>
</tr>
<tr>
<td>$\mu_T$</td>
<td></td>
<td>150.4</td>
<td>139.0</td>
<td>132.4</td>
<td>120.0</td>
</tr>
<tr>
<td>$\sigma_T$</td>
<td></td>
<td>74.49</td>
<td>60.38</td>
<td>55.94</td>
<td>36.16</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td></td>
<td>9.51</td>
<td>7.58</td>
<td>7.35</td>
<td>7.45</td>
</tr>
</tbody>
</table>

The variance criterion is applied following the implementation of the mean criterion too, and the results are recorded in Table 2.

**Table 2.** $\mu_E, \sigma_E, \mu_T, \sigma_T, \sigma_S$, per different SNRs applying the mean and variance criteria

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Amplitude (nv)</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_E$</td>
<td></td>
<td>46.51</td>
<td>94.57</td>
<td>149.8</td>
<td>199.6</td>
</tr>
<tr>
<td>$\sigma_E$</td>
<td></td>
<td>8.53</td>
<td>10.28</td>
<td>5.18</td>
<td>5.19</td>
</tr>
<tr>
<td>$\mu_T$</td>
<td></td>
<td>139.4</td>
<td>131.2</td>
<td>103.6</td>
<td>102.7</td>
</tr>
<tr>
<td>$\sigma_T$</td>
<td></td>
<td>65.55</td>
<td>53.35</td>
<td>7.72</td>
<td>5.68</td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td></td>
<td>3.55</td>
<td>4.66</td>
<td>2.89</td>
<td>2.88</td>
</tr>
</tbody>
</table>

It should be noted the estimated parameters in Table 2 are significantly improved compared with Table 1. The available values in Table 2 are comparable with the ones in Table 1, as the same are exactly similar to the ones from Table 1 whereby the second stage of the algorithm has been implemented upon them. In fact, all the conditions are identical for Tables 1 and 2, but Table 2 concludes the implementation of both stages of the proposed method. It is seen for initial amplitude 150 nv, $\mu_E$ in Table 1, is 138.4 nv and in Table 2, it is equal to 149.8 nv and the ideal value of the same estimation is equal to the first value of the related column being 150 nv, as it is hereby observed the absolute value of the estimation error following the implementation of the first stage of the algorithm to be based on the mean then it defines according to Equation (21) as follows:

$$\left| \frac{138.4 - 150}{150} \right| = 0.0773,$$

and the estimation error following the implementation of the second stage of the algorithm to be the de-noising based on the variance then it defines according to Equation (21) as follows:

$$\left| \frac{149.8 - 150}{150} \right| = 0.0013.$$

Comparing the error rate it is evident that the implementation of the second stage will largely reduce the estimation errors (see Table 4).
4.2. **The proposed method implementation on real data.** Now, it is impossible to calculate the mentioned parameters of the previous sections because the value of the ideal signal is not known beforehand, but a safety margin may be used for the estimated parameters according to the recorded regional noise rate. The field data were recorded by the NUMIS equipment and the proposed method has been applied for these data and the estimated ideal signal is shown before and after the variance application and the noise removal for the MRS data.

![Figure 2](image-url)

**Figure 2.** (a) The real MRS signal and the de-noised signal after applying the mean and variance criteria; (b) the noise removed from the signal

The same data shown with the connected lines in Figure 2(a) are actually the realistic MRS data to have been collected by the NUMIS equipment. As seen the same are intensely noisy, consequently the decaying exponential form is not observable in them. In fact, the level of the signal is so low and even the small noises can affect them. In the outset of the same signal which will have to contain the most initial amplitude, therefore, the amplitude is about 33 nV up to 82 nV. As mentioned the de-noising procedure of the signal will include two stages based on the proposed method. In the first stage by applying the mean criterion it will be proceeded the de-noising and in the second stage by
applying the variance criterion, then the initial amplitude will be corrected as well. The light-colored, dotted line signal is related to the first stage, it shows the initial amplitude without correction. As it is seen the amount of the initial amplitude has been estimated about 33 nv, but the present noise over the signal has been removed as much as possible and the decaying exponential form is easily observable. The dark-colored, bold dotted line signal is in fact the final signal following the implementation of the variance criterion. As it is seen the estimation value of the initial amplitude has been modified in this stage, consequently it has been estimated at approximately 60 nv.

5. **Comparison the Proposed Method with the Hanning Frequency Method.** It is observed that, the frequency methods have some failures, the main failures are the reduced initial amplitude and the increased decay time due to the noise removal (Table 3). Meanwhile, the time-based noise removal changes the decay time and initial amplitude but in this method the ratio of the changes is much less than the frequency de-noising methods (Tables 1 and 2). So, the MRS signal will be de-noised by use of the Hanning frequency filter (a low-pass filter) and the results will hereby be recorded. Firstly, the concerning MRS signal must be transformed into the frequency domain to be de-noised by the Hanning filter. The Discrete Fourier Transform (DFT) is used to transform the signal to the frequency domain, so the MRS signal is seen as a frequency spectrum which its most frequency spectrum density is just close to zero [1]. The concerning noise frequencies must be removed and the main signal frequencies must be preserved after the signal transforming to the frequency domain, thus the frequency content of the ideal signal must be specified to identify the other disturbing frequencies. 10 Hz cutoff frequency is usually used for the MRS signal as the main frequency and the rest are deemed to be the noise frequencies. Of course, this is not correct absolutely because a low percentage of the ideal signal frequencies appear in more than 10 Hz too or vice versa but the concerned values are in fact very trivial and negligible.

Accordingly, the de-noising process does not fully eliminate the noise. Meanwhile, a small proportion of the main signal data will be consequently omitted [1,5,7]. There are various kinds of de-noising methods in the frequency domain such as the Hanning and Legchenko methods. Here we use the Hanning’s window that has a length named \( L \) and it is defined as follows:

\[
 w(n) = 0.5 \left( 1 - \cos \left( \frac{2\pi n}{N} \right) \right) \quad 0 \leq n \leq N, \tag{20}
\]

therefore, the length of the window is \( L = N + 1 \) [19]. With the point to point multiplication of the defined window and the frequency domain signal, the de-noising process and the elimination of the disturbing frequency will be implemented. Then the remaining frequency spectrum is transformed to the time domain to reconstruct the signal using the Inverse Discrete Fourier Transform (IDFT). As mentioned for the time domain 10 different signals with different noises are simulated to study the concerning parameters statistically, then these 10 signals are de-noised according to the above mentioned method, concerning parameters are calculated according to Equations (14) to (18). Consequently, the test is applied and the obtained results are recorded and classified in Table 3.

Then, the relative initial amplitude error as well as the relative decay time error are calculated as follows:

\[
 e_{IT} = \frac{p_{est} - p_{true}}{p_{true}}, \tag{21}
\]

where \( p_{est} \) is the estimated value of the parameter and \( p_{true} \) is the true value of this parameter. The relative error values of Tables 1-3 are calculated for \( \mu_E \) and \( \mu_T \) according to Equation (21), finally, the same are summarized in Table 4 for better comparison.
Table 3. \( \mu_E, \sigma_E, \mu_T, \sigma_T, \sigma_S \), per different SNRs based on the Hanning filter

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Amplitude (nv)</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_E )</td>
<td></td>
<td>26.51</td>
<td>51.64</td>
<td>76.77</td>
<td>101.91</td>
</tr>
<tr>
<td>( \sigma_E )</td>
<td></td>
<td>2.96</td>
<td>2.96</td>
<td>2.96</td>
<td>2.96</td>
</tr>
<tr>
<td>( \mu_T )</td>
<td></td>
<td>618.4</td>
<td>602.5</td>
<td>600.7</td>
<td>600.5</td>
</tr>
<tr>
<td>( \sigma_T )</td>
<td></td>
<td>161.2</td>
<td>80.21</td>
<td>53.98</td>
<td>40.77</td>
</tr>
<tr>
<td>( \sigma_S )</td>
<td></td>
<td>2.55</td>
<td>2.55</td>
<td>2.55</td>
<td>2.55</td>
</tr>
</tbody>
</table>

According to the table, \( e_{E-m} \) and \( e_{T-m} \), \( e_{E-v} \) and \( e_{T-v} \) and \( e_{E-h} \) and \( e_{T-h} \) refers to the estimated relative error of the initial amplitude and the estimated relative error of the decay time after applying the merely mean criterion, the mean and variance criteria together and the Hanning frequency method respectively.

Table 4. The relative error comparison of the estimated initial amplitude, decay time, for the mentioned methods

<table>
<thead>
<tr>
<th>Relative error</th>
<th>Amplitude (nv)</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_{E-m} )</td>
<td></td>
<td>0.19</td>
<td>0.10</td>
<td>0.07</td>
<td>0.062</td>
</tr>
<tr>
<td>( e_{T-m} )</td>
<td></td>
<td>0.50</td>
<td>0.39</td>
<td>0.32</td>
<td>0.20</td>
</tr>
<tr>
<td>( e_{E-v} )</td>
<td></td>
<td>0.06</td>
<td>0.05</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>( e_{T-v} )</td>
<td></td>
<td>0.59</td>
<td>0.31</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>( e_{E-h} )</td>
<td></td>
<td>0.46</td>
<td>0.48</td>
<td>0.48</td>
<td>0.49</td>
</tr>
<tr>
<td>( e_{T-h} )</td>
<td></td>
<td>5.18</td>
<td>5.02</td>
<td>5.00</td>
<td>5.00</td>
</tr>
</tbody>
</table>

It is evident that the time-based noise removal errors of the initial amplitude as well as the decay time are much lower than the de-noised signal errors of the Hanning frequency method. By considering the results (Table 4) it is concluded that the proposed time-based method is more precise and correct than that of the Hanning frequency method and possibly the other concerning frequency methods.

6. Conclusions. In this paper, an innovative method has been proposed for the noise removal from the MRS signals at the time domain. We assert to demonstrate that the mentioned method is the first MRS signal de-noising at the time domain and no other method has ever been developed for the MRS signals at the time domain. Generally, the time domain methods are mostly limited due to the necessity of the previous information on the signal relations. Here the form and the relation of the MRS signal are exponentially decaying thus the limitation is eliminated and the time domain methods will be easily utilized. Another advantage of the proposed method is its adaptability, consequently it may adapt itself with different noise features. Whereas, the system directly records the field noise prior to the signal record thus the de-noising of every signal is based on the previously recorded noise features. In fact, the de-noising parameters are taken from the noise and they adapt themselves to the received noise. For comparison as shown before, the initial amplitude and decay time are intensely affected after the de-noising by the Hanning frequency method and their values are beyond the real values, but these errors are actually much lower in the mentioned time-based method.

Acknowledgments. The authors are grateful to Prof. M. M. Petke, Prof. F. M. Rubio and Prof. J. L. Plata (Spain) who directed us with no hesitation, they are hereby very
much appreciated for their sincere cooperation, especially Prof. J. F. Girard (France) who supported us continually during the research process, so he is appreciated for the dispatch of the effective field data and the required papers.

REFERENCES